Online Appendix: A Dual Mandate Can Support Price Stability*

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A Detailed Derivations of Findings From Section 2

A.1 Outcomes Under Single Price Stability Mandate

The output gap, inflation gap and the policy rule following the equations listed below:

$$x_t = E_t x_{t+1} - \left(i_t - E_t \pi_{t+1} - r_t^n\right) \tag{A.1}$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa \, x_t \tag{A.2}$$
$$i_t = \phi_\pi \pi_t \tag{A.3}$$

Note that $r_t^n = 0$ in periods 1, 3 and 4. However, in period 2 $r_t^n = \Delta$ or $-\Delta$. We solve this model backwards starting from period 4. All agents in the economy are fully aware of the state of the economy and form their expectations on all available knowledge. Agents know the economy ceases to exist after period 4 and therefore $x_5 = \pi_5 = 0$. Additionally, it follows that $E_4 x_5 = E_4 \pi_5 = 0$. Using this, we can now solve for period 4 outcomes. Solving for the the inflation gap yields:

$$\pi_4 = \beta E_4 \pi_5 + \kappa x_4 = \kappa x_4 \tag{A.4}$$

Substituting Equation A.4 into Equation A.1 yields:

$$\begin{aligned} x_4 &= E_4 x_5 - \left(i_4 - E_4 \pi_5 - r_4^n\right) \\ x_4 &= 0 - (\phi_\pi \pi_4 - 0 - 0) \\ x_4 &= -\phi_\pi \pi_4 = -\phi_\pi \kappa x_4 = 0 \end{aligned} \tag{A.5}$$

Taken together, for period 4, we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, we know that $E_3 \pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Solving for the inflation gap in period 3 yields:

$$\pi_3 = \beta E_3 \pi_4 + \kappa x_3 = \kappa x_3 \tag{A.6}$$

Solving for the output gap yields:

$$\begin{aligned} x_3 &= E_3 x_4 - \left(i_3 - E_3 \pi_4 - r_3^n\right) \\ x_3 &= 0 - \left(\phi_\pi \pi_3 - 0 - 0\right) \\ x_3 &= -\phi_\pi \pi_3 = -\phi_\pi \kappa x_3 = 0 \end{aligned} \tag{A.7}$$

Taken together for period 3 we obtain the following outcomes: $x_3 = \pi_3 = 0$ and $E_2\pi_3 = E_2 x_3 = 0$. We now solve for period 2, in the same manner as we did for the previous two periods. Note that in period 2 $r_t^n = \Delta$ or $r_t^n = -\Delta$ with equal probability. We therefore solve for period 2 outcomes twice. Once under the assumption that $r_t^n = \Delta$ and the other when $r_t^n = -\Delta$. Solving for outcomes under the $+\Delta$ case yields us the following.

$$\begin{aligned} \pi_2^\Delta &= \beta E_2 \pi_3 + \kappa x_2^\Delta \\ \pi_2^\Delta &= \kappa\, x_2^\Delta \end{aligned} \tag{A.8}$$

The output gap in the + Δ case is given by:

$$\begin{aligned} x_2^\Delta &= E_2\, x_3 - \Big(i_2 - E_2\, \pi_3 - r_2^n\Big) \\ x_2^\Delta &= 0 - \Big(\phi_\pi \pi_2^\Delta - 0 - \Delta\Big) \\ x_2^\Delta &= -\phi_\pi \pi_2^\Delta + \Delta \\ x_2^\Delta &= -\phi_\pi \kappa\, x_2^\Delta + \Delta \\ x_2^\Delta + \phi_\pi \kappa\, x_2^\Delta &= \Delta \\ x_2^\Delta (1 + \phi_\pi \kappa) &= \Delta \\ x_2^\Delta &= \frac{\Delta}{1 + \phi_\pi\, \kappa} \end{aligned} \tag{A.9}$$

Solving for π_2^Δ yields:

$$\pi_2^\Delta = \frac{\Delta \kappa}{1 + \phi_\pi \kappa} \tag{A.10}$$

For the -Δ case we repeat the same steps as above with one key distinction i.e. r_2^n = -Δ. Since the shocks are symmetric, this yields us the following equations for the inflation and output gap respectively:

$$\pi_2^{-\Delta} = \frac{-\Delta \kappa}{1 + \phi_\pi\, \kappa} \tag{A.11}$$

$$x_2^{-\Delta} = \frac{-\Delta}{1 + \phi_\pi\, \kappa} \tag{A.12}$$

We use the values obtained for x_2^Δ, π_2^Δ, $x_2^{-\Delta}$, and $\pi_2^{-\Delta}$ from above to calculate $E_1\pi_2$ and $E_1 x_2$.

$$\begin{aligned} E_1\pi_2 &= \pi_2^\Delta\, Prob\,(\pi_2^\Delta) + \pi_2^{-\Delta}\, Prob\,(\pi_2^{-\Delta}) \\ E_1\pi_2 &= \Big(\frac{\Delta\kappa}{1 + \phi_\pi\, \kappa} \times \frac{1}{2}\Big) + \Big(\frac{-\Delta\kappa}{1 + \phi_\pi\, \kappa} \times \frac{1}{2}\Big) \\ E_1\pi_2 &= 0 \end{aligned} \tag{A.13}$$

We use the same methodology to calculate $E_1 x_2$.

$$\begin{aligned} E_1 x_2 &= x_2^{\Delta}\, Prob\,(x_2^{\Delta}) + x_2^{-\Delta}\, Prob\,(x_2^{-\Delta}) \\ E_1 x_2 &= \Big(\frac{\Delta}{1+\phi_\pi\,\kappa} \times \frac{1}{2}\Big) + \Big(\frac{-\Delta}{1+\phi_\pi\,\kappa} \times \frac{1}{2}\Big) \\ E_1 x_2 &= 0 \end{aligned} \tag{A.14}$$

In period 1, using the same methodology as we did for the previous periods, we observe the following.

$$\begin{aligned} \pi_1 &= \beta E_1 \pi_2 + \kappa x_1 \\ \pi_1 &= \kappa x_1 \end{aligned} \tag{A.15}$$

Solving for the output gap yields:

$$\begin{aligned} x_1 &= E_1 x_2 - \Big(i_1 - E_1 \pi_2 - r_1^n\Big) \\ x_1 &= 0 - \Big(\phi_\pi \pi_1 - 0 - 0\Big) \\ x_1 &= -\phi_\pi \pi_1 \\ x_1 &= -\phi_\pi \kappa x_1 \\ x_1 &= 0 \end{aligned} \tag{A.16}$$

This implies that $x_1 = \pi_1 = 0$.

A.2 Outcomes Under a Dual Mandate

Under the dual mandate the IS Curve and the Phillips Curve remain the same, but we now assume that the monetary authority follows a dual mandate rule in period 3 only.

$$i_t = \phi_\pi \pi_t + \phi_x x_{t-1} \tag{A.17}$$

We now solve our four period model using this new policy rule. In periods 1, 2 ,and 4 we assume the model follows the policy rule as given by Equation A.3. As before, $r_t^n = 0$ in periods 1, 3, and 4, while in period 2 $r_t^n = \Delta$ or $r_t^n = -\Delta$ with equal likelihood.

Since this is a finite period, agents know the economy ceases to exist after period 4. Therefore, $x_5 = \pi_5 = 0$. Additionally, from our assumption it follows that $E_4 x_5 = E_4 \pi_5 = 0$.

Using this we can now solve for period 4 outcomes. Solving for the the inflation gap yields:

$$\begin{aligned}\pi_4 &= \beta E_4 \pi_5 + \kappa x_4 \\ \pi_4 &= \kappa x_4\end{aligned} \tag{A.18}$$

Solving for the output gap yields:

$$\begin{aligned}x_4 &= E_4 x_5 - \Big(i_4 - E_4 \pi_5 - r_4^n\Big) \\ x_4 &= 0 - \Big(\phi_\pi \pi_4 - 0 - 0) \\ x_4 &= -\phi_\pi \pi_4 \\ x_4 &= -\phi_\pi \kappa x_4 \\ x_4 &= 0\end{aligned} \tag{A.19}$$

Taken together for period 4 we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, from our key assumption we can conclude that $E_3\pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Note that in period 3, the policy rule follows the dual mandate as described in Equation 11. Solving for the inflation gap yields:

$$\begin{aligned}\pi_3 &= \beta E_3 \pi_4 + \kappa x_3 \\ \pi_3 &= \kappa\, x_3\end{aligned} \tag{A.20}$$

Solving for the output gap yields:

$$\begin{aligned}x_3 &= E_3\, x_4 - \Big(i_3 - E_3\, \pi_4 - r_{23}^n\Big) \\ x_3 &= 0 - \Big(\phi_\pi \pi_3 + \phi_x x_2 - 0 - 0\Big) \\ x_3 &= -\phi_\pi \pi_3 - \phi_x x_2 \\ x_3 &= -\phi_\pi \kappa x_3 - \phi_x x_2 \\ x_3 + \phi_\pi \kappa\, x_3 &= -\phi_x x_2 \\ x_3(1 + \phi_\pi \kappa) &= -\phi_x x_2 \\ x_3 &= -\frac{\phi_x}{1 + \phi_\pi \kappa} x_2\end{aligned} \tag{A.21}$$

Substituting Equation (A.21) into Equation (A.20) expresses the period 3 inflation gap as a function of the period 2 output gap:

$$\pi_3 = -\frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2 \tag{A.22}$$

Before we solve for 2nd period, we need to reiterate a key point. Since participants have complete knowledge about the economy, their expectations accurately reflect the state of the economy currently. Therefore, $E_2 x_3 = x_3$ and $E_2 \pi_3 = \pi_3$.

