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# Monetary Policy Regime Shifts and Inflation Persistence

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RESEARCH WORKING PAPERS

# **Monetary Policy Regime Shifts and Inflation Persistence**

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## **Abstract**

Using Bayesian methods, we estimate a Markov-switching New Keynesian (MSNK) model that allows shifts in the monetary policy reaction coefficients and shock volatilities. Using U.S. data, we find that a more-aggressive monetary policy regime was in place after the Volcker disinflation and before 1970 than during the Great Inflation of the 1970s. Our estimates also indicate that a low-volatility regime has been in place during most of the sample period after 1984. We connect the timing of the different regimes to a measure of inflation persistence. In the MSNK model, the population moment describing the serial correlation of inflation is a weighted average of the autocorrelation parameters of the exogenous shocks. A shift to an aggressive monetary regime or a low-volatility regime shuffles the weight from the more-persistent to the less-persistent shocks, resulting in a decline in inflation persistence. The timing of regimes from the estimated MSNK model generates a statistically significant 'low-high-low' pattern of inflation persistence that is consistent with reduced-form empirical models. We discuss relative importance of policy shifts and volatility shifts in explaining this pattern.

**Keywords:** Inflation persistence, Bayesian estimation, Markov-switching, DSGE models

**JEL Classification:** C11, E31, E52

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# 1 Introduction

An important issue for monetary policy is understanding the factors that contribute to persistence in deviations of inflation from its underlying trend. Persistence and predictability are closely related, so given the importance attached to inflation forecasting in central banks, it is important for them to understand the factors driving inflation persistence.<sup>1</sup> Evidence of changes to inflation persistence from empirical models that allow for time-varying coefficients, such as Cogley, Primiceri, and Sargent (2009), often reports a ‘low-high-low’ pattern. This pattern reflects low persistence prior to the Great Inflation of the 1970s, followed by a rise in the 1970s and then a decline beginning in the early 1980s. Recent efforts to interpret changes in inflation persistence through the lens of dynamic stochastic general equilibrium (DSGE) models, such as Benati and Surico (2007), Carlstrom, Fuerst, and Paustian (2008) and Cogley, Primiceri, and Sargent (2009), focus on the role of monetary policy and the decline in persistence that occurs roughly after the Volcker disinflation.<sup>2</sup> To allow for a potential monetary policy regime change, these papers split the sample of U.S. data around the early 1980s.<sup>3</sup> A central conclusion of this work is that monetary policy can reduce inflation persistence by more aggressively adjusting the nominal interest rate in response to inflation.

In this paper, we estimate a New Keynesian model that admits various forms of policy shifts, such as a time-varying inflation target and regime-dependent policy coefficients. We also incorporate regime-dependent heteroskedastic shocks, which is an additional mechanism that affects inflation persistence and, as Sims and Zha (2006) discuss, avoids the statistical bias that results in favoring shifts in policy coefficients due to the failure of accounting for heteroskedasticity. One benefit of estimating MSNK models is that we can let the data speak when exactly regime changes happen, rather than judgementally splitting the sample. Using Bayesian methods, we estimate a few variations of the Markov-switching New Keynesian (MSNK) model and find that a more aggressive monetary policy regime was in place after the Volcker disinflation than during the 1970s. Our estimation results indicate that monetary policy was also more aggressive in the period prior to 1970, which is consistent with that period also having lower inflation persistence. We also find that a low-volatility regime is

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<sup>1</sup>Our notion of inflation persistence does not necessarily imply constant trend inflation. If the underlying trend is time-varying, our notion of inflation persistence is equal to inflation-gap persistence as in Cogley, Primiceri, and Sargent (2009).

<sup>2</sup>The statistical significance of this decline has been debated. While Cecchetti and Debelle (2006), Clark (2006), Levin and Piger (2006), and Pivetta and Reis (2007) question the statistical significance of the decline in inflation persistence, Cogley, Primiceri, and Sargent (2009) provide evidence for changes in inflation-gap persistence using both univariate and multivariate models. Stock and Watson (2007) conclude inflation persistence has fallen from roughly 1984 on because a smaller fraction of the variance of changes in inflation is attributable to persistent shocks.

<sup>3</sup>While Benati and Surico (2007) use only data starting from 1983, Cogley, Primiceri, and Sargent (2009) estimate a similar DSGE model with two subsamples.

primarily in place throughout most of the post-1984 period, or Great Moderation era.<sup>4</sup>

To understand how shifts in monetary and volatility regimes affect inflation persistence, we analytically show the mechanisms that explain how a shift in these regimes affect the serial correlation in inflation. Specifically, we show that the population moment describing its serial correlation is a weighted average of the autocorrelation parameters of the exogenous shocks, which include a technology, monetary policy and markup shock. The weight on the autocorrelation parameter for each shock is a function of the monetary policy coefficient and shock volatilities, both of which depend on the current regime. Changes in regimes then reshuffle weights over the autocorrelation parameters and alter the serial correlation properties of inflation. A shift to a monetary regime that more aggressively fights inflation or a shift to a low-volatility regime reduces the weight on the more persistent shock, which reduces the serial correlation in inflation.

To empirically quantify these channels, the broadest model we estimate is a four-regime model that allows independent regime changes between two monetary regimes and two shock-volatility regimes. We also estimate more basic two-regime models with switching only in monetary policy or shock volatility, but find that the four-regime model is favored by data. Estimation of these two-regime models reveals that it is difficult to explain the increase in inflation persistence during the Great Inflation period of 1970s without appealing to a less aggressive monetary policy. More generally, we find a shift in monetary regime has a larger impact on persistence than a shift in the shock-volatility regime, though shifts in the volatility regime still have a quantitatively meaningful impact on persistence.

We also find that modeling a time-varying inflation target improves model fit when regime-switching is allowed only in volatility or policy coefficients, but that it makes little difference in the four-regime model. This finding indicates that jointly modeling shifts in policy coefficients and volatility captures low-frequency movements in inflation, so leaves little role for also allowing time-variation in the inflation target.

The conclusion that it is difficult to explain the shift in inflation persistence without appealing to monetary policy supports the previous findings in Cogley, Primiceri, and Sargent (2009) and Benati and Surico (2007), who both emphasize policy factors in accounting for changes in inflation persistence.<sup>5</sup> We also find that the active monetary policy following the Volcker disinflation, combined with the low volatility of the Great Moderation, set in place conditions conducive to low inflation persistence. However, many of these conditions were also in place prior to the 1970s, so the

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<sup>4</sup>In a related work, Kang, Kim, and Morley (2009) estimate a multiple breakpoint model for inflation and provide confidence intervals for these breakpoints. However, the model is reduced-form, so cannot address whether the breaks are driven by policy or other factors.

<sup>5</sup>See also Murray, Nikolsko-Rzhevskyy, and Papell (2009)

mechanisms in the MSNK model can replicate the ‘low-high-low’ pattern of inflation persistence that reduced-form models report, such as in Evans and Wachtel (1993) and Cogley, Primiceri, and Sargent (2009).

In the remainder of this paper, we proceed as follows. In Section 2, we describe a MSNK model. Section 3 provides a brief description of data and econometric methodology. Section 4 reports parameter estimates and the estimated timing of regime shifts. In Section 5, we discuss implications of estimation results for inflation persistence. Section 6 concludes. Technical issues on solving and estimating the model are discussed in the appendix.

## 2 A Markov-Switching New Keynesian Model

This section presents a Markov-switching New Keynesian model with a relatively standard private sector specification, following closely the setups in Ireland (2004) and An and Schorfheide (2007). The primary difference relative to these specifications is that the parameters in the monetary rule and shock volatilities are subject to regime shifts.

The basic elements of the model economy include a representative household, a representative firm that produces a final good and a continuum of monopolistically competitive firms that each produce an intermediate good indexed by  $j \in [0, 1]$ .

### 2.1 Households

The representative household chooses  $\{C_t, N_t, B_t\}_{t=0}^{\infty}$  to maximize lifetime utility

$$E_t \sum_{t=0}^{\infty} \beta_t \left( \frac{(C_t/A_t)^{1-\tau}}{1-\tau} - H_t \right),$$

where  $C_t$  denotes consumption of a composite good,  $H_t$  are hours worked,  $A_t$  is a measure of technology,  $\beta \in (0, 1)$  is the discount factor and  $\tau > 0$  is the coefficient of relative risk aversion.<sup>6</sup> Utility maximization is subject to the intertemporal budget constraint

$$P_t C_t + Q_t B_t = B_{t-1} + W_t H_t + P_t D_t - P_t T_t,$$

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<sup>6</sup> As we discuss below, technology follows a non-stationary process and induces a stochastic trend in consumption. Detrending  $C_t$  by  $A_t$  is convenient because the model has a well-defined steady state in terms of detrended variables. Also, an alternative interpretation of  $A_t$  is as a measure of external habit stock.

where  $B_t$  are nominal bond holdings,  $D_t$  are real profits from ownership of firms,  $T_t$  are lump-sum taxes,  $P_t$  is the aggregate price level,  $W_t$  is the nominal wage and  $Q_t$  is the inverse of the gross nominal interest rate.

## 2.2 Firms

Intermediate goods-producing firm  $j$  produces output,  $y_{jt}$ , according to

$$y_{jt} = A_t n_{jt},$$

where  $A_t$  is an exogenous measure of productivity that is the same across firms and  $n_{jt}$  is the labor input hired by firm  $j$ . The labor market is perfectly competitive and firms are able to hire as much as demanded at the real wage.

