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What Does the Yield Curve Tell Us About the Federal Reserve's Implicit Inflation Target?

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Abstract: This paper uses a dynamic stochastic general equilibrium (DSGE) model to explore the additional information that can be extracted from the yield curve about the Federal Reserve's implicit inflation target. In the model, monetary policy follows a nominal interest rate rule with a drifting inflation target and agents have imperfect information about the persistent component of inflation target. When the yield curve information is included, the DSGE model generates inflation expectations that are highly correlated with survey data evidence. In the DSGE model, agents learn quickly inflation target and the gap between the perceived target and the actual target is quantitatively small. This is in contrast to some of existing studies that suggest a persistent role of imperfect information even as long-run inflation expectations has declined and stabilized at a low level since the mid 1980s.

Keywords: Inflation target, DSGE model, term structure of interest rates

JEL classification: C32, E43, G12

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1 Introduction

There are noticeable low-frequency movements in the U.S inflation data. For instance, the inflation rate as measured by the GDP deflator in Figure 1 shows an upward trend during the 1960s and 1970s. This upward trend is reversed after the Volcker disinflation period of the early 1980s. To fit this persistent inflation process, estimated dynamic stochastic general equilibrium (DSGE) models often model the central bank's inflation target as a nonstationary process (e.g., Smets and Wouters (2005), Ireland (2007)).¹ The permanent shifts in the inflation target induce a common trend for nominal interest rates. Unit root tests of inflation and interest rates in Table 1 provide evidence for such a specification. Under this specification, the entire yield curve, not just the short rate, reflects the movement of inflation target, because long-horizon inflation expectations affect long-term rates. Accordingly, using the entire term structure of interest rates can provide additional information in estimating inflation target, which is not directly observed but is a key determinant of long-horizon inflation expectations.

In this paper, I estimate a small-scale New Keynesian DSGE model using yield curve data on top of macro data. In the model, monetary policy follows a nominal interest rate rule with a drifting inflation target. The main focus of this paper is to use the estimated DSGE model to find out the information content of the yield curve about the time-varying inflation target of the central bank. The model features i) imperfect information of private agents about inflation target and ii) time-varying volatility in the macro shocks.²

There are two main findings from this study. First, the estimated target from the DSGE model indeed captures the common trend for nominal interest rates and

¹Of course, the Federal Reserve's inflation target is only implicitly defined because the Federal Reserve has not adopted explicit inflation targeting.

²See Beechey (2004), Dewachter and Lyrio (2008), and Dewachter (2008), Erceg and Levin (2003), and Schorfheide (2005) on the implications of imperfect information for expected inflation. On the other hand, Justiniano and Primiceri (2008) and Liu, Waggoner, and Zha (2007) emphasize the role of the time-varying shock volatility as opposed to changes in monetary policy in explaining macroeconomic fluctuations in the U.S.

inflation. There are two pieces of evidence for this finding. First of all, unit root tests for in-sample nominal interest rates and inflation data detrended by the estimated target reject the existence of unit roots at 5% level. In addition, the estimated target is highly correlated with long-horizon inflation expectations based on survey data that are not used in the estimation. Second, the model estimates imply that agents learn the inflation target of the central bank quickly relative to what is implied by existing studies (e.g., Erceg and Levin (2003), Kozicki and Tinsley (2005)).

This paper is related to the literature which links term structure data with changes in monetary policy.³ Papers most closely related to this work are Dewachter and Lyrio (2008) and Dewachter (2008), who study the role of changing beliefs about the inflation target in small scale New Keynesian models estimated using macro and yield curve data. Dewachter and Lyrio (2008) assume that the actual target is constant or chairman-specific but the perceived target by private agents drifts like a random walk. Their setup implies that the difference between the actual target and the perceived target can be nonstationary, preventing private agents from ever learning the actual target. On the other hand, Dewachter (2008) assumes that the actual target itself drifts, like this paper, but allows for nonstationary real interest rates. However, the unit root test results for interest rates in Table 1 provide little evidence for nonstationary real rates.

Other related papers are Kozicki and Tinsley (2001, 2005) and Cogley (2005) who try to identify shifts in monetary policy using term structure data in reduced-form models. Kozicki and Tinsley (2001) argue that incorporating term structure information into long-horizon inflation expectations reduces the variation of the term premium and supports a more substantial role of short rate expectations in explaining term structure data. Cogley (2005) conveys the same message by using a VAR with drifting coefficients and volatilities for the short rate and a measure of term spread. While this paper also finds a significant time variation of long-horizon inflation expectations, it ties down the law of motion for macro variables and the

³A non-exhaustive list of papers on this topic includes Ang, Dong, and Piazzesi (2007), Bekaert et. al. (2010), Cogley (2005), Dewachter (2008), Dewachter and Lyrio (2008), Kozicki and Tinsley (2001, 2005), Rudebusch and Wu (2008).

learning speed of agents by DSGE restrictions.

The remainder of this paper is organized as follows. Section 2 presents the macro DSGE model and discusses its log-linear approximation and equilibrium nominal bond yields based on the log-linearized model. Section 3 discusses the empirical analysis and Section 4 concludes.

2 Model

The model economy is a standard new Keynesian monetary DSGE model with optimizing households and monopolistically competitive firms that face price stickiness as in Woodford (2003).

2.1 Firms and Production Sector

I assume a continuum of monopolistically competitive firms in the intermediate product markets. Firms in the competitive final-goods market combine the intermediate goods ($Y_t(i)$) into a composite good (Y_t) according to the following technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varpi_t-1}{\varpi_t}} di \right]^{\frac{\varpi_t}{\varpi_t-1}}, \quad \varpi_t > 1. \quad (1)$$

ϖ is the elasticity of substitution among different intermediate goods. The demand for each intermediate good and the expression for the aggregate price index are obtained as follows.