In period 2, when the economy experiences positive shock represented by $+\,\Delta$ the inflation gap takes the following shape:

$$\begin{aligned}
\pi_2^\Delta &= \beta E_2 \pi_3 + \kappa x_2^\Delta \\
\pi_2^\Delta &= \beta\Big(-\frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2^\Delta\Big) + \kappa\, x_2^\Delta \\
\pi_2^\Delta &= \kappa x_2^\Delta \underbrace{\Big(1 - \frac{\beta \phi_x}{1+\phi_\pi \kappa}\Big)}_{\Psi^D} \\
\pi_2^\Delta &= \Psi^D \kappa x_2^\Delta
\end{aligned} \tag{A.23}$$

Solving for the output gap equation yields the following:

$$\begin{aligned}
x_2^\Delta &= E_2\, x_3 - \Big(i_2 - E_2\, \pi_3 - r_2^n\Big) \\
x_2^\Delta &= -\frac{\phi_x}{1+\phi_\pi \kappa} x_2^\Delta - \Big(\phi_\pi \pi_2^\Delta + \frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2^\Delta - \Delta\Big) \\
x_2^\Delta &= -\frac{\phi_x}{1+\phi_\pi \kappa} x_2^\Delta - \phi_\pi \Psi^D \kappa x_2^\Delta - \frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2^\Delta + \Delta \\
x_2^\Delta + \frac{\phi_x}{1+\phi_\pi \kappa} x_2^\Delta + \phi_\pi \Psi^D \kappa x_2^\Delta + \frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2^\Delta &= \Delta \\
\Big(1 + \phi_\pi \kappa \Psi^D + \phi_x \frac{1+\kappa}{1+\phi_\pi \kappa}\Big) x_2^\Delta &= \Delta \\
x_2^\Delta &= \Big(\frac{1}{1 + \phi_\pi \kappa \Psi^D + \phi_x \frac{1+\kappa}{1+\phi_\pi \kappa}}\Big)\Delta
\end{aligned} \tag{A.24}$$

The inflation gap in period 2, therefore takes the following shape:

$$\pi_2^\Delta = \Big(\frac{\Psi^D \kappa}{1 + \phi_\pi \kappa \Psi^D + \phi_x \frac{1+\kappa}{1+\phi_\pi \kappa}}\Big)\Delta \tag{A.25}$$

Since the shocks are symmetric, the output gap and the inflation gap in the - Δ case are as follows:

$$x_2^{-\Delta} = -\Big(\frac{1}{1+\phi_\pi\kappa\Psi^D+\phi_x\frac{1+\kappa}{1+\phi_\pi\kappa}}\Big)\Delta \tag{A.26}$$

$$\pi_2^{-\Delta} = \Big(\frac{-\Psi^D\kappa}{1+\phi_\pi\kappa\Psi^D+\phi_x\frac{1+\kappa}{1+\phi_\pi\kappa}}\Big)\Delta \tag{A.27}$$

We use the values obtained for x_2^{Δ}, π_2^{Δ}, $x_2^{-\Delta}$, and $\pi_2^{-\Delta}$ from above to calculate $E_1\pi_2$ and E_1x_2.

$$\begin{aligned}
E_1\pi_2 &= \pi_2^{\Delta}\,Prob\,(\pi_2^{\Delta}) + \pi_2^{-\Delta}\,Prob\,(\pi_2^{-\Delta})\\
E_1\pi_2 &= \Big(\frac{\Psi^D\kappa\Delta}{1+\phi_\pi\kappa\Psi^D+\phi_x\frac{1+\kappa}{1+\phi_\pi\kappa}}\Big)\times\Big(\frac{1}{2}\Big) + \Big(\frac{-\Psi^D\kappa\Delta}{1+\phi_\pi\kappa\Psi^D+\phi_x\frac{1+\kappa}{1+\phi_\pi\kappa}}\Big)\times\Big(\frac{1}{2}\Big)\\
E_1\pi_2 &= 0
\end{aligned} \tag{A.28}$$

$$\begin{aligned}
E_1x_2 &= x_2^{\Delta}\,Prob\,(x_2^{\Delta}) + x_2^{-\Delta}\,Prob\,(x_2^{-\Delta})\\
E_1x_2 &= \Big(\frac{\Delta}{1+\phi_\pi\kappa\Psi^D+\phi_x\frac{1+\kappa}{1+\phi_\pi\kappa}}\Big)\times\Big(\frac{1}{2}\Big) + \Big(\frac{-\Delta}{1+\phi_\pi\kappa\Psi^D+\phi_x\frac{1+\kappa}{1+\phi_\pi\kappa}}\Big)\times\Big(\frac{1}{2}\Big)\\
E_1x_2 &= 0
\end{aligned} \tag{A.29}$$

In period 1, using the same methodology as we did for the previous periods, we observe the following.

$$\begin{aligned}
\pi_1 &= \beta E_1\pi_2 + \kappa x_1\\
\pi_1 &= \kappa x_1
\end{aligned} \tag{A.30}$$

Solving for the output gap yields:

$$\begin{aligned}
x_1 &= E_1x_2 - \Big(i_1 - E_1\pi_2 - r_1^n\Big)\\
x_1 &= 0 - \Big(\phi_\pi\pi_1 - 0 - 0\Big)\\
x_1 &= -\phi_\pi\pi_1\\
x_1 &= -\phi_\pi\kappa x_1\\
x_1 &= 0
\end{aligned} \tag{A.31}$$

This implies that $\pi_1 = x_1 = 0$.

A.3 Connection with Average Inflation Targeting

Using this framework, we can show that the history-dependence introduced by the dual mandate can be observationally equivalent to average inflation targeting. Instead of a dual mandate, we now assume that monetary policy aims to stabilize the some weighted average of inflation in period 3:

$$i_t = \phi_\pi \pi_t + \phi_\pi^a \pi_{t-1}. \tag{A.32}$$

We solve this model backwards starting from period 4. We hold our key assumption that all participants in the economy are fully aware of the state of the economy and form their expectations based on this all available knowledge. Since this is a finite period, participants know that the economy ceases to exist after period 4. Therefore, we know that $x_5 = \pi_5 = 0$. This implies that $E_4 x_5 = E_4 \pi_5 = 0$. Using this we now solve for period 4 outcomes.

$$\begin{aligned} \pi_4 &= \beta E_4 \pi_5 + \kappa x_4 \\ \pi_4 &= \kappa x_4 \end{aligned} \tag{A.33}$$

Solving for the output gap yields:

$$\begin{aligned} x_4 &= E_4 x_5 - \Big(i_4 - E_4 \pi_5 - r_4^n\Big) \\ x_4 &= 0 - \Big(\phi_\pi \pi_4 - 0 - 0) \\ x_4 &= -\phi_\pi \pi_4 \\ x_4 &= -\phi_\pi \kappa x_4 \\ x_4 &= 0 \end{aligned} \tag{A.34}$$

Taken together for period 4 we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, from our key assumption we can conclude that $E_3 \pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Note that in period 3, the policy rule follows the dual mandate as described in Equation 16. Solving for the inflation gap yields:

$$\begin{aligned} \pi_3 &= \beta E_3 \pi_4 + \kappa x_3 \\ \pi_3 &= \kappa \, x_3 \end{aligned} \tag{A.35}$$

Solving for the output gap yields:

$$
\begin{aligned}
x_3 &= E_3\, x_4 - \Big(i_3 - E_3\, \pi_4 - r^n_{23}\Big) \\
x_3 &= 0 - \Big(\phi_\pi \pi_3 + \phi^a_\pi \pi_2 - 0 - 0\Big) \\
x_3 &= -\phi_\pi \pi_3 - \phi^a_\pi \pi_2 \\
x_3 &= -\phi_\pi \kappa x_3 - \phi^a_\pi \pi_2 \\
x_3 + \phi_\pi \kappa\, x_3 &= -\phi^a_\pi \pi_2 \\
x_3(1 + \phi_\pi \kappa) &= -\phi^a_\pi \pi_2 \\
x_3 &= -\frac{\phi^a_\pi}{1 + \phi_\pi \kappa} \pi_2
\end{aligned}
\tag{A.36}
$$

We substitute this back into the equation for the inflation gap to obtain the values of the inflation gap in period 3 as a function of the inflation gap in period 2.

$$
\pi_3 = -\frac{\phi^a_\pi \kappa}{1 + \phi_\pi \kappa} \pi_2 \tag{A.37}
$$

Before we solve for 2nd period, we need to reiterate a key point. Since participants have complete knowledge about the economy, their expectations accurately reflect the state of the economy currently. Therefore, $E_2 x_3 = x_3$ and $E_2 \pi_3 = \pi_3$.