The monopolistic intermediate goods-producing firms pay a cost of adjusting their price, given by

$$ac_{jt} = \frac{\varphi}{2} \left( \frac{p_{jt}}{\Pi p_{jt-1}} - 1 \right)^2 Y_t, \quad (1)$$

where  $\varphi \geq 0$  determines the magnitude of the price adjustment cost,  $\Pi$  denotes the steady-state inflation which coincides with the central bank's inflation target<sup>7</sup> and  $p_{jt}$  denotes the nominal price set by firm  $j \in [0, 1]$ . The price adjustment cost is in terms of the final good  $Y_t$ . Each intermediate goods-producing firms maximizes the expected present value of profits,

$$E_t \sum_{s=0}^{\infty} \beta^s \Delta_{t+s} \frac{d_{jt+s}}{P_{t+s}}, \quad (2)$$

where

$$\Delta_{t+s} \equiv \left( \frac{C_{t+s}}{C_t} \right)^{-\tau} \left( \frac{A_t}{A_{t+s}} \right)^{1-\tau}$$

is the representative household's stochastic discount factor and  $d_{jt}$  are nominal profits of firm  $j$  at time  $t$ . Real profits are

$$\frac{d_{jt}}{P_t} = \frac{p_{jt}}{P_t} y_{jt} - \psi_t y_{jt} - \frac{\varphi}{2} \left( \frac{p_{jt}}{\Pi p_{jt-1}} - 1 \right)^2 Y_t, \quad (3)$$

where  $\psi_t$  denotes real marginal cost, where  $\psi_t = (W_t/P_t)/A_t$ .

There is a representative final-goods producing firm that purchases the intermediate inputs at nominal prices  $p_{jt}$  and combines them into a final good using the

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<sup>7</sup>When the central bank's inflation target follows a random walk, it creates a stochastic trend of inflation  $\Pi_t^*$ . Accordingly, we replace  $\Pi$  by  $\Pi_t^*$  under the alternative assumption.

following constant-returns-to-scale technology

$$Y_t = \left[ \int_0^1 y_t(j)^{\frac{\theta_{t-1}}{\theta_t}} dj \right]^{\frac{\theta_t}{\theta_t-1}}, \quad (4)$$

where  $\theta_t > 1 \forall t$  is the elasticity of substitution between goods. Variations in  $\theta_t$  translate into shocks to the desired markup, which is the actual markup in the absence of price adjustment costs. The steady-state markup is

$$u = \frac{\theta}{\theta - 1}, \quad (5)$$

and  $\theta$  is the steady-state elasticity of substitution.

The profit-maximization problem for the final-goods producing firm yields a demand for each intermediate good given by

$$y_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta_t} Y_t, \quad (6)$$

where  $p_{jt}$  is the nominal price of good  $j$ . The zero-profit condition for the final goods-producing firm implies  $P_t \equiv \left[ \int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$  is the aggregate price level.

## 2.3 Policy

The monetary authority sets the short-term nominal rate using the following rule

$$R_t = \bar{r}\Pi \left( \frac{\Pi_t}{\Pi} \right)^{\alpha(s_t)} \left( \frac{Y_t}{A_t y^*} \right)^{\gamma(s_t)} (e_t), \quad (7)$$

where  $R_t$  is the gross nominal interest rate,  $\Pi_t = P_t/P_{t-1}$ ,  $\Pi$  is the target rate of inflation,  $\bar{r}$  is the steady-state real rate,  $y^*$  is the steady-state level of the detrended output and the regime,  $s_t$ , is a discrete-valued random variable that follows a two-state Markov chain,

$$P_1 = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}, \quad (8)$$

where  $p_{ii} = \Pr [s_t = i | s_{t-1} = i]$ . The active, or more aggressive, regime corresponds to  $s_t = 1$  and the less-active regime, or possibly passive regime, corresponds to  $s_t = 2$ . This labeling implies  $\alpha(2) < \alpha(1)$ .

The assumption of a constant inflation target may seem at odds with the empirical literature that stresses the importance of allowing for mean shifts when measuring inflation persistence, such as Cecchetti and Debelle (2006), Clark (2006) and Levin

and Piger (2006). For example, we could have followed Schorfheide (2005) and Liu, Waggoner, and Zha (2009) and estimate a DSGE model with a policy rule that has a shifting inflation target. However, our rationale for imposing a constant mean, but shifting reaction coefficients, is to give monetary policy a potential mechanism to affect inflation persistence.<sup>8</sup> A shifting inflation target in the policy rule of this DSGE model does not affect the model implied serial correlation of inflation, so is an inadequate framework to address the issue of how changes in monetary policy affect inflation persistence.

The potential exists, however, that allowing for changes in trend inflation may better capture changes in U.S. inflation dynamics than allowing for shifts in the reaction coefficients in the policy rule or shock volatilities. To capture this possibility, we also estimate versions of the MSNK model that allows trend inflation to follow a driftless random walk,

$$\ln \Pi_t^* = \ln \Pi_{t-1}^* + \varepsilon_{\pi t}, \quad (9)$$

where  $\varepsilon_{\pi t} \sim N(0, \sigma_\pi^2)$ .

Regarding fiscal policy, we assume the fiscal authority passively adjusts lump-sum taxes to satisfy the government's flow budget constraint and transversality condition on government debt.

## 2.4 Exogenous Shock Processes

Aggregate productivity follows

$$\ln A_t = \lambda + \ln A_{t-1} + \ln a_t, \quad (10)$$

where

$$\ln a_t = \rho_a \ln a_{t-1} + \varepsilon_{at}, \quad (11)$$

with  $\varepsilon_{at} \sim N(0, \sigma_a^2(r_t))$  and  $|\rho_a| < 1$  for  $r_t \in \{1, 2\}$ . The process for productivity imposes that it grows at an average rate of  $\lambda$ , but is subject to serially correlated shocks that have varying degrees of volatility depending on the regime.

Shocks to the markup and monetary policy rule follow

$$\ln u_t = (1 - \rho_u) \ln u + \rho_u \ln u_{t-1} + \varepsilon_{ut}, \quad (12)$$

$$\ln e_t = \rho_e \ln e_{t-1} + \varepsilon_{et}, \quad (13)$$

where  $\varepsilon_{ut} \sim N(0, \sigma_u^2(r_t))$ ,  $\varepsilon_{et} \sim N(0, \sigma_e^2(r_t))$ ,  $|\rho_u| < 1$  and  $|\rho_e| < 1$  for  $r_t \in \{1, 2\}$ .

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<sup>8</sup>A subsequent section demonstrates how changes in reaction coefficients affect persistence.

The regime governing the volatility of the shock process,  $r_t$ , also follows a two-state Markov chain,

$$P_2 = \begin{bmatrix} q_{11} & 1 - q_{11} \\ 1 - q_{22} & q_{22} \end{bmatrix}, \quad (14)$$

where  $q_{ii} = \Pr[r_t = i | r_{t-1} = i]$ .

In the four-regime MSNK model, the shock-volatility regime,  $r_t$ , is independent from the monetary regime,  $s_t$ .<sup>9</sup> As a result, the shock-volatility regime can change without requiring a change in the monetary regime. This approach allows the data to indicate whether, say, a period of high inflation volatility is more likely to be caused by a less aggressive monetary policy or higher exogenous shock volatility. In principle, the volatility for each shock could change regime according to their own independent Markov chains. However, it turns out that such a model does not necessarily improve the model fit and create identification issues of each regime. So we focus on the model with synchronized switching in volatilities.

## 2.5 Equilibrium Relations

In a symmetric equilibrium, each intermediate goods-producing firm faces the same marginal cost, so makes the same pricing and production decisions. In equilibrium, we can then eliminate the  $j$  subscripts, yielding  $y_{jt} = Y_t$ ,  $p_{jt} = P_t$ ,  $n_{jt} = N_t$ ,  $ac_{jt} = AC_t$  and  $d_{jt} = D_t$ . The log-linearized first-order conditions for private agents are

$$\hat{y}_t = E_t \hat{y}_{t+1} - \tau^{-1} \left( \hat{R}_t - E_t \hat{\pi}_{t+1} - E_t \hat{a}_{t+1} \right), \quad (15)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\hat{y}_t + \hat{u}_t), \quad (16)$$

where  $\hat{y}_t \equiv \ln(y_t/y^*)$  is a measure of the output gap,  $y_t = (Y_t/A_t)$ ,  $\hat{\pi}_t = \ln(\Pi_t/\Pi)$  and  $\hat{R}_t = \ln(R_t/R)$ , where  $R = \bar{r}\Pi$ . Conditioning on a given regime, the monetary rule and shock processes are linear, given by

$$\hat{R}_t = \alpha(s_t) \hat{\pi}_t + \gamma(s_t) \hat{y}_t + e_t, \quad (17)$$

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{at}, \quad (18)$$

$$\hat{u}_t = \rho_u \hat{u}_{t-1} + \varepsilon_{ut}, \quad (19)$$

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \varepsilon_{et}, \quad (20)$$

where  $\hat{a}_t = \ln a_t$ ,  $\hat{u}_t = \ln(u_t/u)$ , and  $\hat{e}_t = \ln e_t$ . Equations (15) – (20) represent the full MSNK model.

