# Methodology for the Kansas City Fed Model-Based Natural Rate of Interest and Natural Unemployment Rate*

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**Summary**

This technical appendix provides additional information for the construction of the Kansas City Fed model-based natural rate of interest ($r^*$) and natural unemployment rate ($u^*$) described in the Economic Bulletin "Introducing New Monthly Estimates of the Natural Rate of Interest and Natural Unemployment Rate." The two stars are constructed in two steps. The first step estimates a time-varying parameter vector autoregression on monthly series for core personal consumption expenditure inflation, the unemployment rate, and the real federal funds rate. The second step forecasts the real federal funds rate and unemployment rate 60 months forward to generate the current month's estimate of ($r^*$) and ($u^*$).

*The views expressed are those of the authors and do not necessarily reflect the positions of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

## 1. Methodology

Our estimates for KC Fed model-based natural rate of interest ($r^*$) and natural unemployment rate ($u^*$) apply the framework from Lubik and Matthes (2015a). The procedure has two stages. In the first stage, we specify and estimate a Time Varying Parameter Vector Autoregressive with Stochastic Volatility (TVP-VAR-SV), following the approach of Primiceri (2005). In the second stage, the estimated parameters are used to generate iterative forecasts of the model's variables. Furthermore, periods where data is missing, we first use the model's contemporaneous forecast for each missing variable.

### 1.1. *Time-Varying Parameter Bayesian Vector Autoregression (TVP-VAR)*

#### 1.1.1. *TVP-VAR Model*

Time-Varying Parameter (TVP) models allow the coefficients of explanatory variables to evolve over time, providing a flexible framework for capturing dynamic relationships in macroeconomic data. In particular, TVP-BVAR model with stochastic volatility, as proposed by Primiceri (2005) and Del Negro and Primiceri (2015) account for both time-varying coefficients and changing shock variances, enhancing their ability to reflect non-linearities and structural shifts in the economy.

The baseline model is specified as follows:

$$y_t = \beta_{0,t} + \sum_{j=1}^{P} \beta_{j,t} y_{t-j} + \epsilon_t \tag{1}$$

Where $y_t$ is an $n \times 1$ vector of endogenous variables, $\beta_{0,t}$ represents an $n \times 1$ vector of time-varying intercepts, and $\beta_{j,t}$ denotes $n \times n$ matrices of time-varying coefficients corresponding to each lag. The subscript $t$ indicates that these parameters evolve over time, accommodating changes in the underlying economic relationships. The number of variables in the model is denoted by $n$.

To streamline the representation of the model, the lagged variables and can be compactly expressed using matrix notation. The lag structure can be defined as: $X_t' = I \otimes (1, y_{t-1}', \ldots, y_{t-P}')$. Similarly, the time-varying coefficients are collected into a single vector defined as: $\theta_t = \text{vec}([\beta_{0,t}\beta_{1,t}...\beta_{P,t}]')$ This compact form simplifies the estimation process and facilitates the application of Bayesian techniques for parameter inference. We can now write the TVP-VAR as

$$y_t = X_t'\theta_t + \epsilon_t \tag{2}$$

In the TVP model, the coefficient vector $\theta_t$, and the coefficients within, are assumed to follow a random walk as their law of motion

$$\theta_t = \theta_{t-1} + v_t \tag{3}$$

$v_t$ and $\epsilon_t$ are independent. Also, each $u_t$ is independent for each coefficient set within $\theta$.

In addition, as highlighted by Primiceri (2005), time variation in macroeconomic time series can also arise from changes in the second or higher moments of the error terms. This form of variation is captured in the model through stochastic volatility, which reflects time-varying variances and covariances. To incorporate this feature, the model specifies the error covariance matrix as

$$\Omega_t = \Lambda_t^{-1}\Sigma_t\Sigma_t'(\Lambda_t^{-1})' \tag{4}$$

where $\Lambda_t$ represents a lower triangular matrix.

$$\Lambda_t = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \lambda_{21,t} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \lambda_{n1,t} & \cdots & \lambda_{nn-1,t} & 1 \end{bmatrix}$$

and $\Sigma_t$ is the diagonal matrix

$$\Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & \cdots & 0 \\ 0 & \sigma_{2,t} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{n,t} \end{bmatrix}$$