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\varpi_t} Y_t, \quad P_t = \left[\int_0^1 P_t(i)^{1-\varpi_t} di \right]^{\frac{1}{1-\varpi_t}}. \quad (2)$$

$P_t(i)$ is the price of intermediate good i . All firms in the intermediate product markets have production technologies that are linear in labor ($N_t(i)$), which they hire on a competitive market.

$$Y_t(i) = A_t N_t(i). \quad (3)$$

Total factor productivity A_t contains a stochastic trend and the growth rate of A_t follows a stable AR(1) process with time-varying volatility.⁴

$$\begin{aligned} \ln A_t &= \ln A_{t-1} + u_{a,t} , \quad u_{a,t} = (1 - \rho_a)u_a^* + \rho_a u_{a,t-1} + \epsilon_{a,t} , \quad \epsilon_{a,t} \sim iid\mathcal{N}(0, \sigma_{a,t-1}^2) \\ \sigma_{a,t}^2 &= (1 - \nu_a)\sigma_a^2 + \nu_a\sigma_{a,t-1}^2 + \sigma_{w,a}w_{a,t} , \quad w_{a,t} \sim iid\mathcal{N}(0, 1). \end{aligned} \quad (4)$$

The model features a nominal rigidity in the style of Calvo (1983). Each period only $(1 - \theta)$ fraction of the firms can reoptimize their prices while the other firms adjust their prices by the previous period's inflation rate.⁵ The optimal price, $P_t^o(i)$, is determined by maximizing the following sum of discounted future expected profits:

$$E_t \left[\sum_{s=0}^{\infty} \theta^s M_{t,t+s} \left(P_t^o(i) \frac{P_{t+s-1}}{P_{t-1}} Y_{t+s}(i) - W_{t+s} N_{t+s}(i) \right) \right]. \quad (5)$$

where W_{t+s} is the nominal wage, and $M_{t,t+s}$ is a stochastic discount factor that firms use to evaluate their future profit streams. In equilibrium, the stochastic discount factor is identical to the one derived from household optimization problem.

If prices were flexible, the profit maximization of firms in monopolistically competitive markets would make the price markup equal to $f_t = \frac{\bar{\omega}_t}{\bar{\omega}_t - 1}$. This markup determines the equilibrium output level known as the natural rate of output. I assume an AR(1) process with time-varying volatility for the log markup.

$$\begin{aligned} \ln f_t &= (1 - \rho_f)\ln f^* + \rho_f \ln f_{t-1} + \epsilon_{f,t} , \quad \epsilon_{f,t} \sim iid\mathcal{N}(0, \sigma_{f,t-1}^2) \\ \sigma_{f,t}^2 &= (1 - \nu_f)\sigma_f^2 + \nu_f\sigma_{f,t-1}^2 + \sigma_{w,f}w_{f,t} , \quad w_{f,t} \sim iid\mathcal{N}(0, 1). \end{aligned} \quad (6)$$

⁴This specification for time-varying volatility does not guarantee the nonnegativity of the variance. Nonetheless, when the standard deviation of innovation to the volatility ($\sigma_{w,a}$) is small relative to σ_a^2 and ν_a , the chance of hitting the zero bound is practically negligible (less than 5%), which is indeed the case in my estimates. Alternatively we can assume an AR(1) process for the log of the variance, which guarantees the nonnegativity. However, this assumption makes equilibrium bond yields complicated nonlinear functions of time-varying volatility. The Gaussian specification for the variance ensures that equilibrium bond yields are linear with respect to the time-varying volatility.

⁵Christiano, Eichenbaum, and Evans (2005) use the same indexation rule by the lagged inflation. I assume a full indexation scheme to make sure that firms which do not optimize their prices can still catch up with trend inflation.

2.2 Household Optimization

The economy is populated by a continuum of representative households who maximize their expected discounted lifetime utility with respect to consumption of the final good C_t , hours worked H_t , and real money balance $\frac{M_t}{P_t}$:

$$E_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s})^{1-\sigma} - 1}{1-\sigma} - \frac{H_{t+s}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \chi_m \log\left(\frac{M_{t+s}}{A_{t+s}P_{t+s}}\right) \right) \right]. \quad (7)$$

where σ is the constant relative risk aversion, and ν the short-run (Frisch) labor supply elasticity. Consumption is deflated by the current technology level as in Schorfheide (2005). This assumption makes the marginal benefit of working more hours bounded despite a growing real wage and ensures a balanced growth path.⁶

Assuming asset markets are complete, the household is subject to the following period-by-period budget constraint:

$$P_t C_t + \sum_{n=1}^{\infty} P_{n,t} (B_{n,t} - B_{n+1,t-1}) + M_t + T_t = W_t H_t + B_{1,t-1} + M_{t-1} + Q_t + \Pi_t \quad (8)$$

where P_t is the price level, $P_{n,t}$ the price of an n quarter bond, $B_{n,t}$ bond holding, T_t lump-sum tax or subsidy, Q_t the net cash flow from participating in state-contingent security markets, and Π_t the aggregate profit.

The nominal stochastic discount factor between period t and $t+s$ implied by households' optimal behavior is as follows:

$$M_{t,t+s} = \beta^s \frac{U_C(C_{t+s}, H_{t+s}, \frac{M_{t+s}}{P_{t+s}}) P_t}{U_C(C_t, H_t, \frac{M_t}{P_t}) P_{t+s}} = \beta^s \left(\frac{C_{t+s}/A_{t+s}}{C_t/A_t} \right)^{-\sigma} \frac{A_t P_t}{A_{t+s} P_{t+s}}. \quad (9)$$

The government does not make any independent expenditure and its budget constraint is simply $\sum_{n=1}^{\infty} P_{n,t} (B_{n,t} - B_{n+1,t-1}) + M_t - M_{t-1} + T_t = B_{1,t-1}$. Therefore, the market clearing implies that the aggregate consumption will be equal to the aggregate output (i.e. $C_t = Y_t$).

⁶If σ is equal to 1 (the log utility case), this detrending is irrelevant because the marginal utility of consumption grows at the same rate as real wage, making the marginal benefit of working more hours stationary.