In period 2, when the economy experiences positive shock represented by $+\,\Delta$ the inflation gap takes the following shape.

$$
\begin{aligned}
\pi_2^{\Delta} &= \beta E_2 \pi_3 + \kappa x_2^{\Delta} \\
\pi_2^{\Delta} &= \beta\Big(-\frac{\phi_\pi^a \kappa}{1+\phi_\pi \kappa}\pi_2\Big) + \kappa x_2^{\Delta} \\
\pi_2^{\Delta} + \frac{\beta \phi_\pi^a \kappa}{1+\phi_\pi \kappa}\pi_2 &= \kappa x_2^{\Delta} \\
\pi_2^{\Delta}\Big(1 + \frac{\beta \phi_\pi^a \kappa}{1+\phi_\pi \kappa}\Big) &= \kappa x_2^{\Delta} \\
\pi_2^{\Delta}\Big(\frac{1+\phi_\pi\kappa+\beta \phi_\pi^a \kappa}{1+\phi_\pi \kappa}\Big) &= \kappa x_2^{\Delta} \\
\pi_2^{\Delta} = \underbrace{\Big(\frac{1+\phi_\pi \kappa}{1+\phi_\pi\kappa+\beta \phi_\pi^a \kappa}\Big)}_{\Psi^{AIT}} &\kappa x_2^{\Delta} \\
\pi_2^{\Delta} &= \Psi^{AIT}\kappa x_2
\end{aligned}
\tag{A.38}
$$

$$
\begin{aligned}
x_2^{\Delta} &= E_2\, x_3 - \Big(i_2 - E_2\, \pi_3 - r_2^n\Big) \\
x_2^{\Delta} &= -\frac{\phi_\pi^a}{1+\phi_\pi\kappa}\pi_2 - \Big(\phi_\pi \pi_2^{\Delta} + \frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}\pi_2^{\Delta} - \Delta\Big) \\
x_2^{\Delta} &= -\frac{\phi_\pi^a \Psi^{AIT}\kappa}{1+\phi_\pi\kappa}x_2^{\Delta} - \phi_\pi\Psi^{AIT}\kappa x_2^{\Delta} - \frac{\phi_\pi^a\kappa^2\Psi^{AIT}}{1+\phi_\pi\kappa}x_2^{\Delta} + \Delta \\
x_2^{\Delta} + \frac{\phi_\pi^a \Psi^{AIT}\kappa}{1+\phi_\pi\kappa}x_2^{\Delta} + \phi_\pi\Psi^{AIT}\kappa x_2^{\Delta} + \frac{\phi_\pi^a\kappa^2\Psi^{AIT}}{1+\phi_\pi\kappa}x_2^{\Delta} &= \Delta \\
x_2^{\Delta}\Big(1 + \kappa\Psi^{AIT}\Big(\phi_\pi + \phi_\pi^a\Big(\frac{1+\kappa}{1+\phi_\pi\kappa}\Big)\Big)\Big) &= \Delta \\
x_2^{\Delta} &= \frac{1}{1+\kappa\Psi^{AIT}\Big(\phi_\pi + \phi_\pi^a\Big(\frac{1+\kappa}{1+\phi_\pi\kappa}\Big)\Big)}\Delta
\end{aligned}
\tag{A.39}
$$

The inflation gap in period 2, therefore takes the following shape:

$$
\pi_2^{\Delta} = \frac{\Psi^{AIT}\kappa}{1+\kappa\Psi^{AIT}\Big(\phi_\pi + \phi_\pi^a\Big(\frac{1+\kappa}{1+\phi_\pi\kappa}\Big)\Big)}\Delta
\tag{A.40}
$$

Since the shocks are symmetric, the output gap and the inflation gap in the - Δ case are as follows:

$$
x_2^{-\Delta} = -\frac{1}{1+\kappa\Psi^{AIT}\Big(\phi_\pi + \phi_\pi^a\Big(\frac{1+\kappa}{1+\phi_\pi\kappa}\Big)\Big)}\Delta
\tag{A.41}
$$

$$\pi_2^{-\Delta} = -\frac{\Psi^{AIT}\kappa}{1+\kappa\Psi^{AIT}\left(\phi_\pi + \phi_\pi^a\left(\frac{1+\kappa}{1+\phi_\pi\kappa}\right)\right)}\Delta \tag{A.42}$$

We use the values obtained for x_2^{Δ}, π_2^{Δ}, $x_2^{-\Delta}$, and $\pi_2^{-\Delta}$ from above to calculate $E_1\pi_2$ and E_1x_2.

$$\begin{aligned}
E_1\pi_2 &= \pi_2^{\Delta}\, Prob\,(\pi_2^{\Delta}) + \pi_2^{-\Delta}\, Prob\,(\pi_2^{-\Delta}) \\
E_1\pi_2 &= \frac{\Psi^{AIT}\kappa\Delta}{1+\kappa\Psi^{AIT}\left(\phi_\pi + \phi_\pi^a\left(\frac{1+\kappa}{1+\phi_\pi\kappa}\right)\right)} \times \left(\frac{1}{2}\right) + \frac{-\Psi^{AIT}\kappa\Delta}{1+\kappa\Psi^{AIT}\left(\phi_\pi + \phi_\pi^a\left(\frac{1+\kappa}{1+\phi_\pi\kappa}\right)\right)} \times \left(\frac{1}{2}\right) \\
E_1\pi_2 &= 0
\end{aligned} \tag{A.43}$$

$$\begin{aligned}
E_1x_2 &= x_2^{\Delta}\, Prob\,(x_2^{\Delta}) + x_2^{-\Delta}\, Prob\,(x_2^{-\Delta}) \\
E_1x_2 &= \frac{\Delta}{1+\kappa\Psi^{AIT}\left(\phi_\pi + \phi_\pi^a\left(\frac{1+\kappa}{1+\phi_\pi\kappa}\right)\right)} \times \left(\frac{1}{2}\right) + \frac{-\Delta}{1+\kappa\Psi^{AIT}\left(\phi_\pi + \phi_\pi^a\left(\frac{1+\kappa}{1+\phi_\pi\kappa}\right)\right)} \times \left(\frac{1}{2}\right) \\
E_1x_2 &= 0
\end{aligned} \tag{A.44}$$

In period 1, using the same methodology as we did for the previous periods, we observe the following.

$$\begin{aligned}
\pi_1 &= \beta E_1\pi_2 + \kappa x_1 \\
\pi_1 &= \kappa x_1
\end{aligned} \tag{A.45}$$

Solving for the output gap yields:

$$\begin{aligned}
x_1 &= E_1x_2 - \left(i_1 - E_1\pi_2 - r_1^n\right) \\
x_1 &= 0 - \left(\phi_\pi\pi_1 - 0 - 0\right) \\
x_1 &= -\phi_\pi\pi_1 \\
x_1 &= -\phi_\pi\kappa x_1 \\
x_1 &= 0
\end{aligned} \tag{A.46}$$

This implies that $x_1 = \pi_1 = 0$.

A.4 Stabilizing the Economy at the Zero Lower Bound

A.4.1 Single Mandate at the Zero Lower Bound

To illustrate how the dual mandate helps stabilize inflation at the zero lower bound, we now alter the shock process in our simple 4-period model. The natural rate still takes a value of 0 in periods 1, 3 and 4 but the economy now experiences a large negative demand shock that takes the economy to the zero lower bound with certainty. Specifically we assume $r_2^n = -i - \Delta^{ZLB}$ where $\Delta^{ZLB} > 0$. Since the model in Equations (A.1) and (A.2) is linearized around its non-stochastic steady state, the zero lower bound on the nominal policy rate binds whenever deviations of the nominal rate i_t fall below the negative of its steady-state value $(-i)$.