As a standard convention in the literature, the coefficients $\sigma_{j,t}$ are modeled as geometric standard walks:

$$\log \sigma_{j,t} = \log \sigma_{j,t-1} + \eta_{j,t} \tag{5}$$

The coefficients of $\sigma_{j,t}$ can be collated in a vector form as $\sigma_t = [\sigma_{1,t}, ..., \sigma_{n,t}]'$ and $\eta_{j,t}$ in $\eta_t = [\eta_{1,t}, ..., \eta_{n,t}]$ with $\eta_{n,t} \sim \mathcal{N}(0, W)$ and W is diagonal. In the same way, the nonzero and nonunity elements of $\Lambda_t$ are collated in the vector $\lambda_t = [\lambda_{21,t}, ..., \lambda_{nn-1,t}]$ and evolve as random walks:

$$\lambda_t = \lambda_{t-1} + \zeta_t \tag{6}$$

where $\zeta_t \sim \mathcal{N}(0, S)$ and S is block-diagonal. With all these, the error term in equations (1) and (2), $\epsilon_t$, can be decomposed into:

$$\epsilon_t = \Lambda_t^{-1}\Sigma_t\varepsilon_t \tag{7}$$

It is assumed that the error terms in equations (7), (3), (6) and (5) are independent and jointly normally distributed with the variance covariance matrix following this assumptions:

$$V = Var\left(\begin{bmatrix} \varepsilon_t \\ \upsilon_t \\ \zeta_t \\ \eta_t \end{bmatrix}\right) = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix} \tag{8}$$

Where $I_n$ denotes a n-dimensional identity matrix, while Q, S, and W are positive definite matrices. Following Primiceri (2005), this paper adopt a block-diagonal structure for S.

*1.1.2. Bayesian Inference*

Selecting appropriate priors for the parameters is a critical aspect of Bayesian analysis. Since a Gibbs sampler operates iteratively, the initial values of the TVP-VAR parameters must be assigned priors. Following Primiceri (2005), we imposes priors on both the coefficients $(\theta_0, \Lambda_0, \log \Sigma_0)$ and the innovation terms $(Q, W, S)$.

The initial values of the lag coefficient matrices $\theta_0$, the free elements of the loading matrix $\Lambda_0$ in the innovation terms, and the independent innovation variances $\log \Sigma_0$ are assumed to follow a normal distribution:

$$\theta_0 \sim \mathcal{N}(\overline{\theta}, k_\theta V_\theta) \tag{9}$$

$$\Lambda_0 \sim \mathcal{N}(\overline{\Lambda}, k_\Lambda V_\Lambda) \tag{10}$$

$$\log \Sigma_0 \sim \mathcal{N}(\overline{\sigma}, I) \tag{11}$$

Here, $\overline{\theta}, \overline{\Lambda},$ and $\overline{\sigma}$ denote the prior means for the respective parameters, while $V_\theta$ and $V_\Lambda$ are their corresponding covariance matrices. The parameters $k_\theta$ and $k_\Lambda$ act as scaling factors, determining the tightness of the priors.

Additionally, priors are assigned to the innovation variances of the lag coefficient matrices ($Q$), the variance of the shock terms ($W$), and the loading matrix ($S$). Each of these follows an Inverted Wishart distribution:

$$Q \sim \mathcal{IW}(k_Q^2 df_Q V_Q, df_Q) \tag{12}$$

$$W \sim \mathcal{IW}(k_W^2 df_W V_W, df_W) \tag{13}$$

$$S \sim \mathcal{IW}(k_S^2 df_S V_S, df_S) \tag{14}$$

In this framework, the scaling parameters ($k$) determine the flexibility of the prior distributions, $df$ represents the degrees of freedom, and the matrices $V$ define the prior variances. The prior specification is initialized using the first five years of data, with hyperparameters set to $k_Q = 0.005$, $k_S = 0.1$, and $k_W = 0.01$. All other prior settings follow the standard choices in Primiceri (2005).

After specifying the priors, Bayesian estimation proceeds using a block Gibbs sampler as described in Del Negro and Primiceri (2015). The advantage of the Gibbs sampler is its ability to draw from a high-dimensional joint distribution by iteratively sampling from lower-dimensional conditional distributions.