We compute the minimum state variable solution of the model by the method of undetermined coefficients.<sup>10</sup> The resulting solution takes the following form:

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<sup>9</sup>The transition matrix in this case is  $P_1 \otimes P_2$ .

<sup>10</sup>In the model with a nonstationary inflation target, we would replace the steady-state inflation by the stochastic trend to compute  $\hat{\pi}_t$  and  $\hat{R}_t$ .

$$\hat{y}_t = B_y(s_t)x_t, \hat{\pi}_t = B_\pi(s_t)x_t, \hat{R}_t = B_R(s_t)x_t, x_t = [\hat{a}_t, \hat{u}_t, \hat{e}_t]'. \quad (21)$$

Using methods in Davig and Leeper (2007), we check if the parameterization of the model yields a unique equilibrium. Also, since we use a first-order approximation to the equilibrium conditions of households and firms, the solution coefficients depend only on the monetary regime and not the shock-volatility regime. The steady state is also independent of regime-shifts, since only the slope coefficients in the monetary reaction function change and not any variables that affect the deterministic steady state.<sup>11</sup>

One issue that naturally arises when solving the model is determinacy of equilibrium - that is, whether the solution is unique within the class of bounded solutions. An indeterminate equilibrium will have a different representation and depend on a different set of state variables than a determinate one. One approach to account for this complications in the estimation is to follow Lubik and Schorfheide (2004), where posterior weights apply to the determinate and indeterminate regions of the parameter space. This is a project worth pursuing in the context of a MSNK model, but beyond the scope of this paper. Our approach to dealing with the issue of indeterminacy is to require a linear representation of the MSNK model to have a unique solution. The linear representation maps (15) – (20) into a purely linear system of expectational difference equations, where the standard methods of solving these systems are available (e.g. eigenvector-eigenvalue decompositions).<sup>12</sup> This approach permits an active and passive monetary regime, but places limits on ‘how passive’ and ‘how long’ the passive regime can remain in place. In the subsequent empirical analysis, we consider only the region of parameter space where the determinacy restrictions of the linear representation are satisfied.

### 3 Data and Econometric Methodology

The linear structure of the model solution conditional on the current regime makes the application of the approximate Kalman filter of Kim and Nelson (1999) feasible. Given laws of motion for the shock processes and the minimum state variable solutions of inflation, output, and nominal interest rate, we can write down the following regime-

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<sup>11</sup>For an example of how to linearize a model where regime shifts do affect the steady state, see Schorfheide (2005)

<sup>12</sup>Technical appendix provides a brief description of the linear representation used in Davig and Leeper (2007) and some issues related with using the linear representation for checking the determinacy restrictions.

dependent state-space representation<sup>13</sup>

$$Z_t = A_z + B_z(s_t)x_t + [1, 0, 0]' \ln A_t, \quad Z_t = [\ln Y_t, \pi_t, R_t]', \quad (22)$$

$$x_t = \rho x_{t-1} + \varepsilon_t, \quad x_t = [\hat{a}_t, \hat{u}_t, \hat{e}_t]', \quad \varepsilon_t = [\varepsilon_{at}, \varepsilon_{ut}, \varepsilon_{et}]', \quad (23)$$

$$\ln A_t = \lambda + \ln A_{t-1} + \hat{a}_t. \quad (24)$$

$x_t$  is a vector of state variables and  $Z_t$  is a vector of three observed variables consisting of per capita real GDP, inflation (log difference of GDP deflator), and 3 month Treasury bill rate.  $B_z(s_t)$  is a conformable state-dependent matrix with elements arising from the solution of the MSNK model. The sample period is from 1953:Q1 to 2006:Q4. A plot of U.S. inflation, as measure by the GDP deflator, is given in Figure 1. Constructing the likelihood for the MSNK model requires integrating out latent variables, including the history of regimes. Kim and Nelson (1999) note that collapsing some paths of regimes with very small probability is necessary to make the filtering algorithm operable. Otherwise, we have to consider  $\bar{S}^t$  different paths of regimes to evaluate the likelihood value at  $t$ , where  $\bar{S}$  is the number of possible regimes. We allow 4 (16) different paths of regimes in a two (four) regime case. The likelihood for the four-regime model is

$$p(Z_t|Z^{t-1}, \vartheta) = \sum_{r_t, s_t \in \{1, 2\}} p(Z_t|Z^{t-1}, \vartheta, s_t, r_t)p(s_t|Z^{t-1}, \vartheta)p(r_t|Z^{t-1}, \vartheta), \quad (25)$$

where  $\vartheta$  is the vector of structural parameters and  $Z^{t-1}$  denotes observations up to time  $t - 1$ .<sup>14</sup> For the models with two regimes, either  $r_t$  or  $s_t$  is constant and the corresponding probability density collapses to unity.

Using the Bayesian approach, we combine the likelihood with a prior distribution of  $\vartheta$ . From the Bayesian perspective, the resulting posterior distribution of  $\vartheta$  reflects an update to the prior distribution using the information from the likelihood and is a key tool for inference. Incorporating prior information on  $\vartheta$  provides additional curvature for the posterior density and excludes implausible estimates of parameters which may overfit the sample data.<sup>15</sup> The posterior distribution of  $\vartheta$  is hard to characterize analytically, so we use a random-walk Metropolis-Hastings algorithm to obtain the posterior draws.<sup>16</sup> We initialize the Markov chain at the (local) mode of

<sup>13</sup>The representation assumes the model with the constant inflation target of the central bank. We can accommodate a random walk with drift in the inflation target in the same way as incorporating a technological trend.

<sup>14</sup>We increased the number of histories that are considered and found little difference in terms of the likelihood value.

<sup>15</sup>For further discussion of advantages of Bayesian approach in the estimation of DSGE models, see An and Schorfheide (2007).

<sup>16</sup>We use a mixture of normal distribution and  $t$  distribution as a proposal density. The relatively fat-tailed  $t$  distribution makes it more likely for the ratio between the proposal density and the target density to be bounded, which is a pre-requisite for uniform ergodicity necessary for the convergence of Markov chains. For this reason, Geweke (2005) mentions that transition mixtures can be powerful tools in building posterior simulators that are robust to ill-behaved posterior distributions.

the posterior density by using a numerical optimization routine (CSMINWEL provided by Christopher Sims). The inverse of the negative hessian evaluated at the local mode is used as the covariance matrix of the proposal density. After obtaining one million draws from the Markov chain, we compute means and the covariance matrix and update the covariance matrix of the proposal density. Then, we run multiple Markov chains starting around the means of the previous one million draws with the updated covariance matrix. We run those chains until trace plots of parameters and other convergence diagnostic tests confirm that the distribution of Markov Chain Monte Carlo (MCMC) output converges to the stationary distribution.<sup>17</sup>

In constructing the likelihood, we use the filtered probability for each regime to integrate out the latent regimes. Since regimes are not directly observable to the econometrician, we are often interested in computing the estimates of the probability of different regimes conditional on all the observations available. This approach provides an indication of which history of regimes is most probable given the available observations. The smoothed probability of each regime can be obtained by applying the filtering step backwards. In the four-regime model, we compute the smoothed probabilities as follows

$$p(Q_t|Z^T, \vartheta, r_t) = \frac{p(Q_t|Z^t, \vartheta)p(Q_{t+1}|Q_t)p(Q_{t+1}|Z^T, \vartheta)}{\sum_{Q_t \in \{1,2,3,4\}} p(Q_t|Z^t, \vartheta)p(Q_{t+1}|Q_t)p(Q_{t+1}|Z^T, \vartheta)}, \quad (26)$$

where  $Q_t$  is a composite four-state discrete valued random variable that describes both the monetary and volatility regimes.<sup>18</sup> Either  $r_t$  or  $s_t$  replaces  $Q_t$  in (26) for the two-regime models. Since  $p(Q^T|Z^T, \vartheta)$  and  $p(Q_t|Z^t, \vartheta)$  are obtained as byproducts of the likelihood evaluation, this is relatively easy to implement.

To identify the sources of the changes in inflation persistence, we need to compare different regime-switching models. The marginal likelihood of each model provides a coherent framework to compare non-nested models. Conceptually, it is obtained by integrating the posterior kernel over the entire parameter space in each model  $M_i$

$$p(Z^T|M_i) = \int p(Z^T|\vartheta, M_i)p(\vartheta|M_i)d\vartheta. \quad (27)$$

The practical computation of this constant is done by the numerical approximation based on the posterior simulator as in Geweke (1999), for example.<sup>19</sup>

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<sup>17</sup>For details, see technical appendix.

<sup>18</sup>The transition matrix is  $P_1 \otimes P_2$  with elements given by  $p(Q_{t+1}|Q_t)$ .

<sup>19</sup>Sims, Waggoner, and Zha (2008) argue that Geweke (1999)'s method is not robust when the posterior distribution may be non-Gaussian due to regime-switching effects and suggest an alternative method of computing the marginal likelihood based on a family of elliptical distributions. However, this turns out to be numerically unstable in the models we estimate because the measure is quite sensitive to the scaling parameter of the covariance matrix of the proposal density. We provide additional details of this issue in the technical appendix.