Beginning with the outcomes under a single mandate, we incorporate the zero lower bound constraint as follows:

$$i_t = \max\left(\phi_\pi \pi_t, -i\right) \tag{A.47}$$

As before we solve backwards to determine economic outcomes. Since this is a finite period, agents know the economy ceases to exist after period 4. Therefore, $x_5 = \pi_5 = 0$. Additionally, from our assumption it follows that $E_4 x_5 = E_4 \pi_5 = 0$. Using this we can now solve for period 4 outcomes. Solving for the the inflation gap yields:

$$\begin{aligned} \pi_4 &= \beta E_4 \pi_5 + \kappa x_4 \\ \pi_4 &= \kappa x_4 \end{aligned} \tag{A.48}$$

Solving for the output gap yields:

$$\begin{aligned} x_4 &= E_4 x_5 - \left(i_4 - E_4 \pi_5 - r_4^n\right) \\ x_4 &= 0 - \left(\phi_\pi \pi_4 - 0 - 0\right) \\ x_4 &= -\phi_\pi \pi_4 \\ x_4 &= -\phi_\pi \kappa x_4 \\ x_4 &= 0 \end{aligned} \tag{A.49}$$

Taken together for period 4 we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, from our key assumption we can conclude that $E_3 \pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Solving for the inflation gap in period 3 yields:

$$
\begin{aligned}
\pi_3 &= \beta E_3 \pi_4 + \kappa x_3 \\
\pi_3 &= \kappa x_3
\end{aligned} \tag{A.50}
$$

Solving for the output gap yields:

$$
\begin{aligned}
x_3 &= E_3 x_4 - \Big(i_3 - E_3 \pi_4 - r_3^n\Big) \\
x_3 &= 0 - \Big(\phi_\pi \pi_3 - 0 - 0\Big) \\
x_3 &= -\phi_\pi \pi_3 \\
x_3 &= -\phi_\pi \kappa x_3 \\
x_3 &= 0
\end{aligned} \tag{A.51}
$$

Taken together for period 3 we obtain the following outcomes: $x_3 = \pi_3 = 0$. Again, from our key assumption we can conclude that $E_2 \pi_3 = E_2 x_3 = 0$. We now solve for period 2, in the same manner as we did for the previous two periods. In period 2, note that $r_t^n = -i - \Delta^{ZLB}$ with complete certainty. Therefore, we yield the following outcomes. The output gap is given by

$$
\begin{aligned}
x_2^{-\Delta^{ZLB}} &= E_2 x_3 - \Big(i_2 - E_2 \pi_3 - r_2^n\Big) \\
x_2^{-\Delta^{ZLB}} &= 0 - (-i_2 - 0 - (-i_2 - \Delta^{ZLB})) \\
x_2^{-\Delta^{ZLB}} &= -\Delta^{ZLB}
\end{aligned} \tag{A.52}
$$

The inflation gap in this period is given by:

$$
\begin{aligned}
\pi_2^{-\Delta^{ZLB}} &= \beta E_3 \pi_2 + \kappa x_2^{-\Delta^{ZLB}} \\
\pi_2^{-\Delta^{ZLB}} &= \kappa x_2^{-\Delta^{ZLB}} \\
\pi_2^{-\Delta^{ZLB}} &= \kappa \Delta^{ZLB}
\end{aligned} \tag{A.53}
$$

A.4.2 Dual Mandate Outcomes at the ZLB

Beginning with the outcomes under a dual mandate, we incorporate the zero lower bound constraint as follows:

$$
i_t = \max\Big(\phi_\pi \, \pi_t + \phi_x x_{t-1}, \, -i\Big) \tag{A.54}
$$

To keep the model analytically tractable, we assume that the dual mandate is followed only in period 3. As before we solve backwards to determine economic outcomes. Since this is a finite period, agents know the economy ceases to exist after period 4. Therefore, $x_5 = \pi_5 = 0$. Additionally, from our assumption it follows that $E_4 x_5 = E_4 \pi_5 = 0$. Using this we now solve for period 4 outcomes.

$$\begin{aligned} \pi_4 &= \beta E_4 \pi_5 + \kappa x_4 \\ \pi_4 &= \kappa x_4 \end{aligned} \tag{A.55}$$

Solving for the output gap yields:

$$\begin{aligned} x_4 &= E_4 x_5 - \Big(i_4 - E_4 \pi_5 - r_4^n\Big) \\ x_4 &= 0 - \Big(\phi_\pi \pi_4 - 0 - 0) \\ x_4 &= -\phi_\pi \pi_4 \\ x_4 &= -\phi_\pi \kappa x_4 \\ x_4 &= 0 \end{aligned} \tag{A.56}$$

Taken together for period 4 we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, from our key assumption we can conclude that $E_3 \pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Note that in period 3, the policy rule follows the dual mandate as described in Equation 11. Therefore, solving for the inflation gap yields:

$$\begin{aligned} \pi_3 &= \beta E_3 \pi_4 + \kappa x_3 \\ \pi_3 &= \kappa\, x_3 \end{aligned} \tag{A.57}$$

Solving for the output gap yields:

$$\begin{aligned} x_3 &= E_3\, x_4 - \Big(i_3 - E_3\, \pi_4 - r_{23}^n\Big) \\ x_3 &= 0 - \Big(\phi_\pi \pi_3 + \phi_x x_2 - 0 - 0\Big) \\ x_3 &= -\phi_\pi \pi_3 - \phi_x x_2 \\ x_3 &= -\phi_\pi \kappa x_3 - \phi_x x_2 \\ x_3 + \phi_\pi \kappa\, x_3 &= -\phi_x x_2 \\ x_3(1 + \phi_\pi \kappa) &= -\phi_x x_2 \\ x_3 &= -\frac{\phi_x}{1 + \phi_\pi \kappa} x_2 \end{aligned} \tag{A.58}$$

We substitute this back into the equation for the inflation gap to obtain the values of the inflation gap in period 3 as a function of the output gap in period 2.

$$\pi_3 = -\frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2 \tag{A.59}$$

Before we solve for 2nd period, we need to reiterate a key point. Since participants have complete knowledge about the economy, their expectations accurately reflect the state of the economy currently. Therefore, $E_2 x_3 = x_3$ and $E_2 \pi_3 = \pi_3$. We now solve for period 2 outcomes, in the same manner as we did for the previous two periods. In period 2, note that $r_t^n = -i - \Delta^{ZLB}$ with complete certainty. Therefore, we yield the following outcomes. The output gap is given by:

$$\begin{aligned}
x_2^{-\Delta^{ZLB}} &= E_2, x_3 - \Big(i_2 - E_2, \pi_3 - r_2^n\Big) \\
x_2^{-\Delta^{ZLB}} &= -\frac{\phi_x}{1+\phi_\pi \kappa} x_2 - (-i_2 + \frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2 - (-i_2 - \Delta^{ZLB})) \\
x_2^{-\Delta^{ZLB}} + \frac{\phi_x}{1+\phi_\pi \kappa} x_2 + \frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2 &= -\Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}} \Big(1 + \frac{\phi_x}{1+\phi_\pi \kappa} + \frac{\phi_x \kappa}{1+\phi_\pi \kappa}\Big) &= -\Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}} \Big(\frac{1+\phi_\pi \kappa + \phi_x(1+\kappa)}{1+\phi_\pi \kappa}\Big) &= -\Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}} &= \underbrace{\Big(\frac{1+\phi_\pi \kappa}{1+\phi_\pi \kappa + \phi_x(1+\kappa)}\Big)}_{\Phi^D} - \Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}} &= -\Phi^D \Delta^{ZLB}
\end{aligned} \tag{A.60}$$

The inflation gap in this period is given by:

$$\begin{aligned}
\pi_2^{-\Delta^{ZLB}} &= \beta E_2 \pi_3 + \kappa x_2 \\
\pi_2^{-\Delta^{ZLB}} &= -\frac{\beta \phi_x \kappa}{1+\phi_\pi \kappa} x_2 + \kappa x_2 \\
\pi_2^{-\Delta^{ZLB}} &= \kappa x_2 \Big(1 - \frac{\beta \phi_x}{1+\phi_\pi \kappa}\Big) \\
\pi_2^{-\Delta^{ZLB}} &= \kappa x_2 \Psi^D \\
\pi_2^{-\Delta^{ZLB}} &= -\kappa \Phi^D \Psi^D \Delta^{ZLB}
\end{aligned} \tag{A.61}$$

A.4.3 Average Inflation Targeting at the Zero Lower Bound

Instead of a dual mandate, we now assume that monetary policy aims to stabilize the some weighted average of inflation in period 3:

$$i_t = \phi_\pi \pi_t + \phi_\pi^a \pi_{t-1}. \tag{A.62}$$

We solve this model backwards starting from period 4. We hold our key assumption that all participants in the economy are fully aware of the state of the economy and form their expectations based on this all available knowledge. Since this is a finite period, participants know that the economy ceases to exist after period 4. Therefore, we know that $x_5 = \pi_5 = 0$. This implies that $E_4 x_5 = E_4 \pi_5 = 0$. From this it follows that: Using this we can now solve for period 4 outcomes. Solving for the the inflation gap yields:

$$\begin{aligned} \pi_4 &= \beta E_4 \pi_5 + \kappa x_4 \\ \pi_4 &= \kappa x_4 \end{aligned} \tag{A.63}$$

Solving for the output gap yields:

$$\begin{aligned} x_4 &= E_4 x_5 - \Big(i_4 - E_4 \pi_5 - r_4^n \Big) \\ x_4 &= 0 - \Big(\phi_\pi \pi_4 - 0 - 0) \\ x_4 &= -\phi_\pi \pi_4 \\ x_4 &= -\phi_\pi \kappa x_4 \\ x_4 &= 0 \end{aligned} \tag{A.64}$$

Taken together for period 4 we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, from our key assumption we can conclude that $E_3 \pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Note that in period 3, the policy rule follows the dual mandate as described in Equation 11. Solving for the inflation gap yields:

$$\begin{aligned} \pi_3 &= \beta E_3 \pi_4 + \kappa x_3 \\ \pi_3 &= \kappa \, x_3 \end{aligned} \tag{A.65}$$