2.3 Monetary Policy and Inflation Target

The central bank adjusts the nominal interest rate according to a forward-looking Taylor rule with policy inertia. The nominal target interest rate (i_t^*) reacts to expected inflation and the output gap in the following way:

$$\begin{aligned} (1 + i_t) &= (1 + i_t^*)^{1-\rho_i} (1 + i_{t-1})^{\rho_i} \exp\{\epsilon_{i,t}\}, \quad \epsilon_{i,t} \sim iid\mathcal{N}(0, \sigma_{i,t-1}^2) \\ 1 + i_t^* &= ((1 + r^*)(\pi_t^*)) \left(\frac{E_t(\pi_{t+1})}{\pi_t^*}\right)^{\gamma_p} \left(\frac{Y_t}{Y_t^n}\right)^{\gamma_y} \\ \sigma_{i,t}^2 &= (1 - \nu_i)\sigma_i^2 + \nu_i\sigma_{i,t-1}^2 + \sigma_{w,i}w_{i,t}, \quad w_{i,t} \sim iid\mathcal{N}(0, 1) \end{aligned} \quad (10)$$

where r^* is the steady state real interest rate, which is equal to $\frac{e^{u_a^*}}{\beta} - 1$, π_t^* the time-varying inflation target of the central bank, and Y_t^n a natural rate of output, which will prevail in a flexible price economy.

In the model, agents observe the current inflation target but do not distinguish the permanent component ($\pi_t^{P,*}$) from the transitory noise component ($\pi_t^{T,*}$) as in Erceg and Levin (2003). They face a signal extraction problem when forming expectations about the future inflation target.

$$\begin{aligned} \ln \pi_t^* &= \ln \pi_t^{P,*} + \ln \pi_t^{T,*}, \quad \ln \pi_t^{T,*} \sim iid\mathcal{N}(0, \sigma_n^2) \\ \ln \pi_t^{P,*} &= \ln \pi_{t-1}^{P,*} + \epsilon_{\pi^*,t}, \quad \epsilon_{\pi^*,t} \sim iid\mathcal{N}(0, \sigma_{\pi^*,t-1}^2) \\ \sigma_{\pi^*,t}^2 &= (1 - \nu_{\pi^*})\sigma_{\pi^*}^2 + \nu_{\pi^*}\sigma_{\pi^*,t-1}^2 + \sigma_{w,\pi^*}w_{\pi^*,t}, \quad w_{\pi^*,t} \sim iid\mathcal{N}(0, 1). \end{aligned} \quad (11)$$

Under the above assumptions, agents filter out the transitory component by Kalman filtering in order to forecast the future inflation target as follows.

$$\begin{aligned} E_t(\ln \pi_{t+j}^*) &= \ln \pi_t^* + \zeta_t \quad (\forall j \geq 1) \\ E_t(\ln \pi_{t+1}^*) &= E_{t-1}(\ln \pi_t^*) + \frac{\Omega_t}{\Omega_t + \sigma_n^2} (\ln \pi_t^* - E_{t-1}(\ln \pi_t^*)) \\ \zeta_t &= \frac{\sigma_n^2}{\Omega_t + \sigma_n^2} (\zeta_{t-1} - [\ln \pi_t^* - \ln \pi_{t-1}^*]) \\ \Omega_t &= V_t(E_t(\ln \pi_{t+1}^*) - \ln \pi_t^{P,*}), \quad \Omega_{t+1} = \Omega_t + \sigma_{\pi^*,t}^2 - \frac{\Omega_t^2}{\Omega_t + \sigma_n^2} \end{aligned} \quad (12)$$

where Ω_t is the covariance matrix of the filtered estimate. ζ_t is the gap between the expected future inflation target and the current inflation target, which measures the degree of imperfect credibility of the central bank due to the imperfect information of agents.

$$\zeta_t = \frac{\sigma_n^2}{\Omega_t + \sigma_n^2} (\zeta_{t-1} - (\ln \pi_t^* - \ln \pi_{t-1}^*)) \implies \zeta_t = \sum_{j=0}^{\infty} w_{\zeta,j,t} (\ln \pi_{t-j}^* - \ln \pi_{t-j-1}^*). \quad (13)$$

The evolution of the degree of imperfect credibility is tightly linked with the signal to noise ratio ($\frac{\sigma_{\pi^*,t}^2}{\sigma_n^2}$). When the ratio goes to ∞ , $\frac{\Omega_t}{\sigma_n^2}$ also approaches ∞ and the target is fully credible, implying ζ_t is close to 0. On the contrary, if the signal to noise ratio is sufficiently small, the gap between expected and current target behaves like a nonstationary variable. Erceg and Levin (2003) assume that agents use the steady state Kalman gain, which implies the constant covariance matrix of the filtered estimate. In contrast, I consider a time-varying gain. This time-varying gain is more suitable for the environment with time-varying volatility.

2.4 Log-linear Approximation of the Full Model

I log-linearize the system of equations around the deterministic steady state to derive the macro dynamics implied by the model. To induce stationarity, output is detrended by technological level while the inflation rate and the nominal interest rate are detrended by the inflation target. Steady state values for the detrended variables are defined by setting all the exogenous shocks at their unconditional means forever.⁷ The percentage deviation of a detrended variable d_t from the steady state is denoted by \tilde{d}_t .

I define the vector of relevant detrended state variables $x_{1,t}^f$ by $[\tilde{y}_t, \tilde{\pi}_t, \tilde{i}_t, \tilde{y}_{n,t}, u_{a,t}, u_{f,t}, E_t(\tilde{y}_{t+1}), E_t(\tilde{\pi}_{t+1}) + \zeta_t]$. Equilibrium conditions from the log-linearized system result in the following system of equations.⁸

$$\Gamma_0 x_{1,t}^f = \Gamma_1 x_{1,t-1}^f + \Phi \epsilon_t + \Pi \eta_t \quad \epsilon_t = [\epsilon_{a,t}, \epsilon_{f,t}, \epsilon_{i,t}, w_t]' \quad (14)$$

$$w_t = \ln \pi_t^* - \ln \pi_{t-1}^*, \quad \eta_t = [\tilde{y}_t - E_{t-1}(\tilde{y}_t), \tilde{\pi}_t - E_t(\tilde{\pi}_{t+1}) - \zeta_t] \quad (15)$$

The following representation of the dynamics of state variables as the solution of the above linear rational expectations system can be obtained by using a numerical routine explained in Sims (2002):

$$x_{1,t}^f = T_1 x_{1,t-1}^f + T_\epsilon \epsilon_t + \Psi_x \sum_{j=1}^{\infty} \Psi_f^{j-1} \Psi_\epsilon E_t(\epsilon_{t+j}). \quad (16)$$

⁷Without uncertainty, firms do not need to reoptimize their prices. So in the steady state, the actual output would be equal to the natural rate of output.

⁸The details of the solutions procedure can be found in the appendix available upon request.