To estimate the full model with both time-varying parameters and stochastic volatility, the following quantities are sampled: $\theta^T$, $\Lambda^T$, $\Sigma^T$, $Q$, $S$, $W$, and $s^T$. Here, $\theta^T$ denotes the time-varying VAR coefficients, $\Lambda^T$ captures the time-varying Cholesky decomposition elements, $\Sigma^T$ is the covariance matrix of the structural shocks, and $s^T$ is the set of latent state variables used in the stochastic volatility step. The superscript $T$ denotes that these parameters evolve over the sample of length $T$.

The Gibbs sampler involves two main steps:

1. Draw $\Sigma^T$ from:
$$\Sigma^T \sim p(\Sigma^T \mid y^T, \theta^T, \Lambda^T, Q, S, W, s^T)$$

2. Draw the remaining parameters jointly from:
$$(Q, S, W, \Lambda^T, \theta^T, s^T) \sim p(Q, S, W, \Lambda^T, \theta^T, s^T \mid y^T, \Sigma^T)$$

To improve efficiency, the second step is decomposed into a series of conditional draws as follows:

- Draw $\Lambda^T$ from:
$$\Lambda^T \sim p(\Lambda^T \mid y^T, \theta^T, Q, S, W, \Sigma^T)$$

- Draw $Q$, $S$, and $W$ from:
$$(Q, S, W) \sim p(Q, S, W \mid y^T, \theta^T, \Lambda^T, \Sigma^T)$$

- Draw $\theta^T$ from:
$$\theta^T \sim p(\theta^T \mid y^T, \Lambda^T, Q, S, W, \Sigma^T)$$

- Draw $s^T$ from:
$$s^T \sim p(s^T \mid y^T, \theta^T, \Lambda^T, Q, S, W, \Sigma^T)$$

Detailed steps for drawing posterior samples are provided in Del Negro and Primiceri (2015) and Lubik and Matthes (2015b). In this paper, we estimate the TVP-VAR model with stochastic volatility using 2,000 burn-in iterations and 100,000 posterior draws, assuming a lag length of six.

### *1.2. VAR Iterated Forecast Method*

Following the application of the TVP-VAR with SV, as outlined in section 1.1, the model parameters are extracted over time and across multiple draws. These parameters are then used in a step-by-step iterated forecasting approach to estimate the neutral rates. In this paper, we use this approach to estimate both $r^*$ and $u^*$.

At any given time $t$, the forecast horizon, denoted as $h$, represents the number of months into the future for which predictions are made. We forecast horizons from one month to 60 months (equivalent to 5 years). The hat symbol $\hat{y}_{(t+h)|t}$ indicates an $h$-step-ahead forecast value, representing the estimated outcome using all available information up to time $t$. For example, $\hat{y}_{(t+1)|t}$ denotes the one-month-ahead forecast. Also, as discussed in section 1.1, $y_t$ is an $n \times 1$ vector of endogenous variables, $\beta_{0,t}$ represents an $n \times 1$ vector of time-varying intercepts, and $\beta_{j,t}$ denotes $n \times n$ matrices of time-varying coefficients corresponding to each lag. The subscript $t$ indicates that these parameters evolve over time, accommodating changes in the underlying economic relationships. The number of variables in the model is denoted by $n$.

Given a model with six lags, the first six steps of the forecast ($h = 1$ to $6$) are represented as follows:

$$\hat{y}_{(t+1)|t} = \beta_{0,t} + \sum_{i=1}^{6} \beta_{i,t} y_{t+1-i} \tag{15}$$