Solving for the output gap yields:

$$
\begin{aligned}
x_3 &= E_3\, x_4 - \Big(i_3 - E_3\, \pi_4 - r^n_{23}\Big) \\
x_3 &= 0 - \Big(\phi_\pi \pi_3 + \phi^a_\pi \pi_2 - 0 - 0\Big) \\
x_3 &= -\phi_\pi \pi_3 - \phi^a_\pi \pi_2 \\
x_3 &= -\phi_\pi \kappa x_3 - \phi^a_\pi \pi_2 \\
x_3 + \phi_\pi \kappa\, x_3 &= -\phi^a_\pi \pi_2 \\
x_3(1 + \phi_\pi \kappa) &= -\phi^a_\pi \pi_2 \\
x_3 &= -\frac{\phi^a_\pi}{1 + \phi_\pi \kappa} \pi_2
\end{aligned} \tag{A.66}
$$

We substitute this back into the equation for the inflation gap to obtain the values of the inflation gap in period 3 as a function of the inflation gap in period 2.

$$
\pi_3 = -\frac{\phi^a_\pi \kappa}{1 + \phi_\pi \kappa} \pi_2 \tag{A.67}
$$

Before we solve for 2nd period, we need to reiterate a key point. Since participants have complete knowledge about the economy, their expectations accurately reflect the state of the economy currently. Therefore, $E_2 x_3 = x_3$ and $E_2 \pi_3 = \pi_3$. We now solve for period 2 outcomes, in the same manner as we did for the previous two periods. In period 2, note that $r^n_t = -i - \Delta^{ZLB}$ with complete certainty. Therefore, we yield the following outcomes. The inflation gap is given by:

$$
\begin{aligned}
\pi_2^{-\Delta^{ZLB}} &= \beta E_2 \pi_3 + \kappa x_2^{-\Delta^{ZLB}} \\
\pi_2^{-\Delta^{ZLB}} &= -\frac{\beta \phi^a_\pi \kappa}{1 + \phi_\pi \kappa} \pi_2^{-\Delta^{ZLB}} + \kappa x_2^{-\Delta^{ZLB}} \\
\pi_2^{-\Delta^{ZLB}} + \frac{\beta \phi^a_\pi \kappa}{1 + \phi_\pi \kappa} \pi_2^{-\Delta^{ZLB}} &= \kappa x_2^{-\Delta^{ZLB}} \\
\pi_2^{-\Delta^{ZLB}} \Big(\frac{1 + \phi_\pi \kappa + \beta \phi^a_\pi \kappa}{1 + \phi_\pi \kappa}\Big) &= \kappa x_2^{-\Delta^{ZLB}} \\
\pi_2^{-\Delta^{ZLB}} &= \Big(\frac{1 + \phi_\pi \kappa}{1 + \phi_\pi \kappa + \beta \phi^a_\pi \kappa}\Big)\Big(\kappa x_2^{-\Delta^{ZLB}}\Big) \\
\pi_2^{-\Delta^{ZLB}} &= \Psi^{AIT} \kappa x_2^{-\Delta^{ZLB}}
\end{aligned} \tag{A.68}
$$

The output gap is given by:

$$
\begin{aligned}
x_2^{-\Delta^{ZLB}} &= E_2\, x_3 - \Big(i_2 - E_2\, \pi_3 - r_2^n\Big) \\
x_2^{-\Delta^{ZLB}} &= -\frac{\phi_\pi^a}{1+\phi_\pi\kappa}\pi_2^{-\Delta^{ZLB}} - \Big(-i_2 + \frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}\pi_2^{-\Delta^{ZLB}} - (-i_2 - \Delta^{ZLB})\Big) \\
x_2^{-\Delta^{ZLB}} &= -\frac{\phi_\pi^a}{1+\phi_\pi\kappa}\pi_2^{-\Delta^{ZLB}} - \frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}\pi_2^{-\Delta^{ZLB}} - \Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}} + \frac{\phi_\pi^a}{1+\phi_\pi\kappa}\pi_2^{-\Delta^{ZLB}} + \frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}\pi_2^{-\Delta^{ZLB}} &= -\Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}} + \frac{\phi_\pi^a\kappa\Psi^{AIT}}{1+\phi_\pi\kappa}x_2^{-\Delta^{ZLB}} + \frac{\phi_\pi^a\kappa^2\Psi^{AIT}}{1+\phi_\pi\kappa}x_2^{-\Delta^{ZLB}} &= -\Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}}\Big(1 + \frac{\phi_\pi^a\kappa\Psi^{AIT}}{1+\phi_\pi\kappa} + \frac{\phi_\pi^a\kappa^2\Psi^{AIT}}{1+\phi_\pi\kappa}\Big) &= -\Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}}\Big(\frac{1+\phi_\pi\kappa + \phi_\pi^a\kappa\Psi^{AIT}(1+\kappa)}{1+\phi_\pi\kappa}\Big) &= -\Delta^{ZLB} \\
x_2^{-\Delta^{ZLB}} &= \underbrace{\Big(\frac{1+\phi_\pi\kappa}{1+\phi_\pi\kappa + \phi_\pi^a\kappa\Psi^{AIT}(1+\kappa)}\Big)}_{\Phi^{AIT}}(-\Delta^{ZLB}) \\
x_2^{-\Delta^{ZLB}} &= -\Phi^{AIT}\Delta^{ZLB}
\end{aligned}
\tag{A.69}
$$

Substituting equation A.69 into equation A.68, we yield the following:

$$
\pi_2^{-\Delta^{ZLB}} = -\Psi^{AIT}\Phi^{AIT}\kappa\Delta^{ZLB} \tag{A.70}
$$

A.5 Supply Shocks

To highlight the effect of a supply shock, we consider the following four period model.

$$
x_t = E_t x_{t+1} - \Big(i_t - E_t\pi_{t+1}\Big) \tag{A.71}
$$

$$
\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + s_t \tag{A.72}
$$

Here s_t captures fluctuations in firms' desired markups or other inefficient changes on the production side of the economy. We use the same timing setup as before to solve for the economic outcomes under this supply shock. The shock takes a positive value ϵ or a negative value $-\epsilon$ in which both outcomes occur with equal probability. We solve for outcomes and assume that the economy remains away from the zero lower bound.

A.5.1 Supply Shocks under a Single Mandate

Since this is a finite period, agents know the economy ceases to exist after period 4. Therefore, $x_5 = \pi_5 = 0$. Additionally, from our assumption it follows that $E_4 x_5 = E_4 \pi_5 = 0$. Using this we now solve for period 4 outcomes.

$$\begin{aligned} \pi_4 &= \beta E_4 \pi_5 + \kappa x_4 \\ \pi_4 &= \kappa x_4 \end{aligned} \tag{A.73}$$

Solving for the output gap yields:

$$\begin{aligned} x_4 &= E_4 x_5 - \left(i_4 - E_4 \pi_5 - r_4^n\right) \\ x_4 &= 0 - \left(\phi_\pi \pi_4 - 0 - 0\right) \\ x_4 &= -\phi_\pi \pi_4 \\ x_4 &= -\phi_\pi \kappa x_4 \\ x_4 &= 0 \end{aligned} \tag{A.74}$$

Taken together for period 4 we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, from our key assumption we can conclude that $E_3 \pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Solving for the inflation gap in period 3 yields:

$$\begin{aligned} \pi_3 &= \beta E_3 \pi_4 + \kappa x_3 \\ \pi_3 &= \kappa x_3 \end{aligned} \tag{A.75}$$

Solving for the output gap yields:

$$\begin{aligned} x_3 &= E_3 x_4 - \left(i_3 - E_3 \pi_4 - r_3^n\right) \\ x_3 &= 0 - \left(\phi_\pi \pi_3 - 0 - 0\right) \\ x_3 &= -\phi_\pi \pi_3 \\ x_3 &= -\phi_\pi \kappa x_3 \\ x_3 &= 0 \end{aligned} \tag{A.76}$$

Taken together for period 3 we obtain the following outcomes: $x_3 = \pi_3 = 0$. Again, from our key assumption we can conclude that $E_2 \pi_3 = E_2 x_3 = 0$. We solve for period 2 outcomes in the same manner as we did for the previous periods, with one exception: $s_t = \epsilon$ in period

2. This yields us the following outcomes.

$$\begin{aligned}
\pi_2^{\epsilon} &= \beta E_2 \pi_3 + \kappa x_2^{\epsilon} + s_2 \\
\pi_2^{\epsilon} &= 0 + \kappa x_2^{\epsilon} + \epsilon \\
\pi_2^{\epsilon} &= \kappa x_2^{\epsilon} + \epsilon
\end{aligned} \tag{A.77}$$

$$\begin{aligned}
x_2^{\epsilon} &= E_2 x_3 - \Big(i_1 - E_2 \pi_3\Big) \\
x_2^{\epsilon} &= 0 - \Big(\phi_\pi \pi_2^{\epsilon} - 0\Big) \\
x_2^{\epsilon} &= -\phi_\pi \pi_2^{\epsilon} \\
x_2^{\epsilon} &= -\phi_\pi (\kappa x_2^{\epsilon} + \epsilon) \\
x_2^{\epsilon} &= -\phi_\pi \kappa x_2^{\epsilon} - \phi_\pi \epsilon \\
x_2^{\epsilon} + \phi_\pi \kappa x_2^{\epsilon} &= -\phi \pi \epsilon \\
x_2^{\epsilon} &= \frac{-\phi_\pi}{1 + \phi_\pi \kappa} \epsilon
\end{aligned} \tag{A.78}$$

