The Race Between Asset Supply and Asset Demand*

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Abstract

We introduce an asset supply-and-demand approach to analyze the trajectory of U.S. aggregate wealth, real interest rates, and fiscal sustainability. Our framework uses micro-founded and easy-to-implement sufficient statistics to quantify how shifts in demographics, inequality, and other forces affect asset market equilibrium. From 1950 to the present, rapid population aging, rising income inequality, increasing foreign demand for U.S. assets, and declining productivity growth all contributed to a surge in asset demand. Asset supply initially fell, then turned around sharply, mainly driven by increases in government debt and the value of capitalized profits. Overall, asset demand won the race, and interest rates fell. Looking ahead, population aging will continue to push up asset demand, but at current tax and benefit levels, it will drive even larger increases in debt through the rising costs of Social Security and Medicare. Making debt sustainable requires a fiscal consolidation of at least 10% of GDP, but debt could reach 250% of GDP without pushing up interest rates.

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1 Introduction

The U.S. economy has experienced a number of dramatic changes over the last 75 years. The fertility rate fell from over 3 children per woman in the 1950s to 1.6 today, causing the population to grow older, and income inequality has risen sharply since the 1970s. In the last few decades, government debt has also risen along with the value of private assets, while the net foreign asset position has fallen to a record low.

In this paper, we ask two questions. First, how have these shifts affected equilibrium interest rates and wealth over the last 75 years? Second, what will happen going forward, and how will this interact with U.S. fiscal challenges over the next 75 years?

We address these questions using an asset supply-and-demand framework. Our framework is based on the observation that the long-run interest rate r^* equates asset supply $\mathcal{A}^s$, the value of government debt and privately supplied assets, with asset demand $\mathcal{A}^d$, the desired asset holdings by households and foreigners:

$$\mathcal{A}^s(r^*) = \mathcal{A}^d(r^*), \tag{1}$$

where we normalize $\mathcal{A}^s$ and $\mathcal{A}^d$ by GDP. Movements in the natural rate r^* and wealth-to-GDP are driven by shifts in the asset supply and demand curves, as illustrated in figure 1. Higher asset supply raises both wealth-to-GDP and r^*, while higher asset demand raises wealth-to-GDP but pushes down r^*.

The framework implies that changes in r^* are driven by a *race between asset supply and demand*. If asset supply rises more than demand, interest rates increase, threatening fiscal sustainability. If asset demand rises more than supply, interest rates fall, pushing the economy closer to the zero lower bound. The goal of our paper is to understand which side won the race over the last 75 years, why that side won, and what the prospects are for the next 75 years.

We set up a standard general equilibrium model and show how it can be reduced to equation (1). In our model, asset supply $\mathcal{A}^s(r)$ consists of government debt, private capital, and the capitalized value of profits and rents, and it has a simple closed-form expression in terms of standard parameters and the interest rate. Asset demand consists of asset holdings by foreigners and domestic households, where the latter are determined by overlapping generations solving a life-cycle problem. The household side of the model features a realistic age structure, bequest motives, a tax-and-transfer system, and heterogeneous savings by income groups.