For subsequent periods, the forecasts are constructed iteratively using the previously estimated values:

$$\hat{y}_{(t+2)|t} = \beta_{0,t} + \beta_{1,t}\hat{y}_{(t+1)|t} + \sum_{i=2}^{6} \beta_{i,t} y_{t+2-i} \tag{16}$$

$$\hat{y}_{(t+3)|t} = \beta_{0,t} + \sum_{j=1}^{2} \beta_{j,t}\hat{y}_{(t+3-j)|t} + \sum_{i=3}^{6} \beta_{i,t} y_{t+3-i} \tag{17}$$

$$\hat{y}_{(t+4)|t} = \beta_{0,t} + \sum_{j=1}^{3} \beta_{j,t}\hat{y}_{(t+4-j)|t} + \sum_{i=4}^{6} \beta_{i,t} y_{t+4-i} \tag{18}$$

$$\hat{y}_{(t+5)|t} = \beta_{0,t} + \sum_{j=1}^{4} \beta_{j,t}\hat{y}_{(t+5-j)|t} + \sum_{i=5}^{6} \beta_{i,t} y_{t+5-i} \tag{19}$$

$$\hat{y}_{(t+6)|t} = \beta_{0,t} + \sum_{j=1}^{5} \beta_{j,t}\hat{y}_{(t+6-j)|t} + \beta_{6,t} y_t \tag{20}$$

For all $h > 6$, each $h$ is forecasted using:

$$\hat{y}_{(t+h)|t} = \beta_{0,t} + \sum_{j=1}^{6} \beta_{j,t}\hat{y}_{(t+h-j)|t} \tag{21}$$

Once the endpoint of the forecast horizon is reached, the final values from the real rate and unemployment rate equations are stored as the estimated $r^\star$ and $u^\star$, respectively, for period $t$. This procedure is repeated for every period and across all posterior draws, thereby generating a full time series for the KC Fed model-based $r^\star$ and $u^\star$.

## 2. Brief Data and Methodological Comparison with Lubik and Matthes (2015a)

As shown in Table 1, we use monthly data spanning from January 1961 to November 2025. Table 1 compares the similarities and differences in the data transformation we follow and those used by Lubik and Matthes (2015a), while Table 2 highlights the methodological distinctions and their potential implications.

Table 1: Data Sources and Transformations

| **Variables** | **What Lubik and Matthes (2015a) did** | **What we did** | **Data sources** |
|---|---|---|---|
| Unemployment rate | — | Level of unemployment rate [1] | U.S. Bureau of Labor Statistics (Haver Analytics) |
| Real GDP | Percent quarter-over-quarter annualized growth in real GDP | — | U.S. Bureau of Economic Analysis (Haver Analytics) |
| Core PCE (Inflation rate) | Percent quarter-over-quarter annualized core PCE growth | Percent month-over-month annualized core PCE growth | U.S. Bureau of Economic Analysis (Haver Analytics) |
| Nominal interest rate | Quarterly Effective Federal Funds Rate (EFFR) | Monthly EFFR | Federal Reserve Board (Haver Analytics) |
| Inflation expectations | Four quarter moving average of core PCE inflation for quarterly | 12 month moving average of Core PCE inflation for monthly (For instance, Inflation expectations at month t is calculated by taking average of MoM annualized core PCE growth in periods t-12 to t-1) | Calculated by authors |
| Ex-ante real interest rate | EFFR minus quarterly inflation expectations | EFFR minus monthly inflation expectations | Calculated by authors |

[1] Alternatively, we can use the month-to-month change in the unemployment rate, which can be linked to Okun's law Okun (1963). We present results based on this alternative specification in Appendix A.

Table 2: Differences and Similarities in the Model Assumptions

| **Model assumptions** | **What Lubik and Matthes (2015a) did** | **What we did** | **Implications of what we did, if any** |
|---|---|---|---|
| $k_Q$ (Degree of variation in coefficients) | 0.001 | 0.005 | Allows slightly greater time variation in the coefficients, which can better capture structural shifts in monthly data. |
| $k_S$ (Degree of variation in covariance states) | 0.1 | 0.1 | Same as in Lubik and Matthes (2015a), implying similar evolution of the error covariance structure. |
| $k_W$ (Degree of variation in stochastic volatility) | 0.01 | 0.01 | Same specification, ensuring consistency in volatility dynamics across models. |
| Training sample | 20 quarters (5 years) | 60 months (5 years) | Maintains a comparable training window in terms of time length, ensuring stable initialization of the Kalman filter. |
| Burn-in draws | 2000 | 2000 | Same setting; ensures convergence before posterior sampling begins. |
| Forecast real rate | Forecast 20 quarters ahead | Forecast 60 months ahead | Preserves the 5-year horizon of Lubik and Matthes (2015a) but in monthly frequency, producing a more timely estimate of the neutral rate. |