## 4 Empirical Analysis

### 4.1 Prior Distribution

Table 1 provides information on the prior distribution of the parameters. If possible, the prior means are calibrated to match the sample moments of observed variables. For example, the prior mean of the average technology growth rate ( $\lambda$ ) is set to match the average growth rate of per capita real GDP. Similarly, the prior mean of the steady-state inflation rate ( $\Pi$ ) in the model is set to match the average inflation rate in the data. And the prior mean of the discount factor ( $\beta$ ) is then set to match the average nominal interest rate conditional on the prior means of  $\lambda$  and  $\Pi$ . The autocorrelation of technology growth ( $\rho_a$ ) and the standard deviation of the technology shock ( $\sigma_a$ ) are set to match the autocorrelation and the standard deviation of per capita real GDP. Prior distributions of other parameters are mostly set to be consistent with the existing literature on the estimation of New Keynesian models. For example, the prior distribution of the slope of the Phillips curve is from Lubik and Schorfheide (2004). For switching parameters, prior distributions are set to be roughly consistent with split sample (pre-1983, post-1983) estimates in fixed-regime models. This induces the natural ordering of regime-dependent parameters and avoids the potential risk of the ‘label switching’ problem as noted in Hamilton, Waggoner, and Zha (2007).

### 4.2 Posterior Distribution

We estimate three versions of the MSNK model. The first allows switching only in the monetary policy rule and the second allows switching only in the shock volatilities. The third model is the four-regime MSNK model that allows independent switching in monetary policy and the shock-volatility regimes. Table 2 provides prior and posterior probability intervals for all the parameters for the models with a constant inflation target.

For the MSNK model with switching only in the monetary policy rule, the monetary regimes adjust the nominal interest rate differently in response to inflation. The mean of the reaction coefficient to inflation in the active regime is 2.07, which is significantly larger than the .99 coefficient in the less-active regime.<sup>20</sup> The response to output is similar across regimes, although the uncertainty associated with the coeffi-

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<sup>20</sup>We classify the later regime as ‘less active,’ even though the point estimate for the coefficient is less than unity and the 90% credible interval extends down to .86. The reason we do not call it passive is that the mean response to inflation, along with the mean response to the output gap, would not yield indeterminacy in a fixed-regime model. There are parameter values, however, in the 90% credible interval that would yield indeterminacy in a fixed-regime model.

cient in the more-active regime is much higher. The timing of the different regimes are given by the posterior expected values of the smoothed probabilities in Figure 2, which shows two persistent changes in the monetary regime over the sample period. The first occurs when the monetary regime changes from the more-active to less-active stance in late 1970. The second shift occurs in mid-1982 when policy moves back to the more-active stance and remains there until the end of the sample.

The timing and nature of monetary regimes is roughly consistent with estimates from Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004), where both find substantial differences in the reaction of the nominal interest rate to inflation before and after approximately 1980. A key difference between the estimates from the MSNK model and these papers is that policy was active for a significant period before 1970. Similarly, estimates from Bianchi (2008) and Eo (2008), who allow regime shifts in policy coefficients, also find that the active policy stance was in place for much of the time prior to the 1970s.

A key feature of U.S. data is the high and volatile inflation in the 1970s (see Figure 1). In models with a constant steady-state level of inflation, our estimation matches this shift in inflation volatility using whatever switching parameters it has available, which in this first case are the monetary reaction coefficients. The reaction coefficient to inflation plays an important role in determining the volatility of inflation. As this coefficient increases, the volatility of inflation declines. In the limit, monetary policy can completely stabilize inflation. Moving in the other direction, inflation becomes more volatile as the reaction to inflation declines. Davig and Leeper (2007) show that a monetary reaction coefficient less than unity actually has the effect of amplifying shocks. Thus, the MSNK model with switching in only the monetary rule uses a passive monetary regime to generate higher inflation volatility in the 1970s. Alternatively, the volatile inflation in the 1970s could reflect higher shock volatility, so estimating the MSNK model with switching only in monetary policy could incorrectly be attributing the higher volatility in the 1970s to policy. Indeed, Sims and Zha (2006) argue that there is little evidence for changes in the systemic behavior of monetary policy once heteroskedasticity of shocks are properly considered.

To address this concern and assess the role of shifting shock volatilities in explaining U.S. data, we estimate the MSNK model with switching only in the variance of the shocks. Table 2 reports that the standard deviation of each shock roughly doubles in the high-volatility state and that the relative volatility of the markup shock, which is the most persistent one among three shocks, more than triples. Figure 3 reports the timing of the low volatility regime. Again, the high inflation periods in the 1970s and volatile early 1980s stand out as a different regime.

Estimation of two MSNK models suggests that the high and volatile inflation during the 1970s can be explained by either a passive policy or a high volatility regime. To evaluate the relative contribution of each channel, we estimate a broader MSNK model

with switching in both the monetary policy and shock-volatility regimes. Switching in the monetary policy and volatility regimes is independent, so for example, a change in the monetary regime does not require a change in the shock-volatility regime.

For this four-regime MSNK model, the regime-switching parameters are broadly similar to the previous estimates. Table 2 shows the lower bound in the posterior interval of the monetary policy reaction coefficient in the less aggressive monetary regime is greater than unity, implying active policy in both regimes. Also, the difference in the estimates of policy response to inflation across regimes is smaller than the two regime case, indicating that volatility shifts soak up some of the role of shifts in the policy rule. The upper panel in Figure 4 shows the posterior expected values of the smoothed probabilities for the active monetary policy regime and high-volatility regime. These estimates indicate that monetary policy was aggressive in responding to inflation throughout the latter half of the 1950s and most of the 1960s. Beginning in the late 1960s, however, policy began responding less aggressively and maintained this stance until the early 1980s. Policy reverts to its less-active stance preceding the 1990-91 recession and throughout the relatively sluggish recovery in the early 1990s. Policy then turns again to a more-active stance in the mid-1990s, showing indications again of switching to the less active regime around the time of the 2001 recession. The period of less-active policy following the 2001 recession also corresponds to the period following the technology bubble collapse and subsequent period of relatively low inflation.

The posterior expected values of the smoothed probabilities for the shock-volatility regimes from the four-regime model are given in the lower panel in Figure 4. The high volatility regime corresponds to the 1970s and early 1980s, with also a brief interlude in the late 1950s and early 1960s. The low-volatility regime is in place throughout most of post-1984 period, or Great Moderation era, with the brief exception of the 2001 recession. Also, estimates characterize the Volcker disinflation period with the high-volatility regime, rather than the more aggressive monetary regime. Policy is active in both regimes, so policy still responded systematically more than one-for-one to inflation during the Volcker disinflation. However, the dramatic shift in policy during this time is reflected initially more as a volatile policy shock before being recognized as a shift in the systematic behavior of policy.

As an initial check of the fit of the four-regime MSNK model, Figure 1 shows a reasonably accurate in-sample fit for the inflation series. More formally, Table 3 reports the log marginal likelihood values for each model and indicates that the data prefers the model with switching shock volatility over the model with switching monetary policy. However, the model that best fits the data is the four-regime MSNK model. Since the marginal likelihood penalizes overparameterization, the better fit of the four-regime model is not driven by the increase in the number of parameters. We also consider the fit of MSNK models with the random-walk inflation target of the central bank. While including random-walk drift in inflation target improves fit

for models with switching only in policy coefficients or volatilities, there is little gain in the four-regime model. The random-walk inflation target is introduced mainly as a tool to capture low-frequency variations in monetary policy. Our finding suggests that once shifts in policy coefficients and shock volatilities are allowed, these low-frequency variations in monetary policy are properly captured, leaving little room for shifts in the central bank's inflation target.

## 5 Changes in Inflation Persistence

### 5.1 Measure of Inflation Persistence

This section demonstrates how changes in the monetary and volatility regimes affect a measure of inflation persistence. Specifically, we compute the population moment for the autocorrelation of the inflation. For the four-regime MSNK model, this statistic conditional on a given regime is

$$\rho_\pi(\pi_t | s_t = i, r_t = j, \forall t) = w_a(i, j)\rho_a + w_u(i, j)\rho_u + (1 - w_a(i, j) - w_u(i, j))\rho_e, \quad (28)$$

where

$$w_a(i, j) = B_{\pi,a}(i)^2 W(i, j) \left( \frac{\sigma_a^2(j)}{1 - \rho_a^2} \right), \quad (29)$$

$$w_u(i, j) = B_{\pi,u}(i)^2 W(i, j) \left( \frac{\sigma_u^2(j)}{1 - \rho_u^2} \right), \quad (30)$$

and

$$W(i, j) = \left[ B_{\pi,a}(i)^2 \frac{\sigma_a^2(j)}{1 - \rho_a^2} + B_{\pi,u}(i)^2 \frac{\sigma_u^2(j)}{1 - \rho_u^2} + B_{\pi,e}(i)^2 \frac{\sigma_e^2(j)}{1 - \rho_e^2} \right]^{-1}, \quad (31)$$

for  $i, j = 1, 2$ .<sup>21</sup> Equation (28) shows the serial correlation of inflation is a weighted average of the autocorrelation parameters of the exogenous shocks. A change in the monetary or shock-volatility regime reshuffles the weights across these autocorrelation parameters. A regime change that shifts weight from more persistent to less persistent shocks will decrease this measure of inflation persistence. Using the posterior mean estimates from the four-regime MSNK model, Figure 5 shows how the weights on the mark-up shock and policy shock vary with changes in the monetary policy reaction to inflation. The figure shows how a shift to the more aggressive monetary regime

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<sup>21</sup>This construction of the model-implied inflation persistence has similarities to Carlstrom, Fuerst, and Paustian (2008). They suggest a more aggressive monetary policy and a decline in the relative volatility of more persistent shocks are possible explanations for the decline in inflation persistence. However, they do not emphasize the mechanism that a more aggressive monetary policy essentially increases the relative importance of less persistent shocks in the persistence measure.

transfers weight to the less persistent shocks and thereby, reduces inflation persistence. In contrast, if we allow regime shifts in the central bank’s inflation target, but not in policy coefficients or shock volatility, these weights in the above measure of inflation persistence do not vary across regimes.