While household asset demand does not have a simple closed-form expression like

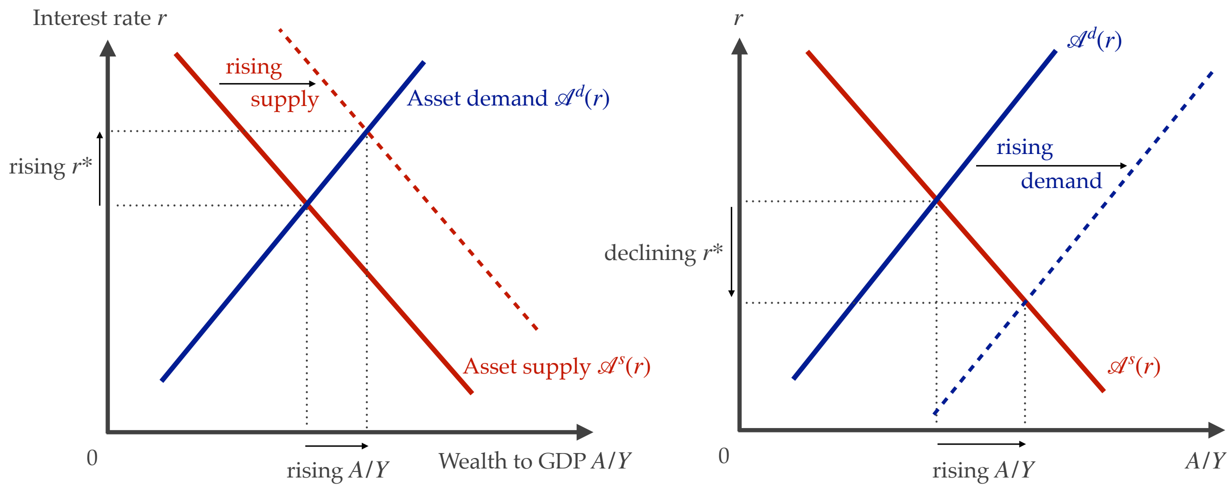


Figure 1: Equilibrium effects of asset supply and demand shifts

asset supply, we show that the slope of the asset demand curve and the effects of key shifters can be expressed using measurable sufficient statistics. For example, as in Auclert, Malmberg, Martenet and Rognlie (2024), we can capture the main effect of an aging population on asset demand with a simple shift-share calculation, which interacts the profile of assets by age with the changing age composition of the population. Since older households tend to hold more assets, an aging population increases asset demand. We similarly model the effects of inequality, showing that since rich households have higher wealth relative to income (Dynan, Skinner and Zeldes 2004, Straub 2019), higher inequality increases asset demand by increasing the share of income accruing to these households. We derive an analogous formula for the effects of productivity growth, and also obtain sufficient statistics for the effects of changes in social security, taxes, and the labor share.

Armed with our framework, we replay the race between asset supply and asset demand over the last 75 years. We first observe that asset demand decisively won the race: aggregate wealth to GDP in the U.S. rose to unprecedented levels while interest rates fell significantly, consistent with the right panel of figure 1. Backing out the implied shifts in each curve, we find that the asset demand curve shifted to the right by a remarkable 415% of GDP; that is, at unchanged interest rates, households and foreigners would have been willing to hold an additional 415% of GDP in assets relative to 1950. This far exceeded the 31% of GDP rightward shift in the asset supply curve. In our framework, this large imbalance explains the long-term decline in interest rates.

Next, we use our characterizations of asset supply and demand shifters to decompose these overall shifts into contributing factors. On the asset demand side, the factors we study jointly explain a shift of 310% of GDP. Demographics, inequality, falling productivity growth, and foreign asset demand all strongly increased demand for U.S. assets. This increase was partially offset by two smaller, negative factors: the expansion of social security and a reduced labor share. On the asset supply side, the contributions from different factors have varied significantly since 1950. Until about 1980, asset supply fell—but since then it has increased by 180% of GDP, mainly due to a rise in government debt and a higher value of capitalized profits and rents.

Next, we turn to the future. Asset demand outpaced supply over the last 75 years, but what can we say about the next 75 years? While the direction of many forces is unclear, one trend that almost surely will continue is population aging. Fertility is on a downward trend, and even if this trend stops and fertility stabilizes, a growing share of the population will still be over 65 in the decades to come. This has implications for both asset demand and supply.

For asset demand, population aging means that the upward trend from recent decades will continue. Using demographic projections to extend our historical analysis, we project that aging will raise asset demand by an additional 200% of GDP between 2024 and 2100. On its own, this would imply a continued decline in real interest rates.

But demographics also play an important role in asset supply, working through government finances. As the population ages, the fiscal burden of Social Security and Medicare will rise dramatically. Applying a simple shift-share analysis to the age profile of entitlement spending, we project that these costs will reach nearly 20% of GDP by 2100, up from 11% today. Absent a large fiscal adjustment, the resulting deficits will sharply increase debt and therefore asset supply. This potential rise in asset supply accelerates throughout the century, surpassing 100% of GDP by 2060 and exploding to nearly 500% of GDP by 2100.

Thus, until fiscal consolidation occurs, there will be a race between the rising asset demand of an older population and the rising debt issuance needed to finance the associated increase in government expenditures. The higher asset demand creates fiscal space for the government to expand debt without pushing up interest rates, consistent with Japan's recent experience. The scale of the fiscal challenge, however, means that without large adjustments, the supply of debt will eventually outrun demand, forcing interest rates to rise.