## 3. Data Input During Shutdowns or Other Disruptions

In the event that a government shutdown or other disruption delays the release of key macroeconomic indicators, we first use the model's forecasts to estimate the missing variables and then proceed to stage 2. We generate forecasts one month at a time because some components of the model remain available on schedule. For instance, the nominal Federal Funds rate is available every month without interruption, and inflation expectations are computed each month using a moving average procedure. Producing forecasts incrementally ensures that the most recent information from these available variables is incorporated into the estimation each time the model is updated.

Formally, after estimating the TVP-VAR-SV model using all data available up to period $t$, the model produces the predictive distribution for $y_{t+1}$, representing the forecast for the next month. Conditioning on the most recent $p$ lags of the endogenous variables, $y_t, y_{t-1}, \ldots, y_{t-p+1}$, the model simulates the next observation within the Bayesian framework. For each posterior draw of the model parameters, comprising the time-varying coefficients $\theta_t$, the time-varying Cholesky decomposition elements $\Lambda_t$, and the covariance matrix of the structural shocks $\Sigma_t$, the model generates one realization of the next observation according to

$$y_{t+1}^{(r)} = X_t' \theta_t^{(r)} + \Lambda_t^{-1(r)} \Sigma_t^{(r)} \varepsilon_{t+1}^{(r)}, \qquad \varepsilon_{t+1}^{(r)} \sim \mathcal{N}(0, I), \tag{22}$$

where $r$ indexes posterior draws, and other variables are as discussed earlier.

Repeating this procedure for all retained posterior draws $\{\theta^{(r)}\}_{r=1}^{R}$ yields a collection of predictive samples $\{y_{t+1}^{(r)}\}_{r=1}^{R}$ that jointly form the posterior predictive distribution of $y_{t+1}$. The point forecast is given by the posterior median of these simulated draws, while the credible intervals are constructed from their posterior quantiles. This distribution summarizes the expected value and forecast uncertainty for period $t + 1$, conditional on the information available through period $t$.

## References

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## Appendix

### A. Alternative $r^\star$ and $u^\star$ Using Changes in the Unemployment Rate

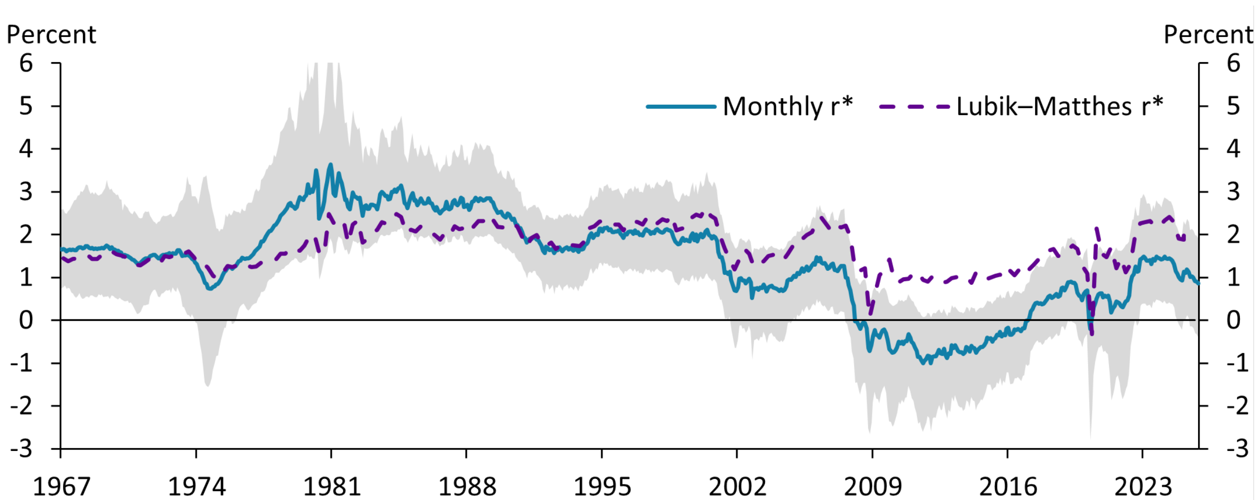
The approach used to construct this alternative measure closely mirrors the methodology described earlier. The key distinction is that, instead of using the level of the unemployment rate, we follow the Okun's law approach Okun (1963) and employ month-to-month changes in the unemployment rate as a proxy for economic activity.