Solving for the inflation gap in this period yields:

$$\begin{aligned}
\pi_2^{\epsilon} &= \kappa x_2^{\epsilon} + \epsilon \\
\pi_2^{\epsilon} &= \frac{-\phi_\pi \kappa \epsilon}{1 + \phi_\pi \kappa} + \epsilon \\
\pi_2^{\epsilon} &= \frac{-\phi_\pi \kappa \epsilon + \epsilon + \phi_\pi \kappa \epsilon}{1 + \phi_\pi \kappa} \\
\pi_2^{\epsilon} &= \frac{\epsilon}{1 + \phi_\pi \kappa}
\end{aligned} \tag{A.79}$$

For the -ϵ case we repeat the same steps as above with one key distinction i.e. $s_2 = -\epsilon$. Since the shocks are symmetric, this yields us the following equations for the output and inflation gap respectively:

$$x_2^{\epsilon} = \frac{\phi_\pi}{1 + \phi_\pi \kappa} \epsilon \tag{A.80}$$

$$\pi_2^{-\epsilon} = -\frac{\epsilon}{1 + \phi_\pi \kappa} \tag{A.81}$$

We use the values obtained for x_2^{ϵ}, π_2^{ϵ}, $x_2^{-\epsilon}$, and $\pi_2^{-\epsilon}$ from above to calculate $E_1\pi_2$ and E_1x_2.

$$\begin{aligned}
E_1\pi_2 &= \pi_2^{\epsilon}\, Prob\,(\pi_2^{\epsilon}) + \pi_2^{-\epsilon}\, Prob\,(\pi_2^{-\epsilon}) \\
E_1\pi_2 &= \Big(\frac{\epsilon}{1+\phi_\pi\kappa} \times \frac{1}{2}\Big) + \Big(\frac{-\epsilon}{1+\phi_\pi\kappa} \times \frac{1}{2}\Big) \\
E_1\pi_2 &= 0
\end{aligned} \tag{A.82}$$

We use the same methodology to calculate E_1x_2.

$$\begin{aligned}
E_1x_2 &= x_2^{\epsilon}\, Prob\,(x_2^{\epsilon}) + x_2^{-\epsilon}\, Prob\,(x_2^{-\epsilon}) \\
E_1x_2 &= \Big(\frac{\phi_\pi\epsilon}{1+\phi_\pi\kappa} \times \frac{1}{2}\Big) + \Big(\frac{-\phi_\pi\epsilon}{1+\phi_\pi\kappa} \times \frac{1}{2}\Big) \\
E_1x_2 &= 0
\end{aligned} \tag{A.83}$$

In period 1, using the same methodology as we did for the previous periods, we observe the following.

$$\begin{aligned}
\pi_1 &= \beta E_1\pi_2 + \kappa x_1 + s_1 \\
\pi_1 &= \kappa x_1
\end{aligned} \tag{A.84}$$

Solving for the output gap yields:

$$\begin{aligned}
x_1 &= E_1x_2 - \Big(i_1 - E_1\pi_2\Big) \\
x_1 &= 0 - \Big(\phi_\pi\pi_1 - 0 - 0\Big) \\
x_1 &= -\phi_\pi\pi_1 \\
x_1 &= -\phi_\pi\kappa x_1 \\
x_1 &= 0
\end{aligned} \tag{A.85}$$

This implies that $x_1 = \pi_1 = 0$.

A.5.2 Supply shocks under a Dual Mandate

Since this is a finite period, agents know the economy ceases to exist after period 4. Therefore, $x_5 = \pi_5 = 0$. Additionally, from our assumption it follows that $E_4x_5 = E_4\pi_5 = 0$. Using this we can now solve for period 4 outcomes. Solving for the the inflation gap yields:

$$
\begin{aligned}
\pi_4 &= \beta E_4 \pi_5 + \kappa x_4 \\
\pi_4 &= \kappa x_4
\end{aligned}
\tag{A.86}
$$

Solving for the output gap yields:

$$
\begin{aligned}
x_4 &= E_4 x_5 - \Big(i_4 - E_4 \pi_5 - r_4^n\Big) \\
x_4 &= 0 - \Big(\phi_\pi \pi_4 - 0 - 0) \\
x_4 &= -\phi_\pi \pi_4 \\
x_4 &= -\phi_\pi \kappa x_4 \\
x_4 &= 0
\end{aligned}
\tag{A.87}
$$

Taken together for period 4 we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, from our key assumption we can conclude that $E_3 \pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Note that in period 3, the policy rule follows the dual mandate as described in Equation 11. Solving for the inflation gap yields:

$$
\begin{aligned}
\pi_3 &= \beta E_3 \pi_4 + \kappa x_3 \\
\pi_3 &= \kappa \, x_3
\end{aligned}
\tag{A.88}
$$

Solving for the output gap yields:

$$
\begin{aligned}
x_3 &= E_3 \, x_4 - \Big(i_3 - E_3 \, \pi_4 - r_{23}^n\Big) \\
x_3 &= 0 - \Big(\phi_\pi \pi_3 + \phi_x x_2 - 0 - 0\Big) \\
x_3 &= -\phi_\pi \pi_3 - \phi_x x_2 \\
x_3 &= -\phi_\pi \kappa x_3 - \phi_x x_2 \\
x_3 + \phi_\pi \kappa \, x_3 &= -\phi_x x_2 \\
x_3(1 + \phi_\pi \kappa) &= -\phi_x x_2 \\
x_3 &= -\frac{\phi_x}{1 + \phi_\pi \kappa} x_2
\end{aligned}
\tag{A.89}
$$

We substitute this back into the equation for the inflation gap to obtain the values of the inflation gap in period 3 as a function of the output gap in period 2.

$$\pi_3 = -\frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2 \tag{A.90}$$

Before we solve for 2nd period, we need to reiterate a key point. Since participants have complete knowledge about the economy, their expectations accurately reflect the state of the economy currently. Therefore, $E_2 x_3 = x_3$ and $E_2 \pi_3 = \pi_3$. We solve for period 2 outcomes in the same manner as we did for the previous periods, with one exception: $s_t = \epsilon$ in period 2. This yields us the following outcomes.

$$\begin{aligned}
\pi_2^\epsilon &= \beta E_2 \pi_3 + \kappa x_2^\epsilon + s_2 \\
\pi_2^\epsilon &= \frac{-\beta \phi_x \kappa}{1+\phi_\pi \kappa} x_2^\epsilon + \kappa x_2^\epsilon + \epsilon \\
\pi_2^\epsilon &= \kappa x_2^\epsilon \Big(1 - \frac{\beta \phi_x}{1+\phi_\pi \kappa}\Big) + \epsilon \\
\pi_2^\epsilon &= \Psi^D \kappa x_2^\epsilon + \epsilon
\end{aligned} \tag{A.91}$$

Solving for the output gap equation yields the following Equation.

$$\begin{aligned}
x_2^\epsilon &= E_2\, x_3 - \Big(i_2 - E_2\, \pi_3\Big) \\
x_2^\epsilon &= -\frac{\phi_x}{1+\phi_\pi \kappa} x_2^\epsilon - \Big(\phi_\pi \pi_2^\epsilon + \frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2^\epsilon\Big) \\
x_2^\epsilon &= -\frac{\phi_x}{1+\phi_\pi \kappa} x_2^\epsilon - \Psi^D \kappa \phi_\pi x_2^\epsilon - \phi_\pi \epsilon - \frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2^\epsilon \\
x_2^\epsilon + \frac{\phi_x}{1+\phi_\pi \kappa} x_2^\epsilon + \Psi^D \kappa \phi_\pi x_2^\epsilon + \frac{\phi_x \kappa}{1+\phi_\pi \kappa} x_2^\epsilon &= -\phi_\pi \epsilon \\
x_2^\epsilon = \Big(1 + \frac{\phi_x}{1+\phi_\pi \kappa} + \frac{\phi_x \kappa}{1+\phi_\pi \kappa} + \Psi^D \kappa \phi_\pi\Big) &= -\phi_\pi \epsilon \\
x_2^\epsilon \Big(\frac{1+\phi_\pi \kappa + \phi_x (1+\kappa)}{1+\phi_\pi \kappa} + \Psi^D \kappa \phi_\pi\Big) &= -\phi_\pi \epsilon \\
x_2^\epsilon \Big(\frac{1}{\Phi^D} + \Psi^D \phi_\pi \kappa\Big) &= -\phi_\pi \epsilon \\
x_2^\epsilon \Big(\frac{1 + \Psi^D \Phi^D \phi_\pi \kappa}{\Phi^D}\Big) &= -\phi_\pi \epsilon \\
x_2^\epsilon &= -\frac{\Phi^D \phi_\pi}{1 + \Phi^D \Psi^D \phi_\pi \kappa} \epsilon
\end{aligned} \tag{A.92}$$