Given that fiscal adjustment is eventually necessary, we explore different scenarios for this adjustment, and how they interact with the asset market. In our baseline, it is possible to push long-run debt to 250% of GDP without raising interest rates—even taking into account the negative effect on asset demand from the higher taxes needed to stabilize debt. Lower-than-expected fertility has mixed implications: by raising asset demand, it creates more space for debt to rise, but it also makes the long-run fiscal adjustment even larger. If working life can be extended, and old-age benefits delayed, the fiscal gap shrinks—but even if the minimum retirement age is 75, a 6% of GDP adjustment remains. Finally, if adjustment is achieved entirely by slashing benefit levels, rather than extending working life, the fiscal cost is attenuated by asset market feedback, as interest rates are pushed down by the savings response to smaller benefits. The required benefit cuts, however, remain very large, on the order of 50%.

One important question is whether government debt can sustain its low interest rate relative to other assets even as the debt burden grows. We consider an extension that relaxes our baseline assumption of a fixed interest rate spread between government debt and other assets, and instead assumes that this gap shrinks as debt rises, in line with Krishnamurthy and Vissing-Jorgensen (2012). In this case, the government can no longer issue 250% of GDP in debt without facing interest rate consequences: while there is sufficient demand for assets in the aggregate, the market cannot absorb such a large increase in government debt without eroding the government's funding advantage. With an endogenous spread, very high debt levels also become catastrophic. For example, to stabilize at 400% debt-to-GDP, the primary surplus required would be 17% of GDP.

Finally, we use our framework to analyze the asset market and fiscal implications of various long-run scenarios. For example, we show that if the profitability of private businesses continues to rise, this generates a *reverse crowding-out effect*: growth in private asset supply pushes up interest rates, making it harder to sustain high government debt. On the other hand, if labor income inequality continues to increase, the resulting rise in asset demand will push interest rates down. Rising productivity growth has mixed effects, but is ultimately a net fiscal benefit: interest rates rise, but the rise in growth decreases the net cost $(r-g)B/Y$ of sustaining government debt, and the cost of old-age benefits also falls.

Related literature. Our paper builds most directly on Auclert, Malmberg, Martenet and Rognlie (2024), who study the effect of demographics on interest rates and wealth levels using an asset supply and demand framework, together with sufficient statistics based on the population age distribution and life-cycle profiles of assets, consumption, and labor supply. We extend their framework along two dimensions. Historically, we study a larger set of drivers, including changes in inequality, the labor share, productivity growth, and foreign asset demand. Prospectively, we incorporate richer fiscal dynamics to study how asset supply and demand interact with fiscal sustainability. Unlike their paper, however, we restrict our analysis to the U.S., and rely entirely on sufficient statistics rather than a calibrated model.

More generally, we relate to two main branches of the literature: one on drivers of the equilibrium real interest rate (often labeled r^*), and one on fiscal sustainability.

A common framework for measuring r^* is the representative-agent Euler equation (Laubach and Williams 2003, Holston, Laubach and Williams 2017). In this framework, asset demand is infinitely elastic, so that movements in r^* only reflect changes in trend growth and the rate of time preference. Our framework, by contrast, builds on a tradition of overlapping-generations models, which feature imperfectly elastic asset demand and therefore consider a richer set of drivers of r^* (Rachel and Summers 2019, Eggertsson, Mehrotra and Robbins 2019, Peruffo and Platzer 2024).

One strand of the literature specifically considers the role of demographics (Carvalho, Ferrero and Nechio 2016, Lisack, Sajedi and Thwaites 2021, Gagnon, Johannsen and Lopez-Salido 2021, Jones 2023). These papers show the importance of falling fertility and rising longevity in increasing desired aggregate asset holdings. Our sufficient statistics for life cycle models also relate to the national transfer accounts literature (Lee and Mason, 2011).

Other drivers of asset demand considered by the literature include productivity growth (Blanchard 2023), inequality (Auclert and Rognlie 2018, Straub 2019), and foreign asset demand (Bernanke 2005, Caballero, Farhi and Gourinchas 2008, Mendoza, Quadrini and Rios-Rull 2009, Obstfeld 2025). We provide sufficient statistics for each of these. For instance, leveraging Blanchard (2023)'s insight that productivity growth does not affect life-cycle decisions when growth loads on cohort effects, we show that aggregate asset demand is determined by a compositional effect across cohorts, which can be calculated similarly to the compositional effect of demographics in Auclert et al. (2024).

Several papers have computed the compositional effects of demographics on the flow of saving, instead of the stock of assets (e.g. Summers, Carroll and Blinder 1987, Auerbach and Kotlikoff 1990, Bosworth, Burtless, Sabelhaus, Poterba and Summers 1991, Mian, Straub and Sufi 2021). We view the stock approach as more accurate, especially over the long run (see section 5 in Auclert et al. 2024). This is why our results for demographics differ from those in Mian, Straub and Sufi (2020), though we confirm their results for inequality.