Now, define a new set of state variables $x_{1,t}$ by substituting $E_t(\tilde{\pi}_{t+1})$ for $E_t(\tilde{\pi}_{t+1}) + \zeta_t$ in $x_{1,t}^f$. Notice $E_t(w_{t+1}) = \zeta_t$ and use the law of motion for ζ_t in equation (13). Finally, we obtain the following solution of the log-linear model in terms of $x_{1,t}$.

$$x_{1,t} = T_{c,t} + T_1 x_{1,t-1} + T_{\epsilon,t} \epsilon_t, \quad \epsilon_t = [\epsilon_{a,t}, \epsilon_{f,t}, \epsilon_{i,t}, w_t]'. \quad (17)$$

2.5 No-arbitrage Term Structure Model

Following Jermann (1998) and Wu (2006), I combine the log-linear approximation to the DSGE model, with asset pricing methods based on the log-normality of the stochastic discount factor. The log stochastic discount factor implied by the macro model is linear with respect to the detrended macro variables. By taking the log of both sides in equation (9), we obtain the following expression:

$$m_{t,t+1} = \ln M_{t,t+1} = \ln \beta - \sigma(y_{t+1} - y_t) - u_{a,t+1} - \pi_{t+1} = E_t(m_{t,t+1}) + \Lambda_t \epsilon_{t+1} \quad (18)$$

where Λ_t denotes a vector of market prices of risk, which is entirely restricted by the structural parameters. Learning about inflation target introduces time variation in the market prices of risk through the time-varying gain Ω_t .⁹

From households' optimal asset allocation, the risk-adjusted return on bonds of different maturities must be equal to 1.

$$1 = E_t(e^{m_{t,t+1} + p_{n-1,t+1} - p_{n,t}}). \quad (19)$$

Here $p_{n,t}$ is the log price of the the constant maturity n quarter bond. I define the vector of nonstationary variables by $x_{2,t} = [\ln A_t, \ln \pi_t^*]$. The normality of innovations implies the following equations for bond prices:

$$\begin{aligned} p_{1,t} &= E_t(m_{t,t+1}) + \frac{V_t(m_{t,t+1})}{2} = -i_t \\ p_{n,t} &= E_t(m_{t,t+1} + p_{n-1,t+1}) + \frac{V_t(m_{t,t+1} + p_{n-1,t+1})}{2} \\ &= p_{1,t} + E_t(p_{n-1,t+1}) + Cov_t(m_{t,t+1}, p_{n-1,t+1}) + \frac{V_t(p_{n-1,t+1})}{2} \quad (n \geq 2) \end{aligned} \quad (20)$$

⁹For this reason, there is still a tight link between the level of volatility and market prices of risk in the learning version unlike the essentially affine term structure models in Duffee (2002).

Based on the above equations, I can recursively define log bond prices as functions of $[x_{1,t}, x_{2,t}, \sigma_{t,l}^2]$ in which $\sigma_{t,l}^2$ is $[\sigma_{a,t}^2, \sigma_{f,t}^2, \sigma_{i,t}^2, \sigma_{\pi^*,t}^2 + \sigma_n^2 + \Omega_t]'$.¹⁰

$$\begin{aligned} p_{n,t} &= a_{n,t} + b_{n,1}x_{1,t} + b_{n,2}x_{2,t} + c_{n,t}\sigma_{t,l}^2 \\ y_{n,t} &= -\frac{a_{n,t}}{n} - \frac{b'_{n,1}}{n}x_{1,t} - \frac{b'_{n,2}}{n}x_{2,t} - \frac{c_{n,t}}{n}\sigma_{t,l}^2. \end{aligned} \quad (21)$$

3 Empirical Analysis

3.1 Estimation Methodology and Data

I use Bayesian estimation methods which combine prior information on the model parameters with the likelihood generated by sample data. On parameters for which existing literature provides some guidance, I take informative priors. For others, I take fairly diffuse priors. One popular method in the Bayesian estimation of a stochastic volatility model is to use a multiple block Metropolis-Hastings algorithm, which iteratively draws volatilities and parameters conditional on each other.¹¹ Instead of integrating out stochastic volatilities in the likelihood evaluation, I obtain the joint posterior draws of parameters and stochastic volatilities for empirical analysis.

I apply the econometric methods outlined in the previous section to U.S. macro and treasury bond data. The macro variables are taken from the Federal Reserve Database (FRED) at Saint Louis. The measure of output is per-capita real GDP, which is obtained by dividing real GDP (GDPC1) by total population (POP). For the inflation rate, the log difference of the GDP price index (GDPCTPI) is used. The nominal interest rate is from the Fama CRSP risk free rate file. I select the average quote of 3-month treasury bill rate. Five bond yields (1, 2, 3, 4, 5 year) are

¹⁰The derivation involves some approximation. The appendix provides the details of the derivation of log bond prices.

¹¹This method has been used for Bayesian estimation of time-varying VARs in Cogley (2005) and Benati (2007). Justiniano and Primiceri (2008) apply a similar method to estimate a large scale DSGE model with time-varying volatility. The details of the estimation procedure are available from the author upon request.

from Fama CRSP zero coupon bond yields files. The estimation uses observations from 1960:Q1 to 2004:Q4.¹²

Table 1 presents the sample moments of the nominal variables in my dataset. Inflation and bond yields are highly persistent but the term spread between the long rate and the short rate is not as persistent as bond yields. Actually, we can reject a unit root for the term spread but not for bond yields. Also, we can reject a unit root for the real short rate but not for inflation. These statistics are consistent with our assumption of a nonstationary inflation target as a common trend for both inflation and nominal interest rates.

3.2 Prior and Posterior Distribution of Parameters

Posterior means and 90% probability intervals for all the parameters with the corresponding 90% prior probability intervals are reported in Table 2.¹³ There are some parameters whose posterior distributions differ much from prior counterparts. For example, the estimated volatility of noise is much smaller relative to the estimated volatility of signal, although the two volatility parameters have the same prior distribution. The average signal to noise ratio ($\frac{\sigma_{\pi^*}}{\sigma_{n,\pi^*}}$) at the posterior mean is 3.55. If I set the signal to noise ratio in order to match the constant gain in Kalman filtering used in Erceg and Levin (2003)'s calibration, it is roughly 0.14, which is much lower than the estimate based on posterior draws. Erceg and Levin (2003) obtain this number by minimizing the distance between the model implied expected inflation and survey evidence over the period 1980:Q4 to 1985:Q4. This low signal to noise ratio makes inflation much more volatile than observed over the period after the mid 1980s.¹⁴

¹²To match the frequency of bond yields with that of the macro data, the monthly observations of the treasury bill rate and bond yields are transformed into quarterly data by averaging the three monthly observations per quarter.