## 5.2 Empirical Evidence for Shifts in Inflation Persistence

Before assessing implications of the estimated MSNK models for inflation persistence, we first want to establish an empirical benchmark that describes changes in persistence. For the benchmark, we estimate the following simple Markov-switching time series model using Bayesian methods

$$\pi_t = \bar{\pi}(S_t) + \gamma(S_t)\pi_{t-1} + \varepsilon_t, \quad (32)$$

where  $\varepsilon_t \sim N(0, \sigma(S_t)^2)$  and  $S_t$  evolves according to a Markov chain.<sup>22</sup> Figure 6 plots the persistence of the inflation based on this model, which is given by regime probability-weighted values of  $\gamma(S_t)$ . Most notably, there is a clear, statistically significant rise in the persistence measure beginning in the late 1960s and extending through to the mid-1980s. The ‘low-high-low’ pattern matches the pattern coming from more sophisticated time-series models, such as Cogley, Primiceri, and Sargent (2009), although they focus on the decline after the early 1980s. Our focus on the ‘low-high-low’ pattern is to see whether a similar pattern emerges from the MSNK model. Given the results in the previous section, the timing of monetary regimes from the MSNK model and changes in persistence from the empirical Markov-switching model do roughly coincide. The similar timing suggests a link between inflation persistence and shifts in monetary policy. The next step, however, is to establish if the full four-regime MSNK model, which is the model preferred by the data, can generate shifts in inflation persistence that are similar in magnitude and timing to the empirical benchmark.

## 5.3 Structural Interpretation

Table 4 and Figure 7 report the model-implied inflation persistence statistic for each MSNK model across regimes. The regimes with the more-active policy (i.e.  $s_t = 1$ ) or lower volatility (i.e.  $r_t = 2$ ) have lower persistence than the regimes with less-active policy (i.e.  $s_t = 2$ ) or higher volatility (i.e.  $r_t = 1$ ). Focusing on the four-regime MSNK model (i.e. specification P3 in Table 4), the lowest degree of inflation persistence is in the regime with aggressive monetary policy and low shock

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<sup>22</sup>Evans and Wachtel (1993) use a similar reduced-from Markov-switching model for inflation, except they impose a unit root in one regime. They show a shift occurs to the regime with the unit root around 1968 that lasts until 1984.

volatility (i.e.  $s_t = 1$  and  $r_t = 2$ ). Comparing this regime with the one having less aggressive monetary policy and higher volatility (i.e.  $s_t = 2$  and  $r_t = 1$ ) reveals significant differences. For example, the 90% credible interval for persistence in the more-active monetary policy regime with low volatility is [.74,.82]. In the less-active monetary policy regime with high-volatility regime, the interval is [.84,.91]. Compared to models with only policy shifts or volatility shifts, the four-regime model generates somewhat tighter posterior intervals for the model-implied inflation persistence. For example, in the model with only policy shifts, the 90% credible interval for persistence is [.73,.85] in the more-active monetary policy regime, which is wider than [.74,.82] in the more-active monetary policy regime with low volatility from the four-regime model.

A more complete picture of the changes in model implied inflation persistence is given in Figure 7, which plots the measure of persistence from the model given in (28). The more-active policy combined with some periods in the low-volatility regime generates relatively low persistence early in the sample. Persistence increases beginning with the shift to less-active policy in the late 1960s and then peaks around 1980 due to the shift to the high-volatility regime. The switch to the more-active monetary policy, and then to the low-volatility regime, decreases persistence throughout the early 1980s. The upward movements that occur around 1990 and 2000 correspond to monetary policy shifts back to the less-active regime, which occur roughly around the NBER recessions in 1990-91 and 2001.

The pattern of rising persistence during the 1960s and 1970s, followed by a decline starting around the Volcker disinflation, is consistent with the empirical benchmark and existing evidence on changes in inflation persistence, as in Cogley, Primiceri, and Sargent (2009). The results from the DSGE analysis in Cogley et. al. primarily attributes the decline in inflation persistence to a decline in the variability of the inflation target. In our framework, the variability of the serially correlated monetary shock has a similar interpretation, which we also estimate to have fallen by over 50 percent. One difference, however, from the results in Cogley et. al. is that we find monetary policy to be more aggressive in fighting inflation in the 1960s than in the 1970s, which has the added effect of further lowering inflation persistence. Cogley et. al. restrict the monetary policy response to inflation to be the same throughout the 1960s and 1970s, so do not attempt to identify the source of the relatively low persistence in the 1960s relative to the 1970s. As a consequence, Cogley et. al. find little difference between the monetary reaction coefficients in the 1960-1979 sample and the 1980-2006 sample. Our results suggest that combining the 1960s and 1970s into one sample essentially mixes two regimes. The implication is that the policy coefficients estimated from the 1960-1979 sample will be closer to estimates based on a post-1980 sample. In contrast, the difference between policy coefficients becomes larger if estimates from a post-1980 sample is compared to a sample that just uses

the 1970s.<sup>23</sup>

Overall, the four-regime MSNK model captures the qualitative shifts in persistence quite well, but does not fully capture the magnitude. In particular, the decline in persistence is smaller in the MSNK model compared to the empirical benchmark. However, the MSNK model does capture the run-up in inflation persistence during the 1970s and its' peak level.

How can we evaluate the relative importance of policy shifts vs. volatility shifts in explaining the rise and fall of inflation persistence? We plot the model implied persistence from the model with only volatility shifts in Figure 8. While the qualitative pattern is similar to the four-regime case, the magnitude of rise in inflation persistence is smaller. As an implication, the model without monetary regime change has difficulty reproducing the swings in persistence implied by the empirical benchmark. To further illustrate the impact monetary policy has on persistence, relative to shifts in volatility, Figure 9 plots the model-implied measure of inflation persistence conditioning on both the low- and high-volatility regimes. For example, the right-most vertical dashed line corresponds to the reaction to inflation in the more-active monetary regime. A shift in the volatility regime causes a rise in inflation equal to the vertical distance between the red-dashed curve and solid black curve. In contrast, a shift in policy causes a movement along a curve. Conditioning on the low-volatility regime (i.e. the red-dashed curve), a shift in monetary policy from the more- to less-active regime would cause persistence to rise along the red-dashed curve. The result of the monetary regime change is a rise in persistence that is roughly double the rise caused by a shift in the volatility regime. The general conclusion is that shift in the monetary rule, conditional on the posterior means of the structural parameters, plays a greater role than shifts in volatility. However, the impacts on persistence of shifts in volatility are non-trivial and volatility shifts still contribute, though more modestly, to movements in inflation persistence.

## 6 Conclusion

This paper reports the results of Bayesian estimation of MSNK models with regime switching in monetary policy and shock volatility. Overall, U.S. data favors the model with independent switching in both the monetary policy and shock-volatility regimes. We show that the population moment describing the serial correlation of the inflation is a weighted average of the autocorrelation parameters of the exogenous shocks, where the weights depend on the different monetary and shock-volatility regimes.

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<sup>23</sup>Of course, estimating a single-regime model just using the 1970s period is likely to yield indeterminacy. However, the linear system representing the regime-switching MSNK model enlarges the determinate parameter space, so permits brief excursions into passive monetary policy without inducing indeterminacy.

Consequently, changes in either of the regimes reshuffle the weights over these serial correlation parameters and alter the serial correlation properties of inflation. A shift to the more-active monetary regime reduces the weight on the more persistent shocks, so lowers the serial correlation of inflation. Similarly, a shift to the low-volatility regime reduces the weight on the more persistent shocks and also contributes to reducing inflation persistence. Estimates indicate that inflation persistence began rising in the late 1960s and peaked around the Volcker disinflation. This ‘low-high-low’ pattern of inflation persistence also emerges from a reduced-form econometric model and is consistent with empirical evidences documented in Cogley, Primiceri, and Sargent (2009). Our general conclusion is that shifts in monetary policy play a larger role in affecting inflation persistence, though changes in volatility still play a non-trivial role.

## 7 Technical Appendix

### 7.1 Determinacy Restrictions

The MSNK model is inherently nonlinear, which complicates the conditions for determinacy. Following Davig and Leeper (2007), we obtain the linear representation of the MSNK model and restrict our attention to the uniqueness of bounded solutions in the linear representation. For the illustrative purpose, first assume all shocks are i.i.d. and there are only two regimes. Then we can rewrite expectations as follows:

$$E_t \pi_{t+1} = E[\pi_{t+1} | s_t = i, \Omega_t^{-s}] = p_{i1} E[\pi_{1t+1} | \Omega_t^{-s}] + p_{i2} E[\pi_{2t+1} | \Omega_t^{-s}], \quad (33)$$

$$E_t y_{t+1} = E[y_{t+1} | s_t = i, \Omega_t^{-s}] = p_{i1} E[y_{1t+1} | \Omega_t^{-s}] + p_{i2} E[y_{2t+1} | \Omega_t^{-s}], \quad (34)$$

where  $\pi_{it} = \pi_t(s_t = i, \varepsilon_t)$ ,  $y_{it} = y_t(s_t = i, \varepsilon_t)$ , and  $\varepsilon_t = [\hat{a}_t \ \hat{u}_t \ \hat{e}_t]'$  for  $i = 1, 2$ .<sup>24</sup> The information set,  $\Omega_t^{-s} = s_{t-1}, \dots$ , excludes the current regime, so  $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$ . Distributing probability mass across the different conditional expectations for inflation, as in (33) – (34), is the same approach as in Gordon and St-Amour (2000) and Bansal and Zhou (2002).