The literature on imperfectly elastic asset demand has also stressed forces on the asset supply side, such as markups (Eggertsson, Robbins and Wold 2021), capital intensity (Moll, Rachel and Restrepo 2022), and government debt. We provide a decomposition of asset supply into each.

A related literature points out that there are multiple r^*'s, with wedges between various measures driven by liquidity premia (Del Negro, Giannone, Giannoni and Tambalotti

2017), safety premia, and risk premia (Farhi and Gourio 2018, Marx, Mojon and Velde 2021, Kopecky and Taylor 2022).[1] Our framework includes a distinction between the "safe" rate paid by the government and the "risky" rate on capital, but maintains a simple structure for asset market equilibrium, in which an *average* of these two rates matters. We abstract from aggregate risk, and instead model the return spread as coming from household preferences for holding different types of assets; shifts in this wedge then enter as shifters of asset supply.

Our paper also relates to the literature on fiscal sustainability. Part of this literature considers the implications of low and falling risk-free rates for the long-term government budget constraint (e.g. Blanchard 2019, Reis 2022, Mian, Straub and Sufi 2022). The basic observation from this literature is that the primary deficit consistent with a constant ratio of debt to GDP B/Y is $(g - r^*)B/Y$—but importantly, that raising B/Y also increases asset supply and therefore r^*. While this effect is present in our model, it has a much smaller impact on fiscal adjustment compared to the effect of demographic change on Medicare and Social Security spending. Our focus on the balance between asset supply and demand is related to the idea in Mian et al. (2022) that there is a "Goldilocks zone" where government debt is neither too high (becoming unsustainable), nor too low (causing the zero lower bound to bind).

A separate literature focuses on trends in entitlement program spending, such as Social Security and Medicare, and uses projected demographics and other factors to calculate the pressure that these programs put on public deficits (Congressional Budget Office 2025, Social Security and Medicare Boards of Trustees 2025). We use similar calculations to forecast asset supply at constant policy. Without any fiscal adjustment, asset supply would diverge and be guaranteed to win the race; we therefore consider fiscal adjustment required to stabilize the debt in the long run. In this, our approach is similar in spirit structural dynamic equilibrium models used to study the consequences of demographic change or the implications of fiscal policy changes (Auerbach and Kotlikoff 1987, Penn Wharton Budget Model 2019).

2 An asset supply and demand framework

Figure 2 plots the average expected real rate of return r on U.S. assets, as well as the total value of assets held in the U.S., relative to GDP from 1950 through 2024. We construct the

[1] Other papers point out the difficulty of observing even a single r^*, as nominal rigidities drive a wedge between the safe short-term interest rate and the equilibrium safe rate.