¹³In computing prior intervals, I throw away draws, which imply the indeterminacy of solutions. Because I impose a huge penalty for a draw implying indeterminacy when I evaluate likelihood in running MCMC chains, all the posterior draws belong to the determinacy region by construction.

¹⁴The DSGE model in this paper allows the time variation of only the volatility of the signal. In principle, this restriction on the variation of the signal to noise ratio can create a poor fit for

In addition, reoptimizing of prices is found to be more frequent in the posterior distribution than the prior one. A more frequent reoptimizing means a lower degree of price rigidity. This finding is consistent with Dewachter and Lyrio (2008) who find the estimated slope of Phillips curve is much steeper when term structure data is included. The parameter is important in evaluating implications of a disinflation experiment. When the nominal rigidity is low, the decline of inflation will lead to a less severe output loss because prices rather than quantities adjust rapidly.

Table 3 shows prior and posterior means of standard deviations of measurement errors for interest rates. The posterior mean of the standard deviation of the measurement error of the short rate is 40 - 50 basis points but the posterior means of standard deviations of other interest rates are much lower, ranging from 6 to 12 basis points.¹⁵ Those values are much lower than comparable estimates in Ang, Dong, and Piazzesi (2007) who estimate different Taylor rules with term structure data by using a no-arbitrage affine term structure model without DSGE restrictions. The comparison suggests that the DSGE model achieve a reasonable degree of the in-sample fit.

3.3 Learning, Inflation Target and Expected Inflation

The DSGE model in this paper assumes that the nonstationary inflation target creates a common trend for inflation and nominal interest rates. Therefore, we can test the plausibility of the estimated inflation target using various implications of this assumption. First, interest rates and inflation rate detrended by inflation target must be stationary. I perform unit root tests for the detrended short rate and the detrended inflation rate using the estimated inflation target from the DSGE model. Test statistics suggest that I can reject the existence of a unit root at 5% level in inflation after the mid 1980s if the volatility of the noise indeed changed. However, Stock and Watson (2007) show that the volatility of the transitory component of inflation measured by GDP deflator did not change much over the period 1953:2004 while its permanent component changed a lot, supporting my assumption.

¹⁵We have to multiply 4 to the numbers reported in the Table 3 in order to compute the annualized percentage.

both cases. This finding implies that the estimated inflation target can be a reliable measure of the trend component of inflation and nominal interest rates.¹⁶

Second, if the estimated inflation target correctly captures the trend component of inflation, the time variation of the target should be in line with the time variation of long-horizon inflation expectations. Here, I use a measure of long-horizon inflation expectations constructed by Clark and Nakata (2008). This measure of long-horizon inflation expectations splices 10-year-ahead expectations from the Survey of Professional Forecasters (1990-2007), 5- to 10-year-ahead expectations of the survey of financial market participants (1981-89), and econometric estimates using term structure data (1960-80).¹⁷ Correlation coefficients between model implied inflation expectations and survey data in Table 4 confirm that the model generates inflation expectations highly correlated with survey data. Moreover, correlation becomes stronger when the yield curve data are used in the estimation. While near term inflation expectations are relatively well matched by the Bayesian Vector Autoregression of the order (1) (BVAR (1)), which does not have trend inflation, long-horizon inflation expectations are better matched by the DSGE model. In particular, BVAR (1) does not generate the volatile movements of long-horizon inflation expectations during the 1970-80s as shown in Figure 2.

Kozicki and Tinsley (2005) model the imperfect information and learning of agents about the inflation target of the central bank in a VAR model estimated with output gap, inflation, federal funds rate, and 10-year bond yield. In the model, inflation has a trend component. They distinguish central bank's actual target from the perceived target by private agents and assume agents do not directly observe the actual target but try to adjust the perceived target toward the actual target based on the deviation of the short rate from their expectations.

¹⁶In contrast, if I estimate inflation target without using yield curve data, unit roots for nominal interest rates detrended by the estimated target are not statistically rejected.

¹⁷Splicing together different sources of information may increase the uncertainty of this measure but the qualitative pattern of the time variation of this measure seems to be robust to this issue as pointed by Clark and Nakata (2008). In fact, the same measure is used for the Federal Reserve Board's FRB/US model as a proxy for long run inflation expectations.

Table 4 shows that the perceived target in Kozicki and Tinsley (2005) is more highly correlated with mean one-year ahead inflation forecasts from survey data than the estimated inflation target from the DSGE model.¹⁸ However, the correlation with long-horizon inflation expectations is weaker. The difference boils down to the fact that the perceived target in Kozicki and Tinsley (2005) continuously declined during the mid-1980s, while the long-horizon inflation expectations and the perceived target from the DSGE model defined by the permanent component of the inflation target $E_t(\pi_{t+1}^*)$, showed a temporary upward shift, as shown in Figure 2.

Figure 3 shows the gap between the perceived target and the actual target for both Kozicki and Tinsley (2005)'s model and the DSGE model in this paper. Compared to the estimates in the DSGE model, the estimates of Kozicki and Tinsley (2005) are highly persistent and much larger. This difference is attributed to a fairly slow learning about the actual target. Indeed, we cannot reject a unit root for the estimated gap between the two targets while they are assumed to be cointegrated in the model of Kozicki and Tinsley (2005). While the actual target and the perceived target are cointegrated in the model, estimated gap between the two targets are close to a random-walk.¹⁹ Such a big inertia of learning in Kozicki and Tinsley (2005) produces observations somewhat at odds with the direct evidence on long-run inflation expectations from survey data. For example, Figure 2 shows that survey data on long run inflation expectations essentially stabilized at around 2% in the late 1990s but the estimated gap from Kozicki and Tinsley (2005)'s model is still high in the same period and neither estimate seems to stabilize. In contrast, estimates of inflation target from the DSGE model exhibit smaller fluctuations from the late 1990s. Unlike Kozicki and Tinsley (2005), the DSGE model implies that agents learn relatively quickly. One motivation of the existing literature to introduce the imperfect information about the inflation target is to explain the sluggish adjustment of inflation expectations relative to actual inflation during the Volcker disinflation period. In the DSGE model, this relatively sluggish adjustment is explained mainly

¹⁸I use data from 1959:Q2 to 2004:Q4 to estimate the same model in Kozicki and Tinsley (2005).