Next, define the forecast errors

$$\eta_{jt+1}^\pi = \pi_{jt+1} - E[\pi_{jt+1} | \Omega_t^{-s}], \quad (35)$$

$$\eta_{jt+1}^x = x_{jt+1} - E[x_{jt+1} | \Omega_t^{-s}], \quad (36)$$

for  $j = 1, 2$ . Substituting expectations, (33) – (34), and the policy rule, (17), into (15) – (16) yields the stacked system

$$AY_t = BY_{t-1} + A\eta_t + Cu_t, \quad (37)$$

where

$$Y_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ x_{1t} \\ x_{2t} \end{bmatrix}, \quad \eta_t = \begin{bmatrix} \eta_{1t}^\pi \\ \eta_{2t}^\pi \\ \eta_{1t}^x \\ \eta_{2t}^x \end{bmatrix}, \quad u_t = \begin{bmatrix} \varepsilon_{at} \\ \varepsilon_{ut} \\ \varepsilon_{et} \end{bmatrix}, \quad (38)$$

and  $A$ ,  $B$  and  $C$  are conforming matrices consisting of private sector parameters, policy parameters and the transition matrix. The stacked system has constant coefficient matrices, yet captures the impact potential regime changes in monetary policy

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<sup>24</sup>Whether shocks are i.i.d. or serially correlated does not matter for determinacy, so is made here for convenience.

have on expectation formation. Further, standard methods for solving linear rational expectations systems are applicable to (37), such as Blanchard and Kahn (1980).<sup>25</sup>

Necessary and sufficient conditions for determinacy, which is the existence of a unique bounded solution to (37), is that all the generalized eigenvalues of  $(B, A)$  lie inside the unit circle. The determinacy conditions are intuitive. First, the passive monetary regime cannot be too passive, meaning the response to inflation can be less than one, but still has to be above some minimum threshold. And second, the passive regime cannot be too persistent, meaning that the expected duration of the regime must be below a given threshold. The determinacy conditions are joint restrictions over both monetary regimes, so the parameters governing the active regime affect the determinacy restrictions over the passive regime. Therefore, the more persistent or active the active regime is, the more persistent or passive the passive regime can be.

One caveat using the linear representation to check the determinacy conditions is that there might be non-MSV solutions of the MSNK model even if determinacy restrictions are satisfied in the linear representation.<sup>26</sup> However, there are no known (necessary and sufficient) determinacy conditions for the class of bounded solutions of MSNK models. Farmer, Waggoner, and Zha (2009b) and Cho (2009) suggest to consider mean-square stable solutions rather than bounded solutions and provide determinacy conditions within the class of mean-square stable solutions of a quasi-linear version of MSNK models.<sup>27</sup> While these advances are certainly interesting, restricting our attention to the class of bounded solutions is of interest not only because of the uniqueness of equilibrium, but also because we can justify the log-linear approximation of the original fully nonlinear model only in the local neighborhood of the steady state by the implicit function theorem.<sup>28</sup> For this reason, we believe that the linear representation is useful for the analysis of MSNK models given our current knowledge.

## 7.2 Convergence Diagnostics of MCMC Output

We start a MCMC chain initialized at the (local) posterior mode found by a numerical optimization routine. We compute the mean ( $m$ ) and the covariance ( $c$ ) matrix based

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<sup>25</sup>McCallum (2004) proves the equivalence between MSV solutions and determinate (i.e. unique and non-explosive) solutions from solving a system of linear expectations difference equations for purely forward looking models. Davig and Leeper (2007) show the equivalence between the MSV solution and determinate solution of the stacked system in regime-switching rational expectation models.

<sup>26</sup>See Farmer, Waggoner, and Zha (2009a).

<sup>27</sup>The terminology is from Davig and Leeper (2009). The quasi-linear version is based on the log-linear approximation of the original MSNK model but keep nonlinearities due to regime-switching parts unlike the linear representation.

<sup>28</sup>See Woodford (2003) for this point.

on 1 million draws from the MCMC chain. Then we run multiple MCMC chains initialized at widely dispersed points around the mean ( $\bar{\vartheta}$ ) by using the covariance matrix ( $C$ ) as the scaling matrix in the proposal density of the Chain. We burn-in the first 100,000 draws and use the remaining 1,400,000 draws for the posterior inference in the four-regime model.<sup>29</sup> We compute potential scale reduction factors (PSRFs) as suggested by Brooks and Gelman (1998) to assess the convergence of MCMC output. PSRFs below 1.1 or 1.2 are regarded as an evidence for the convergence of MCMC output. Using three chains starting from overly dispersed points, I obtain values below 1.1 for all the parameters in the four-regime model and the model with only policy shifts. In the model with only volatility shifts, the PSRF for the persistence of markup shock is 1.27 but all the other parameters have PSRFs below 1.1. Other convergence diagnostics such as tests of the equality of split-sample means or visual check on trace plots also indicate that MCMC output stabilize.

### 7.3 Marginal Likelihood

Marginal likelihood can be computed by the Monte Carlo integration of a probability density function of  $\vartheta$  as follows.

$$p(Z^T)^{-1} = \int \frac{h(\vartheta)}{p(Z^T|\vartheta)p(\vartheta)} p(\vartheta|Z^T) d\vartheta \longrightarrow \hat{p}(Z^T)^{-1} = \frac{1}{N} \sum_{i=1}^N \frac{h(\vartheta_i)}{p(Z^T|\vartheta_i)p(\vartheta_i)}$$

where  $\vartheta_i$  is a posterior draw. Geweke (1999) proposes an implementation with  $h(\cdot)$  a Gaussian density around the posterior mean. Sims, Waggoner, and Zha (2008) show that the Gaussian approximation for the proposal density to compute the marginal likelihood can be misleading and numerically unstable due to the non-Gaussian posterior distribution of parameters in models with regime-switching. They suggest the following elliptical distribution as an alternative.

$$g(\vartheta) = \frac{\Gamma(k/2)}{(2\pi)^{k/2} |\det(\bar{\Omega})|} \frac{f(r)}{r^{k-1}}, \quad r = \sqrt{(\vartheta - \bar{\vartheta})' \bar{\Omega}^{-1} (\vartheta - \bar{\vartheta})}, \quad \bar{\Omega} = c \sqrt{\Omega}$$

where  $f(r)$  is any one-dimensional density defined on the positive reals which we can estimated based on posterior draws.  $\bar{\Omega}$  is the sample covariance matrix computed by centering out posterior draws from the posterior mode. And  $c$  is a scaling parameter. Then the weighting function can be constructed as a truncated  $g(\cdot)$  where the area with a very low posterior probability is truncated.

$$h(\vartheta) = \frac{\Xi_{\Theta_L}}{q_L} g(\vartheta), \quad \Theta_L = \{\vartheta : p(Z^T|\vartheta)p(\vartheta) > L\}$$

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<sup>29</sup>We burn-in the first 100,000 draws and use the remaining 1,900,000 draws in the model with only volatility shifts while we burn-in 500,000 draws use the remaining 500,000 draws in the model with only policy shifts.

where  $\Xi_{\Theta_L}$  is an indicator function which is 1 if  $\vartheta$  belong to  $\Theta_L$  and 0 otherwise. While Sims, Waggoner, and Zha (2008) provide an example where this methods is more robust than Geweke (1999), it does not work well for our purpose. Particularly,  $g(\cdot)$  turns out to be very sensitive to the choice of a scaling parameter of  $\bar{\Omega}$ .<sup>30</sup> In contrast, Geweke (1999)'s method provides more reliable estimates of marginal likelihood in our cases. And the marginal likelihood computed by Sims, Waggoner, Zha (2008)'s method also indicates the the best fitting model is the four regime model with a constant inflation target as implied by the marginal likelihood computed by Geweke (1999)'s method.

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<sup>30</sup>Sims, Waggoner, and Zha (2008) acknowledge that the choice of the scaling parameter is importatnt in the implementation of their method.