We repeat the same modeling framework and estimation procedure to obtain the alternative measure of $r^\star$. However, the construction of $u^\star$ from the model requires a slightly different procedure. Because the model includes the change in the unemployment rate rather than its level, we recover $u^*$ by accumulating the forecasted changes. Specifically, as shown in Equation A.1, we add the current level of unemployment at time $t$ to the cumulative sum of the forecasted changes in the unemployment rate from horizons 1 to 60:

$$u_t^* = u_t + \sum_{h=1}^{60} \hat{y}^{ur}{}_{(t+h)|t} \tag{A.1}$$

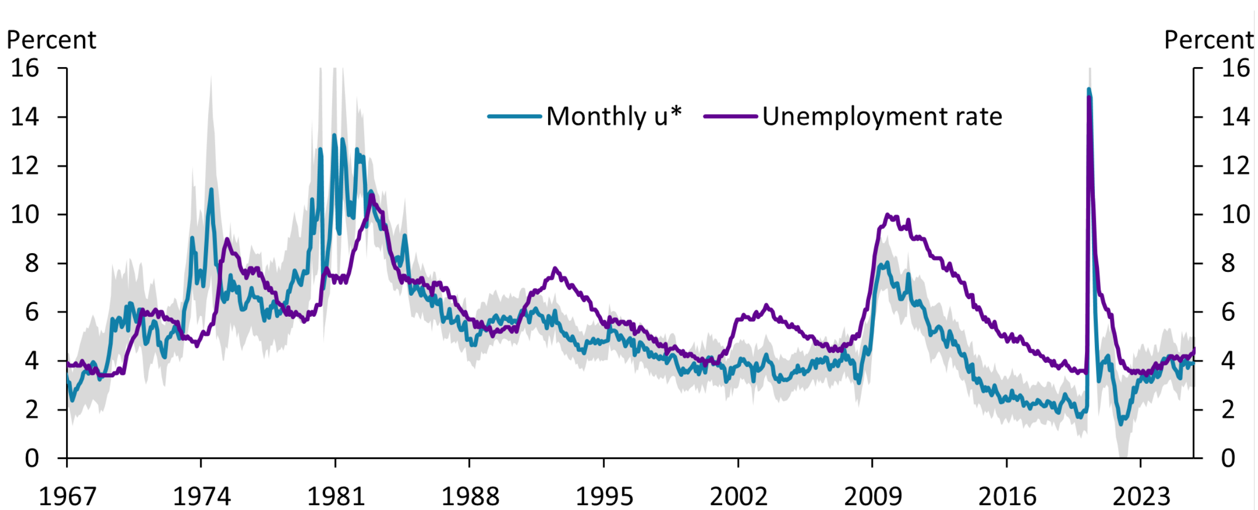
where $u_t$ is the unemployment rate at time $t$, $\hat{y}^{ur}{}_{(t+h)|t}$ is the $h$-step-ahead forecast (from the model) of the change in the unemployment rate at time $t+h$, conditional on information at time $t$.

Figure A.1: Monthly $r^{\star}$ Compared with Quarterly Lubik-Matthes




*Notes: Blue line is the median of estimates of the five-year-ahead forecast of the real interest rate from 100,000 simulations. Gray shaded area shows the upper and lower bounds for 68 percent of estimates from these simulations. We used month-to-month changes in the unemployment rate as a proxy for economic activity instead of level of unemployment rate in the model.*

Figure A.2: Monthly $u^{\star}$ Compared with Current Unemployment Rate




*Note: Blue line is the median of estimates of the forecast generated using Equation A.1 from 100,000 simulations. Gray shaded area shows the upper and lower bounds for 68 percent of estimates from these simulations.We used month-to-month changes in the unemployment rate as a proxy for economic activity instead of level of unemployment rate in the model.*