¹⁹The same issue arises in Dewachter (2008) who estimates a macro-finance model with imperfect information on the nonstationary inflation target.

by the time varying volatility of macro shocks rather than learning.²⁰

4 Conclusion

This paper incorporates information from long-term interest rates to better understand the Federal Reserve’s (implicit) inflation target in a microfounded DSGE setup. I identify the drifting inflation target of the central bank as the common trend component for inflation and nominal interest rates. The model incorporates the imperfect information and learning by agents about the inflation target of the central bank.

The resulting estimates of inflation target are consistent with not only the trend component of nominal interest rates and inflation used in the estimation but also with long-horizon inflation expectations from survey data which are not used in the estimation. The estimated volatility of shocks in the model implies that agents learn more quickly than implied by the calibration in Erceg and Levin (2003) or estimation results in Kozicki and Tinsley (2005). The slow learning speed in the existing literature tends to generate a very persistent gap between the actual target and the perceived target even after the Volcker disinflation period of the early 1980s. In contrast, the estimated learning speed from the DSGE model is more consistent with the decline and stabilization of long-horizon inflation expectations since the Volcker disinflation period.

5 Appendix

Under learning, market prices of risk are time-varying and we have to compute coefficients in log bond prices at each period. Strictly speaking, time-varying market prices of risk should be part of a model’s state vector. It can be achieved by including

²⁰In fact, if we assume the constant volatility of macro shocks, the DSGE model generates the slow adjustment of inflation expectations in expense of overpredicting actual inflation. For example, the constant volatility version of the DSGE model overpredicts inflation by 0.51% at the third quarter of 1981 compared to the stochastic volatility version.

the covariance matrix of the filtered estimate Ω_t in the state vector. However, doing so would result in a quite complicated non-affine term structure model because Ω_t follows a nonlinear law of motion. I assume that agents update market prices of risk each period but treat the updated values as if it would remain constant going forward in time. Cogley (2005) uses a similar approximation in the context of making multi-step forecasts in a VAR with time-varying parameters and argues that the first order impacts of the approximation are small unless precautionary motives are strong. It turns out that this approximation does not create any significant differences in bond yields.²¹ With this approximation, we can obtain coefficients of log bond prices as follows:

$$\begin{aligned}
a_{1,t} &= 0, \quad b'_{1,1} = [0, 0, -1, 0, \dots, 0], \quad c_{1,t} = \frac{1}{2}[\Lambda_1^2, \Lambda_2^2, \Lambda_3^2, \Lambda_{4,t}^2] \\
a_{n,t} &= a_{1,t} + a_{n-1,t} + b'_{n-1,1}T_{c,t} + c_{n-1,t}((I - \nu)\sigma_l^2 + \Omega_{t+1} - \nu_{\pi^*}\Omega_t) + \frac{1}{2}c_{n-1,t}D_w c'_{n-1,t} \\
b'_{n,1} &= [0, 0, -1, 0, \dots, 0] + b'_{n-1,1}T_1, \quad b'_{n,2} = [0, -n], \quad D_w = \text{diag}([\sigma_{w,a}^2, \sigma_{w,f}^2, \sigma_{w,i}^2, \sigma_{w,\pi^*}^2]) \\
c_{n,t} &= 0.5(\lambda_{m,t} + \lambda_{b,t} + 2\lambda_{b,m,t}) + c_{n-1,t}\nu, \\
\lambda_{m,i,t} &= \Lambda_{i,t}^2, \quad \lambda_{b,i,t} = (b_{n-1,1}T_{\epsilon,i,t})^2, \quad \lambda_{b,m,i,t} = \Lambda_{i,t}(b_{n-1,1}T_{\epsilon,i,t})(n \geq 2) \\
\sigma_l^2 &= [\sigma_a^2, \sigma_f^2, \sigma_i^2, \sigma_{\pi^*}^2 + \sigma_n^2].
\end{aligned} \tag{22}$$

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²¹Unless signal to noise ratio is very low, the approximation error does not make meaningful differences. However, the low value of signal to noise ratio generates a too high volatility of inflation which is at odds with data.

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Table 1: SAMPLE MOMENTS

Variables	mean	standard deviation	AR (1)	ADF test statistic
inflation(π_t)	3.439	2.397	0.891	-2.066
short rate(i_t)	5.210	2.911	0.947	-1.933
1 year bond yield($y_{4,t}$)	5.599	2.911	0.952	-1.809
2 year bond yield($y_{8,t}$)	5.809	2.859	0.958	-1.660
3 year bond yield($y_{12,t}$)	5.984	2.787	0.962	-1.520
4 year bond yield($y_{16,t}$)	6.116	2.752	0.965	-1.447
5 year bond yield($y_{20,t}$)	6.198	2.714	0.968	-1.388
term spread ($y_{20,t} - i_t$)	0.989	1.007	0.829	-4.077*
ex post real rate($i_t - \pi_t$)	1.770	2.234	0.822	-4.116*

Notes: Statistics for the sample observations from 1960:Q1 to 2004:Q4. * denotes the rejection of a unit root at the 5% level.