Substituting Equation 92 into Equation 91 yields:

$$
\begin{aligned}
\pi_2^{\epsilon} &= \Psi^D \kappa x_2^{\epsilon} + \epsilon \\
\pi_2^{\epsilon} &= -\frac{\Phi^D \phi_\pi \Psi^D \kappa \epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa} + \epsilon \\
\pi_2^{\epsilon} &= -\frac{\Phi^D \phi_\pi \Psi^D \kappa \epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa} + \frac{\Phi^D \phi_\pi \Psi^D \kappa \epsilon + \epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa} \\
\pi_2^{\epsilon} &= \frac{\epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa}
\end{aligned}
\tag{A.93}
$$

For the -ϵ case we repeat the same steps as above with one key distinction i.e. $s_2 = -\epsilon$. Since the shocks are symmetric, this yields us the following equations for the output and inflation gap respectively:

$$
x_2^{-\epsilon} = \frac{\Phi^D \phi_\pi}{1 + \Phi^D \Psi^D \phi_\pi \kappa} \epsilon \tag{A.94}
$$

$$
\pi_2^{-\epsilon} = -\frac{\epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa} \tag{A.95}
$$

We use the values obtained for x_2^{ϵ}, π_2^{ϵ}, $x_2^{-\epsilon}$, and $\pi_2^{-\epsilon}$ from above to calculate $E_1 \pi_2$ and $E_1 x_2$.

$$
\begin{aligned}
E_1 \pi_2 &= \pi_2^{\epsilon}\, Prob\,(\pi_2^{\epsilon}) + \pi_2^{-\epsilon}\, Prob\,(\pi_2^{-\epsilon}) \\
E_1 \pi_2 &= \Big(\frac{\epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa} \times \frac{1}{2}\Big) + \Big(\frac{-\epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa} \times \frac{1}{2}\Big) \\
E_1 \pi_2 &= 0
\end{aligned}
\tag{A.96}
$$

We use the same methodology to calculate $E_1 x_2$.

$$
\begin{aligned}
E_1 x_2 &= x_2^{\epsilon}\, Prob\,(x_2^{\epsilon}) + x_2^{-\epsilon}\, Prob\,(x_2^{-\epsilon}) \\
E_1 x_2 &= \Big(-\frac{\Phi^D \phi_\pi \epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa} \times \frac{1}{2}\Big) + \Big(\frac{\Phi^D \phi_\pi \epsilon}{1 + \Phi^D \Psi^D \phi_\pi \kappa} \times \frac{1}{2}\Big) \\
E_1 x_2 &= 0
\end{aligned}
\tag{A.97}
$$

In period 1, using the same methodology as we did for the previous periods, we observe the following.

$$
\begin{aligned}
\pi_1 &= \beta E_1 \pi_2 + \kappa x_1 + s_1 \\
\pi_1 &= \kappa x_1
\end{aligned}
\tag{A.98}
$$

Solving for the output gap yields:

$$
\begin{aligned}
x_1 &= E_1 x_2 - \Big(i_1 - E_1 \pi_2\Big) \\
x_1 &= 0 - \Big(\phi_\pi \pi_1 - 0\Big) \\
x_1 &= -\phi_\pi \pi_1 \\
x_1 &= -\phi_\pi \kappa x_1 \\
x_1 &= 0
\end{aligned}
\tag{A.99}
$$

This implies that $x_1 = \pi_1 = 0$.

A.5.3 Supply Shocks under Average Inflation Targeting

Since this is a finite period, participants know that the economy ceases to exist after period 4. Therefore, we know that $x_5 = \pi_5 = 0$. This implies that $E_4 x_5 = E_4 \pi_5 = 0$. From this it follows that: Using this we can now solve for period 4 outcomes. Solving for the the inflation gap yields:

$$
\begin{aligned}
\pi_4 &= \beta E_4 \pi_5 + \kappa x_4 \\
\pi_4 &= \kappa x_4
\end{aligned}
\tag{A.100}
$$

Solving for the output gap yields:

$$
\begin{aligned}
x_4 &= E_4 x_5 - \Big(i_4 - E_4 \pi_5 - r_4^n\Big) \\
x_4 &= 0 - \Big(\phi_\pi \pi_4 - 0 - 0) \\
x_4 &= -\phi_\pi \pi_4 \\
x_4 &= -\phi_\pi \kappa x_4 \\
x_4 &= 0
\end{aligned}
\tag{A.101}
$$

Taken together for period 4 we obtain the following outcomes: $x_4 = \pi_4 = 0$. Again, from our key assumption we can conclude that $E_3 \pi_4 = E_3 x_4 = 0$. We now solve for period 3, in the same manner as we did for period 4. Note that in period 3, the policy rule follows the dual mandate as described in Equation 11. Solving for the inflation gap yields:

$$
\begin{aligned}
\pi_3 &= \beta E_3 \pi_4 + \kappa x_3 \\
\pi_3 &= \kappa \, x_3
\end{aligned}
\tag{A.102}
$$

Solving for the output gap yields:

$$
\begin{aligned}
x_3 &= E_3\, x_4 - \Big(i_3 - E_3\, \pi_4 - r^n_{23}\Big) \\
x_3 &= 0 - \Big(\phi_\pi \pi_3 + \phi^a_\pi \pi_2 - 0 - 0\Big) \\
x_3 &= -\phi_\pi \pi_3 - \phi^a_\pi \pi_2 \\
x_3 &= -\phi_\pi \kappa x_3 - \phi^a_\pi \pi_2 \\
x_3 + \phi_\pi \kappa\, x_3 &= -\phi^a_\pi \pi_2 \\
x_3(1 + \phi_\pi \kappa) &= -\phi^a_\pi \pi_2 \\
x_3 &= -\frac{\phi^a_\pi}{1 + \phi_\pi \kappa} \pi_2
\end{aligned}
\tag{A.103}
$$

We substitute this back into the equation for the inflation gap to obtain the values of the inflation gap in period 3 as a function of the inflation gap in period 2.

$$
\pi_3 = -\frac{\phi^a_\pi \kappa}{1 + \phi_\pi \kappa} \pi_2 \tag{A.104}
$$

Before we solve for 2nd period, we need to reiterate a key point. Since participants have complete knowledge about the economy, their expectations accurately reflect the state of the economy currently. Therefore, $E_2 x_3 = x_3$ and $E_2 \pi_3 = \pi_3$. We solve for period 2 outcomes in the same manner as we did for the previous periods, with one exception: $s_t = \epsilon$ in period 2. This yields us the following outcomes.

$$
\begin{aligned}
x^\epsilon_2 &= E_2\, x_3 - \Big(i_2 - E_2\, \pi_3\Big) \\
x^\epsilon_2 &= -\frac{\phi^a_\pi}{1 + \phi_\pi \kappa} \pi^\epsilon_2 - \phi_\pi \pi^\epsilon_2 - \frac{-\phi^a_\pi \kappa}{1 + \phi_\pi \kappa} \pi^\epsilon_2 \\
x^\epsilon_2 &= \left(\phi_\pi + \frac{\phi^a_\pi (1 + \kappa)}{1 + \phi_\pi \kappa}\right) - \pi^\epsilon_2 \\
x^\epsilon_2 &= \Bigg(\phi_\pi \underbrace{\left(1 + \frac{\phi^a_\pi}{\phi_\pi} \Big(\frac{1 + \kappa}{1 + \phi_\pi \kappa}\Big)\right)}_{\Upsilon^{AIT}}\Bigg)(-\pi^\epsilon_2) \\
x^\epsilon_2 &= -\Upsilon^{AIT} \phi_\pi \pi^\epsilon_2
\end{aligned}
\tag{A.105}
$$

Solving for the inflation gap in this period yields:

$$\begin{aligned}
\pi_2^\epsilon &= \beta E_2\pi_3 + \kappa x_2^\epsilon + s_2 \\
\pi_2^\epsilon &= -\frac{\beta\phi_\pi^a\kappa}{1+\phi_\pi\kappa}\pi_2^\epsilon - \kappa\Upsilon^{AIT}\phi_\pi\pi_2^\epsilon + \epsilon \\
\pi_2^\epsilon + \frac{\beta\phi_\pi^a\kappa}{1+\phi_\pi\kappa}\pi_2^\epsilon + \kappa\Upsilon^{AIT}\phi_\pi\pi_2^\epsilon &= \epsilon \\
\Big(1 + \frac{\beta\phi_\pi^a\kappa}{1+\phi_\pi\kappa} + \kappa\Upsilon^{AIT}\phi_\pi\Big)\pi_2^\epsilon &= \epsilon \\
\pi_2^\epsilon &= \frac{\epsilon}{1+\kappa\Upsilon^{AIT}\phi_\pi + \beta\frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}}
\end{aligned} \tag{A.106}$$