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Table 1: PRIOR DISTRIBUTION

Parameters	Domain	Density	Para(1)	Para(2)	Regime Spec
$\alpha$	$\mathbb{R}^+$	Gamma	1.5	.25	P2
$\alpha_1$	$\mathbb{R}^+$	Gamma	2	.25	P1, P3
$\alpha_2$	$\mathbb{R}^+$	Gamma	1	.1	P1, P3
$\gamma$	$\mathbb{R}^+$	Gamma	.08	.05	P2
$\gamma_1$	$\mathbb{R}^+$	Gamma	.1	.05	P1, P3
$\gamma_2$	$\mathbb{R}^+$	Gamma	.1	.05	P1, P3
$\kappa$	$\mathbb{R}^+$	Gamma	.5	.2	P1, P2, P3
$\beta$	[0,1)	Beta	.998	.001	P1, P2, P3
$\tau$	$\mathbb{R}^+$	Gamma	1.5	.4	P1, P2, P3
$\lambda$	$\mathbb{R}^+$	Gamma	.005	.001	P1, P2, P3
$\Pi$	$\mathbb{R}^+$	Gamma	.0086	.001	P1, P2, P3
$\rho_a$	[0,1)	Beta	.3	.2	P1, P2, P3
$\rho_u$	[0,1)	Beta	.7	.2	P1, P2, P2
$\rho_e$	[0,1)	Beta	.5	.2	P1, P2, P3
$\sigma_a$	$\mathbb{R}^+$	Inverse Gamma	.004	4	P1
$\sigma_{a,1}$	$\mathbb{R}^+$	Inverse Gamma	.006	4	P2, P3
$\sigma_{a,2}$	$\mathbb{R}^+$	Inverse Gamma	.003	4	P2, P3
$\sigma_u$	$\mathbb{R}^+$	Inverse Gamma	.003	4	P1
$\sigma_{u,1}$	$\mathbb{R}^+$	Inverse Gamma	.004	4	P2, P3
$\sigma_{u,2}$	$\mathbb{R}^+$	Inverse Gamma	.002	4	P2, P3
$\sigma_e$	$\mathbb{R}^+$	Inverse Gamma	.003	4	P1
$\sigma_{e,1}$	$\mathbb{R}^+$	Inverse Gamma	.004	4	P2, P3
$\sigma_{e,2}$	$\mathbb{R}^+$	Inverse Gamma	.002	4	P2, P3
$\ln A_0$	$\mathbb{R}$	Normal	9.546	.1	
$y^*$	$\mathbb{R}$	Normal	-0.067	.01	P1, P2, P3
$p_{11}$	[0,1)	Beta	.9	.05	P1, P2, P3
$p_{22}$	[0,1)	Beta	.9	.05	P1, P2, P3
$q_{11}$	[0,1)	Beta	.9	.05	P3
$q_{22}$	[0,1)	Beta	.9	.05	P3

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions;  $s$  and  $\nu$  for the Inverse Gamma distribution, where  $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$ ,  $a$  and  $b$  for the Uniform distribution from  $a$  to  $b$ . P1 allows switching only in monetary policy coefficients while P2 allows switching coefficients only in variance parameters of shocks. P3 allows switching for both policy coefficients and variances.

Table 2: POSTERIOR DISTRIBUTION

Parameters	Prior 90% Interval	Posterior 90% Interval		
		P1	P2	P3
$\alpha$	[1.10,1.91]		[1.41,1.68]	
$\alpha_1$	[1.59,2.41]	[1.76,2.37]		[1.56,2.05]
$\alpha_2$	[0.84,1.16]	[0.86,1.12]		[1.04,1.25]
$\gamma$	[0.008,0.150]		[0.019,0.124]	
$\gamma_1$	[0.025,0.174]	[0.056,0.283]		[0.022,0.160]
$\gamma_2$	[0.024,0.174]	[0.017,0.123]		[0.021,0.137]
$\kappa$	[0.181,0.804]	[0.291,0.661]	[0.258,0.552]	[0.281,0.617]
$\beta$	[0.9965,0.9995]	[0.9984,0.9997]	[0.9984,0.9998]	[0.9986,0.9997]
$\tau$	[0.851,2.134]	[2.952,4.596]	[3.653,4.836]	[3.364, 4.848]
$\lambda$	[0.0033,0.0066]	[0.0036,0.0051]	[0.0040,0.0058]	[0.0042,0.0058]
$\Pi$	[0.0070,0.0103]	[0.0061,0.0078]	[0.0070,0.0090]	[0.0063,0.0081]
$\rho_a$	[0.001,0.592]	[0.000,0.076]	[0.000,0.143]	[0.000,0.097]
$\rho_u$	[0.407,0.999]	[0.929,0.967]	[0.923,0.941]	[0.932,0.967]
$\rho_i$	[0.172,0.828]	[0.610,0.712]	[0.662,0.756]	[0.635,0.739]
$\sigma_a$	[0.0022,0.0080]	[0.0095,0.0112]		
$\sigma_{a,1}$	[0.0031,0.0112]		[0.0131,0.0178]	[0.0124,0.0181]
$\sigma_{a,2}$	[0.0016,0.0060]		[0.0065,0.0087]	[0.0060,0.0080]
$\sigma_u$	[0.0027,0.0099]	[0.0016,0.0026]		
$\sigma_{u,1}$	[0.0022,0.0079]		[0.0034,0.0052]	[0.0024,0.0041]
$\sigma_{u,2}$	[0.0011,0.0039]		[0.0011,0.0016]	[0.0009,0.0014]
$\sigma_e$	[0.0016,0.0059]	[0.0042,0.0055]		
$\sigma_{e,1}$	[0.0022,0.0079]		[0.0055,0.0076]	[0.0051,0.0072]
$\sigma_{e,2}$	[0.0010,0.0039]		[0.0029,0.0037]	[0.0027,0.0038]
$\ln A_0$	[9.381,9.719]	[9.517,9.548]	[9.524,9.529]	[9.513,9.546]
$y^*$	[-0.0832,-0.0504]	[-0.0847,-0.0535]	[-0.0654,-0.0589]	[-0.0836,-0.0511]
$p_{11}$	[0.82,0.98]	[0.95,0.99]		[0.95,0.99]
$p_{22}$	[0.82,0.98]	[0.88,0.95]		[0.92,0.98]
$q_{11}$	[0.82,0.98]		[0.84,96]	[0.85,0.96]
$q_{22}$	[0.82,0.98]		[0.91,0.98]	[0.91,0.97]

Notes: P1 allows switching only in monetary policy coefficients while P2 allows switching coefficients only in variance parameters of shocks. P3 allows switching for both policy coefficients and variances.

Table 3: LOG MARGINAL DATA DENSITIES

P1	P2	P3	Inflation Target
2,699.2	2,719.5	2,727.8	Constant
2,706.9	2,724.6	2,723.2	Random Walk

Table 4: MSNK MODEL-IMPLIED INFLATION PERSISTENCE

Model	Description	P1	P2	P3
$\rho_\pi(\pi s_t = 1, r_t = 1, \forall t)$	[ High $\alpha$ / High $\sigma$ ]	[.73,.85]	[.81,.87]	[.77,.86]
$\rho_\pi(\pi s_t = 1, r_t = 2, \forall t)$	[ High $\alpha$ / Low $\sigma$ ]	-	[.76,.82]	[.74,.82]
$\rho_\pi(\pi s_t = 2, r_t = 1, \forall t)$	[ Low $\alpha$ / High $\sigma$ ]	[.83,.92]	-	[.84,.91]
$\rho_\pi(\pi s_t = 2, r_t = 2, \forall t)$	[ Low $\alpha$ / Low $\sigma$ ]	-	-	[.79,.87]

*Notes:* The posterior 90% probability interval of the implied autocorrelation of inflation is reported in [ ].  $s_t = 1$  corresponds to the more-active monetary policy and  $r_t = 1$  corresponds to the high-volatility regime.

Figure 1: ACTUAL VS. PREDICTED INFLATION FROM FOUR-REGIME MSNK MODEL (GDP DEFLATOR)

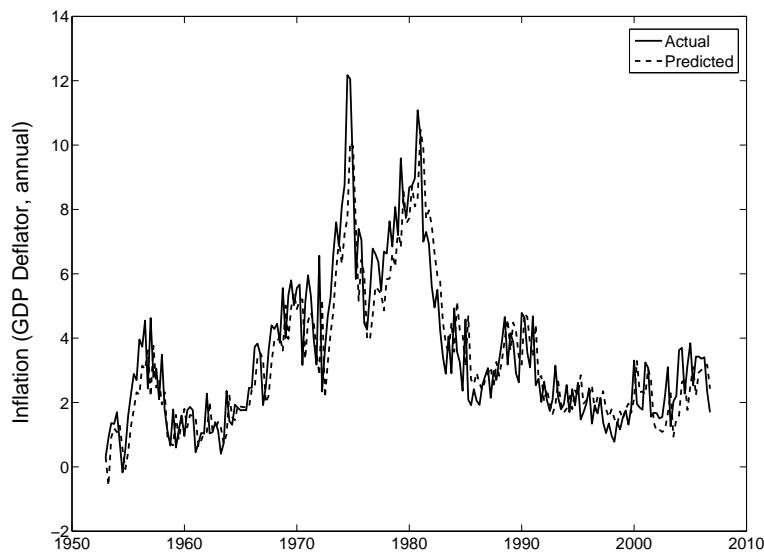


Figure 2: POSTERIOR EXPECTED VALUES OF THE SMOOTHED PROBABILITY FOR THE ACTIVE MONETARY POLICY REGIME (2-REGIME MODEL)