Table 2: PRIOR AND POSTERIOR DISTRIBUTION

Parameter	Prior	P	osterior
	90% Interval	Mean	90% Interval
σ	[1.19, 2.80]	2.48	[2.44, 2.52]
β	[0.9965, 0.9995]	0.9994	[0.9993, 0.9995]
ν	[1.19, 2.80]	2.30	[2.22, 2.38]
$\ln f^*$	[0.052, 0.147]	0.105	[0.101, 0.110]
θ	[0.593, 0.913]	0.262	[0.248, 0.274]
u_a^*	[0.003, 0.007]	0.007	[0.0069, 0.0073]
γ_p	[1.19, 2.79]	1.496	[1.424, 1.584]
γ_y	[0.237, 0.558]	0.467	[0.447, 0.484]
ρ_a	[0.136, 0.461]	0.032	[0.022, 0.046]
ρ_f	[0.647, 0.958]	0.869	[0.861, 0.876]
ρ_i	[0.338, 0.669]	0.265	[0.251, 0.277]
$100\sigma_a$	[0.217, 0.795]	0.325	[0.292, 0.369]
$100\sigma_f$	[0.213, 0.791]	0.459	[0.428, 0.489]
$100\sigma_i$	[0.054, 0.199]	0.441	[0.441, 0.441]
$100\sigma_{\pi^*}$	[0.053, 0.198]	0.103	[0.089, 0.118]
ν_a	[0.101, 0.999]	0.9986	[0.9973, 0.9999]
ν_f	[0.008, 0.906]	0.994	[0.989, 0.999]
ν_i	[0.001, 0.899]	0.267	[0.208, 0.331]
ν_{π^*}	[0.078, 0.976]	0.996	[0.993, 0.999]
$100^2\sigma_{w,a}$	[0.053, 0.198]	0.0248	[0.0226, 0.0269]
$100^2\sigma_{w,f}$	[0.054, 0.199]	0.0266	[0.0247, 0.0287]
$100^2\sigma_{w,i}$	[0.000, 0.003]	0.0002	[0.0002, 0.0003]
$100^2\sigma_{w,\pi^*}$	[0.000, 0.003]	0.0004	[0.0004, 0.0004]
$\ln A_0$	[9.24, 9.90]	9.72	[9.70, 9.75]
$100 \ln \pi_0^*$	[0.104, 0.703]	0.599	[0.567, 0.626]
$100\sigma_{n,\pi^*}$	[0.053, 0.198]	0.029	[0.025, 0.032]

Notes: Prior intervals are computed based on 100,000 draws. I use 50,000 draws to compute posterior means and intervals in the learning version.

Table 3: STANDARD DEVIATIONS OF MEASUREMENT ERRORS

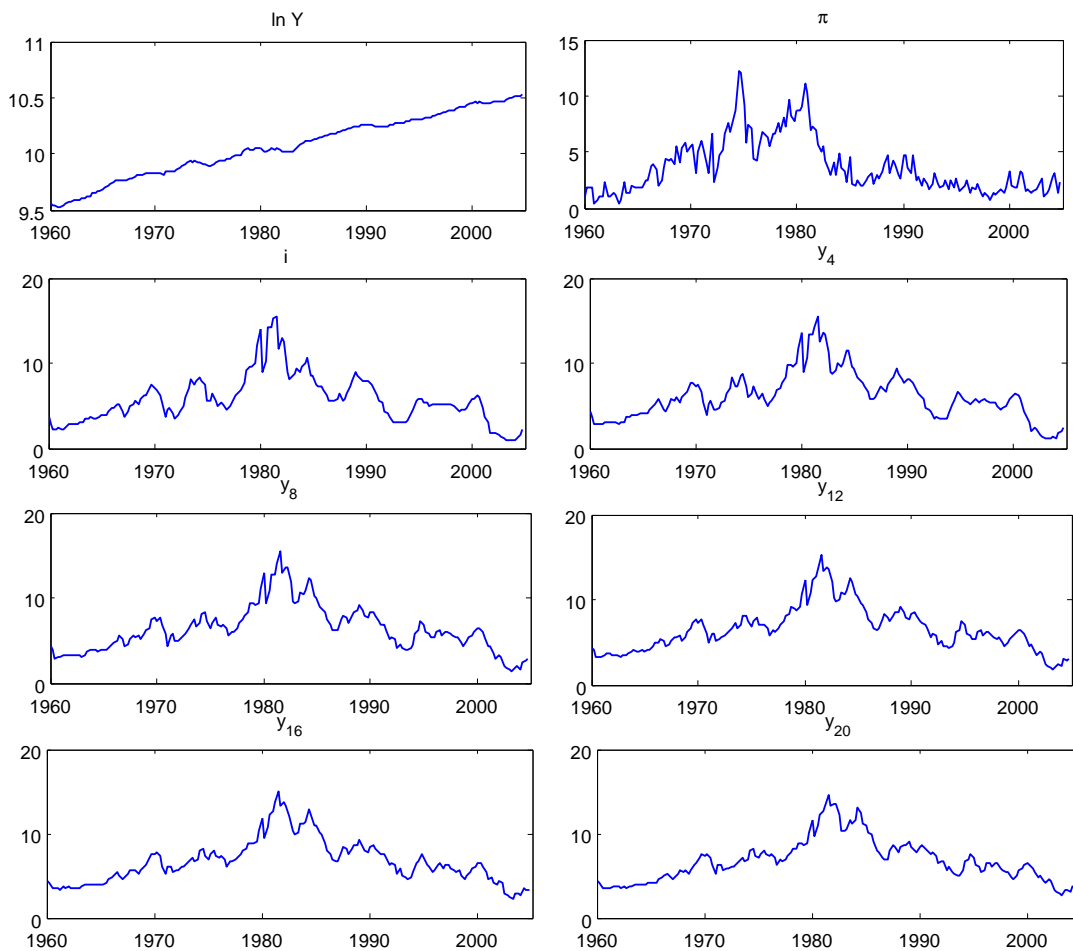
	$100\sigma_{u,1}$	$100\sigma_{u,2}$	$100\sigma_{u,3}$	$100\sigma_{u,4}$	$100\sigma_{u,5}$	$100\sigma_{u,6}$
Prior	0.063 (0.033)	0.063 (0.033)	0.063 (0.033)	0.063 (0.033)	0.063 (0.033)	0.063 (0.033)
Posterior	0.130 (0.003)	0.031 (0.002)	0.017 (0.001)	0.017 (0.001)	0.016 (0.001)	0.019 (0.002)
Ang, Dong, and Piazzesi (2007)	0.177 (0.009)	0.111 (0.006)	0.0056 (0.004)	0.034 (0.002)	0.046 (0.003)	0.064 (0.004)

Table 4: CORRELATION BETWEEN MODEL-IMPLIED INFLATION EXPECTATIONS AND SURVEY DATA

Horizon	DSGE	DSGE (No Yield Data)	BVAR (1)	KT (2005)
1 year	0.9244	0.7990	0.8442	0.9568
Long run	0.8727	0.6999	0.1720	0.7597