We substitute the inflation gap from this period back into the output gap and obtain the following:

$$\begin{aligned}
x_2^\epsilon &= -\Upsilon^{AIT}\phi_\pi\pi_2^\epsilon \\
x_2^\epsilon &= -\Upsilon^{AIT}\phi_\pi\Big(\frac{\epsilon}{1+\kappa\Upsilon^{AIT}\phi_\pi + \beta\frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}}\Big) \\
x_2^\epsilon &= -\frac{\phi_\pi}{\frac{1}{\Upsilon^{AIT}} + \phi_\pi\kappa + \frac{\beta}{\Upsilon^{AIT}}\frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}}\epsilon
\end{aligned} \tag{A.107}$$

For the -ϵ case we repeat the same steps as above with one key distinction i.e. $s_2 = -\epsilon$. Since the shocks are symmetric, this yields us the following equations for the output and inflation gap respectively:

$$x_2^{-\epsilon} = \frac{\phi_\pi}{\frac{1}{\Upsilon^{AIT}} + \phi_\pi\kappa + \frac{\beta}{\Upsilon^{AIT}}\frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}}\epsilon \tag{A.108}$$

$$\pi_2^{-\epsilon} = \frac{-\epsilon}{1+\kappa\Upsilon^{AIT}\phi_\pi + \beta\frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}} \tag{A.109}$$

We use the values obtained for x_2^ϵ, π_2^ϵ, $x_2^{-\epsilon}$, and $\pi_2^{-\epsilon}$ from above to calculate $E_1\pi_2$ and E_1x_2.

$$\begin{aligned}
E_1\pi_2 &= \pi_2^\epsilon\, Prob\,(\pi_2^\epsilon) + \pi_2^{-\epsilon}\, Prob\,(\pi_2^{-\epsilon}) \\
E_1\pi_2 &= \Big(\frac{\epsilon}{1+\kappa\Upsilon^{AIT}\phi_\pi + \beta\frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}} \times \frac{1}{2}\Big) + \Big(\frac{-\epsilon}{1+\kappa\Upsilon^{AIT}\phi_\pi + \beta\frac{\phi_\pi^a\kappa}{1+\phi_\pi\kappa}} \times \frac{1}{2}\Big) \\
E_1\pi_2 &= 0
\end{aligned} \tag{A.110}$$

We use the same methodology to calculate E_1x_2.

$$
\begin{aligned}
&E_1 x_2 = x_2^{\epsilon}\, Prob\,(x_2^{\epsilon}) + x_2^{-\epsilon}\, Prob\,(x_2^{-\epsilon}) \\
&E_1 x_2 = \Bigg(-\frac{\phi_\pi \epsilon}{\frac{1}{\Upsilon^{AIT}} + \phi_\pi \kappa + \frac{\beta}{\Upsilon^{AIT}} \frac{\phi_\pi^a \kappa}{1+\phi_\pi \kappa}} \times \frac{1}{2}\Bigg) + \Bigg(\frac{\phi_\pi \epsilon}{\frac{1}{\Upsilon^{AIT}} + \phi_\pi \kappa + \frac{\beta}{\Upsilon^{AIT}} \frac{\phi_\pi^a \kappa}{1+\phi_\pi \kappa}} \times \frac{1}{2}\Bigg) \\
&E_1 x_2 = 0
\end{aligned}
\tag{A.111}
$$

In period 1, using the same methodology as we did for the previous periods, we observe the following.

$$
\begin{aligned}
\pi_1 &= \beta E_1 \pi_2 + \kappa x_1 + s_1 \\
\pi_1 &= \kappa x_1
\end{aligned}
\tag{A.112}
$$

Solving for the output gap yields:

$$
\begin{aligned}
x_1 &= E_1 x_2 - \Big(i_1 - E_1 \pi_2\Big) \\
x_1 &= 0 - \Big(\phi_\pi \pi_1 - 0\Big) \\
x_1 &= -\phi_\pi \pi_1 \\
x_1 &= -\phi_\pi \kappa x_1 \\
x_1 &= 0
\end{aligned}
\tag{A.113}
$$

This implies that $x_1 = \pi_1 = 0$.

B Data

The sources for the empirical data used in the main analysis on the unemployment rate, Fed Funds rate, headline and core inflation, and hours worked are as follows.

Unemployment rate: U.S. Bureau of Labor Statistics, Unemployment Rate [UNRATE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNRATE, December 17, 2024.

Fed Funds rate: Board of Governors of the Federal Reserve System (US), Federal Funds Effective Rate [FEDFUNDS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/FEDFUNDS, December 17, 2024.

Inflation:

- U.S. Bureau of Economic Analysis, Personal Consumption Expenditures: Chain-type Price Index [PCEPI], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PCEPI, December 17, 2024
- U.S. Bureau of Economic Analysis, Personal Consumption Expenditures Excluding Food and Energy (Chain-Type Price Index) [PCEPILFE], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PCEPILFE, December 17, 2024

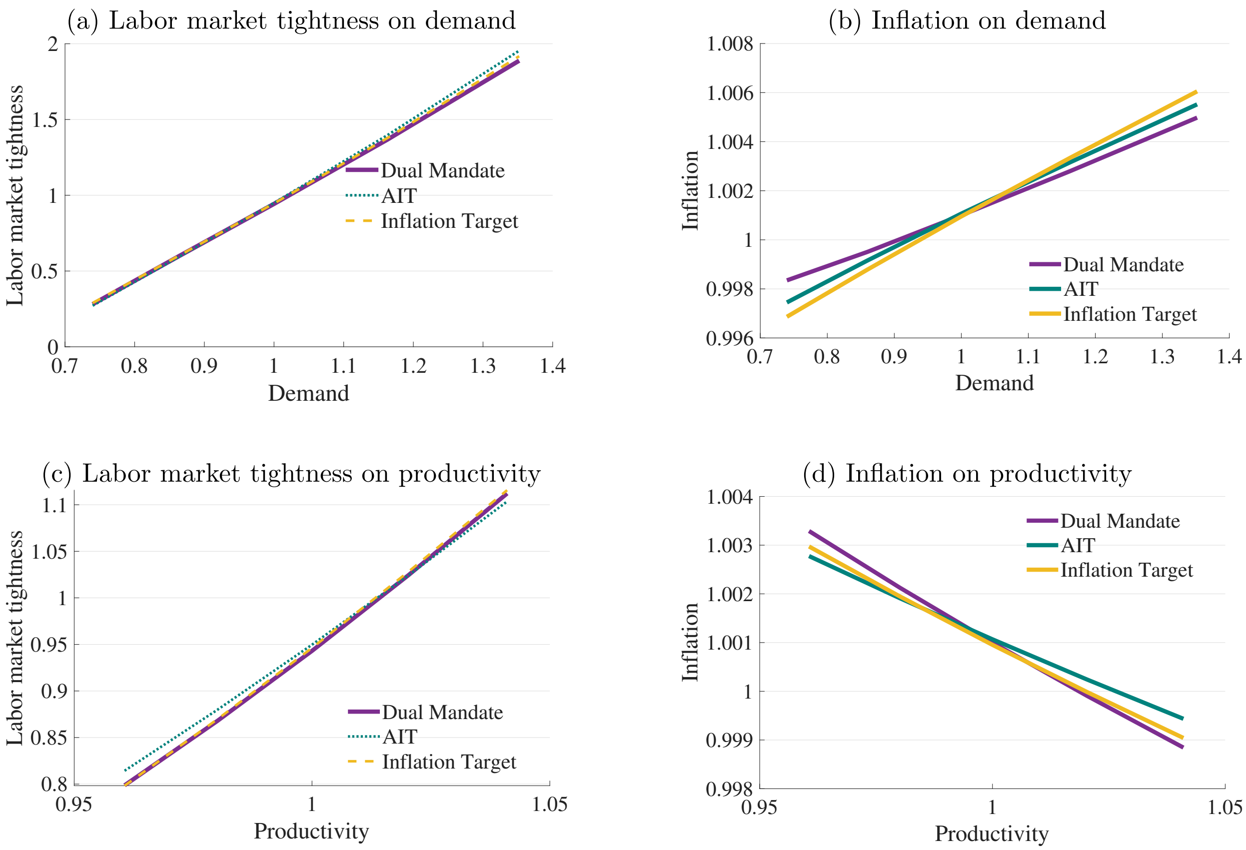
Hours: U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Average Weekly Hours for All Workers [PRS85006023], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PRS85006023, December 17, 2024

C Additional Results Quantitative Model

This section contains the policy functions of the quantitative model and the simulated distribution of outcomes under all three policy rules considered in the main text.

C.1 Model Policy Functions

Figure C.1: Model Policy Functions Under A Dual Mandate & Inflation Targeting



C.2 Comparing Distributions of Outcomes

Figure C.2: Model Distributions Under A Dual Mandate & Inflation Targeting

(a) Unemployment