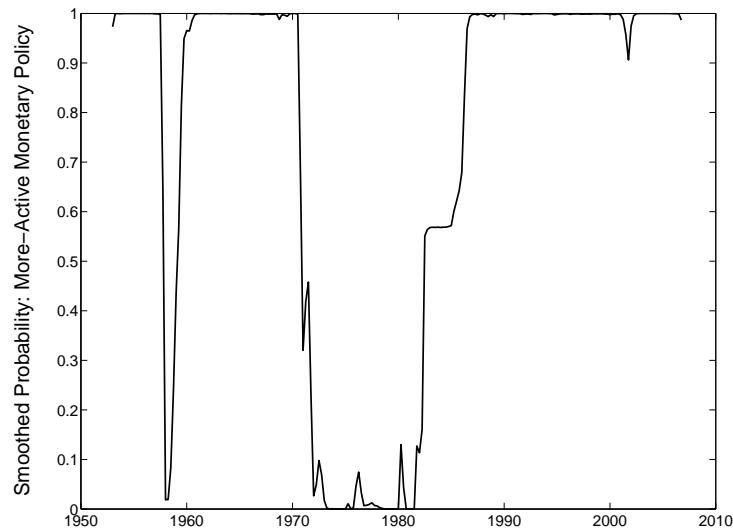


Figure 3: POSTERIOR EXPECTED VALUES OF THE SMOOTHED PROBABILITY FOR THE LOW-VOLATILITY REGIME (2-REGIME MODEL)

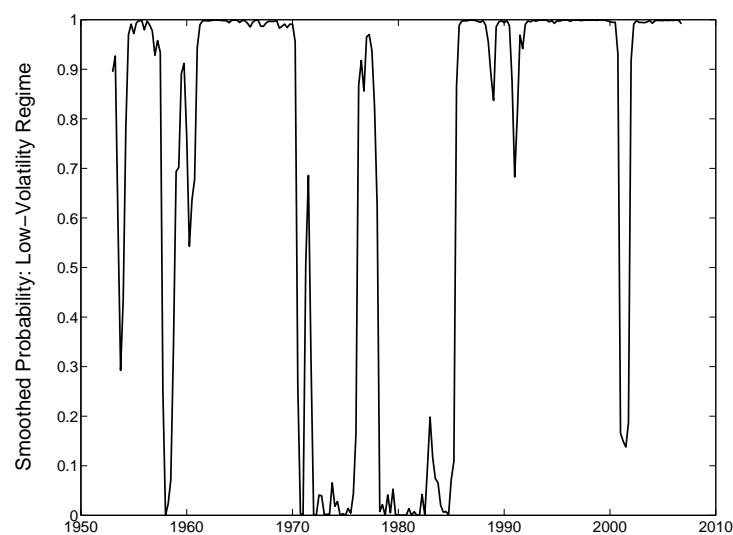


Figure 4: POSTERIOR EXPECTED VALUES OF THE SMOOTHED PROBABILITIES  
(FOUR-REGIME MODEL)

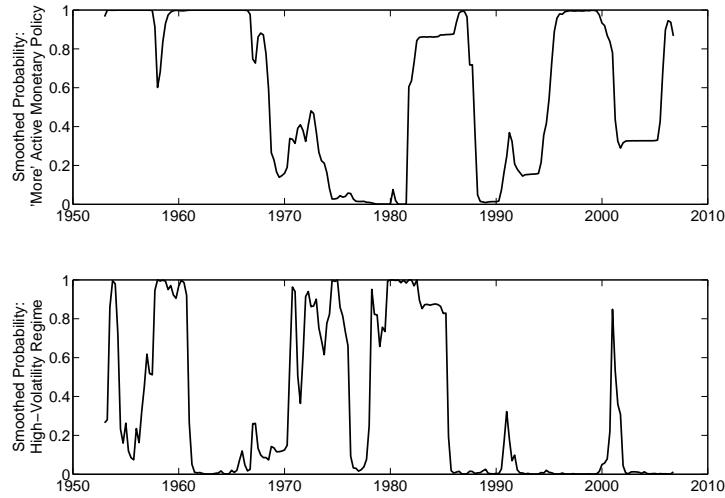


Figure 5: WEIGHTS ON PERSISTENCE PARAMETERS IN INFLATION AUTOCORRELATION FUNCTION

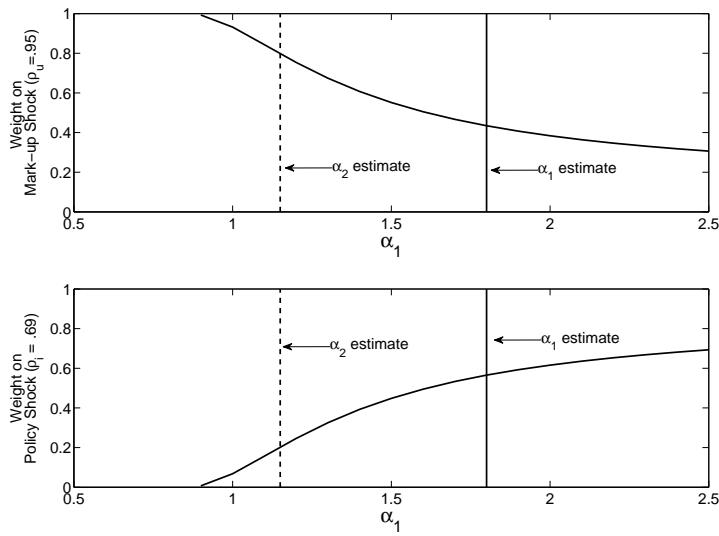


Figure 6: INFLATION PERSISTENCE BASED ON MARKOV-SWITCHING TIME SERIES MODEL (90% BANDS)

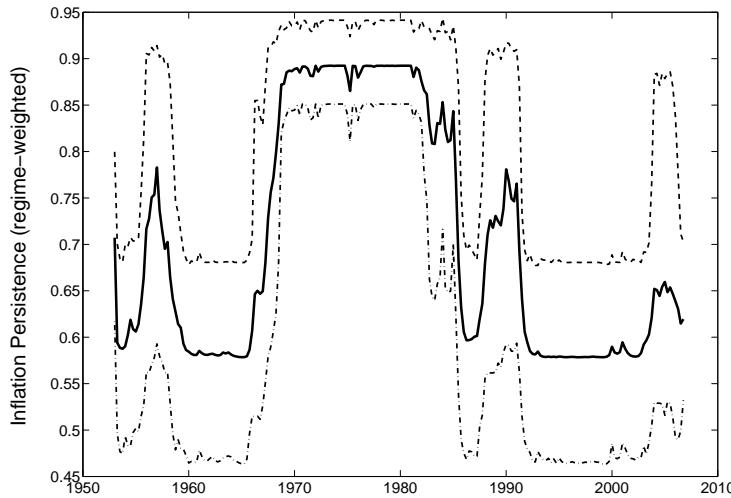


Figure 7: MSNK MODEL-IMPLIED INFLATION PERSISTENCE (90% BANDS, 4-REGIME MODEL)

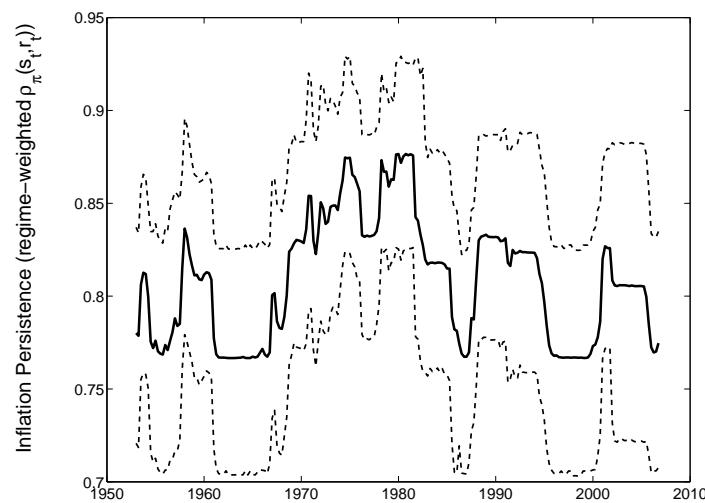


Figure 8: MSNK MODEL-IMPLIED INFLATION PERSISTENCE (90% BANDS, 2-VOLATILITY REGIME MODEL)

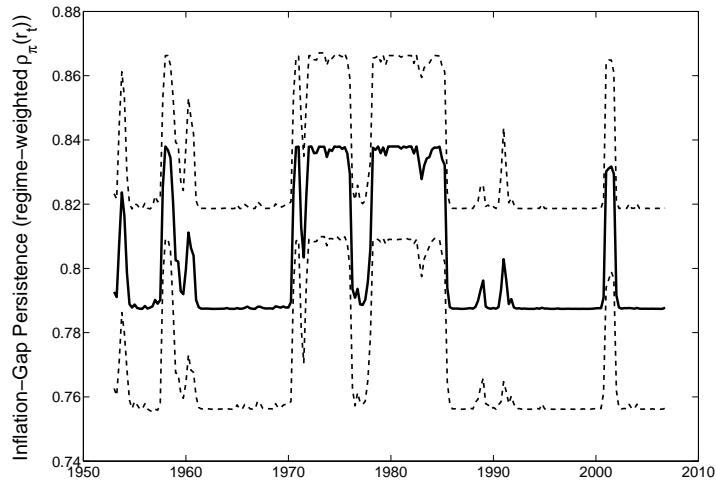


Figure 9: MSNK MODEL-IMPLIED INFLATION PERSISTENCE (PARAMETERS ARE SET TO POSTERIOR MEANS, EXCEPT  $\alpha_1$  WHICH VARIES)