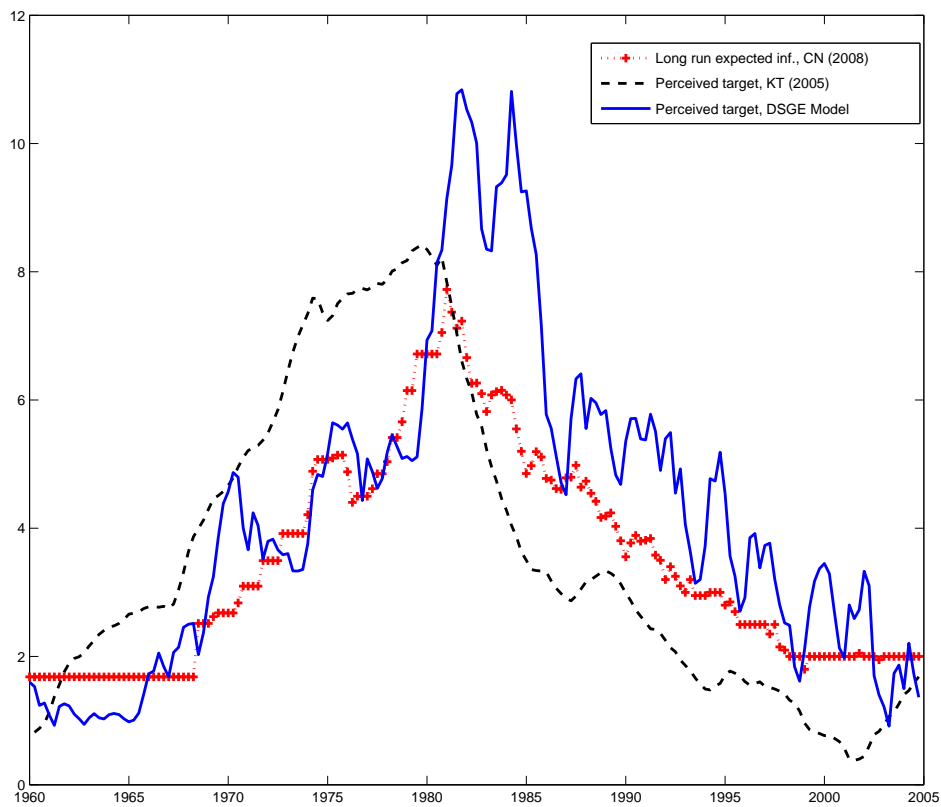
Notes: “For 1 year-ahead inflation forecasts, I use the mean forecasts of GDP deflator from the Survey of Professional Forecasters (SPF) published by the Federal Reserve Bank of Philadelphia over the time period 1981:Q3 -2004:Q4. Long run inflation forecasts are from Clark and Nakata (2008). In the last column, I compute the correlation between the perceived inflation target in Kozicki and Tinsley (2005) and survey data.

Figure 1: TIME SERIES PLOTS OF DATA



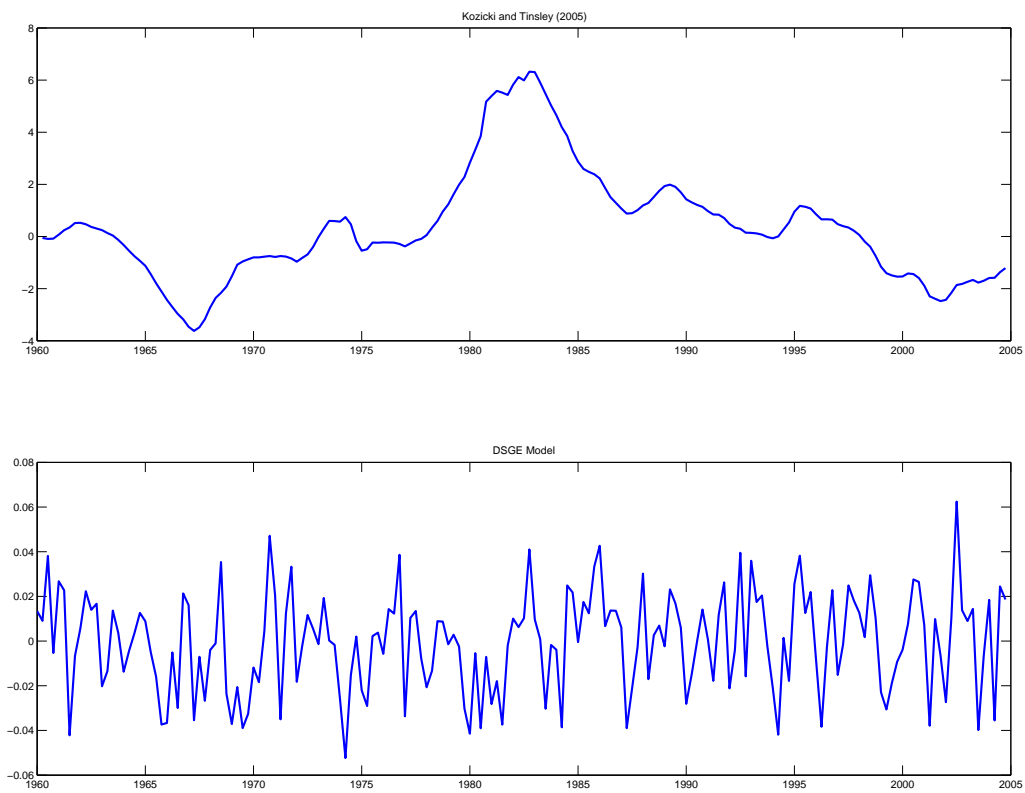
Notes: Per capita real GDP, inflation from GDP deflator, 3 month Treasury bill rate, 1, 2, 3, 4, 5 year Treasury bond yields from 1960:Q1 to 2004:Q4 are plotted.

Figure 2: SMOOTHED ESTIMATES OF PERCEIVED INFLATION TARGET



Notes: Long run expected inflation is from Clark and Nakata (2008) and KT (2005) denotes estimates from using the same model as Kozicki and Tinsley (2005).

Figure 3: SMOOTHED ESTIMATES OF THE GAP BETWEEN PERCEIVED TARGET AND ACTUAL TARGET



Notes: For the DSGE model, the gap is defined by $E_t(\pi_{t+1}^*) - \pi_t^*$.