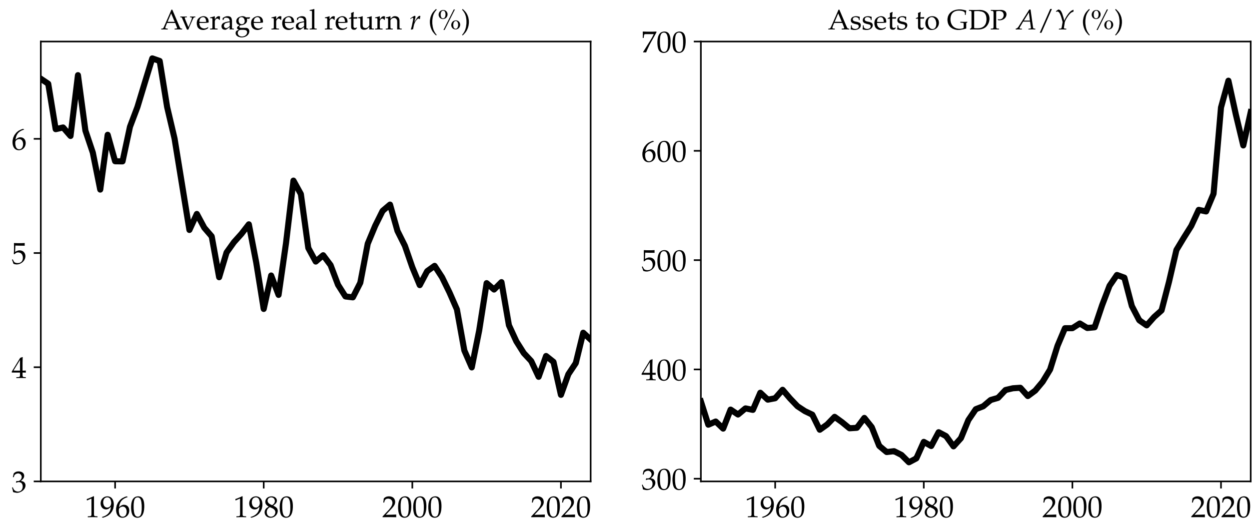


Figure 2: Falling returns and rising wealth, 1950–2024

average return as the weighted average $r = \bar{s}r^f + (1-\bar{s})r^r$ between a safe return r^f on government bonds and a risky return r^r on private assets, where $\bar{s} = 0.11$ is a benchmark safe portfolio share.[2] The average return has fallen by about 2 percentage points since the 1950s, while the value of assets has grown by around 300% of GDP—a rise that began in the 1980s. This section shows how the patterns in figure 2 can be interpreted through the lens of an equilibrium framework, where the return r is determined as the price that equilibrates the supply and demand for assets relative to GDP.

2.1 Asset market equilibrium

We postulate a simple long-run asset supply-and-demand framework, which we microfound in section 3. In this framework, the equilibrium return r^* is determined by the intersection of an *asset supply* curve $\mathcal{A}^s(r)$ and an *asset demand* curve $\mathcal{A}^d(r)$.[3] The equilibrium condition is:

$$\mathcal{A}^s(r^*) = \mathcal{A}^d(r^*). \tag{2}$$

[2] The safe return r^f is the smoothed real yield of a U.S. 5-year Treasury, while the risky return r^r is obtained by calculating the ratio of net capital income, taken from the national accounts, to the total value of privately supplied assets. $\bar{s} = 0.11$ is the average share of government bonds in total U.S. asset supply over the period 1950–2024. See appendix A for more details and plots of both return series.

[3] With our microfoundation in section 3, only the average rate r matters for asset demand $\mathcal{A}^d(r)$. For asset supply, the risky rate r^r matters, but it moves one-for-one with r conditional on the risk premium $rp \equiv r^r - r^f$, which we take as an exogenous shifter of the $\mathcal{A}^s$ curve, and leave implicit here. We later endogenize rp in section 4.

Asset supply $\mathcal{A}^s$ is the value of domestic assets, relative to GDP, available for households and foreigners to hold at a given interest rate. Total assets are the sum of government bonds B/Y, capital K/Y, and the capitalized value Π/Y of profits:

$$\mathcal{A}^s = \frac{B}{Y} + \frac{K}{Y} + \frac{\Pi}{Y}. \tag{3}$$

Asset demand $\mathcal{A}^d$ is the net value of domestic assets, relative to GDP, that households and foreigners desire to hold at a given interest rate. We write it as the sum of household asset demand A^h/Y and net asset demand by foreigners, which is minus the net foreign asset position of the US, $-NFA/Y$:

$$\mathcal{A}^d = \frac{A^h}{Y} - \frac{NFA}{Y}. \tag{4}$$

In our model, $\mathcal{A}^d(r)$ will generally be upward-sloping in r, as higher returns encourage households to hold more assets, and $\mathcal{A}^s(r)$ will be downward-sloping in r, as higher ex-ante returns push down the optimal capital stock and the capitalized value of profits. These curves are represented by the solid lines in figure 1 from the introduction.

Figure 3 plots the components of asset supply and demand that have been realized in equilibrium over time in the U.S. We build these figures by taking B, K, and Y from the national accounts, and A^h and NFA from the financial accounts, then determining Π residually so that $\mathcal{A}^s = \mathcal{A}^d$. (See appendix A.1 for details.) On the supply side, assets have risen due to an expansion of government debt B/Y and the value of profits Π/Y, with a smaller role for capital K/Y.[4] On the demand side, there has been a rise in both domestic and foreign asset holdings.

2.2 Supply and demand shifts

Through the lens of this framework, changes in r^* and A/Y must reflect shifts in the asset supply curve $\mathcal{A}^s(r)$, shifts in the asset demand curve $\mathcal{A}^d(r)$, or both.

Asset supply is shifted up by forces that raise the value of existing assets relative to GDP at given interest rates. These forces include increases in government debt, rising profitability, and increases in the capital intensity of production. The left panel of figure 1 shows that in equilibrium, these shifts raise the asset-to-GDP ratio, but also raise the

[4] As discussed in appendix A.1, our residual approach to calculating Π/Y means that it includes both the capitalized value of pure profits and of any other rents, such as land rents. The latter likely also contributed to its rise.

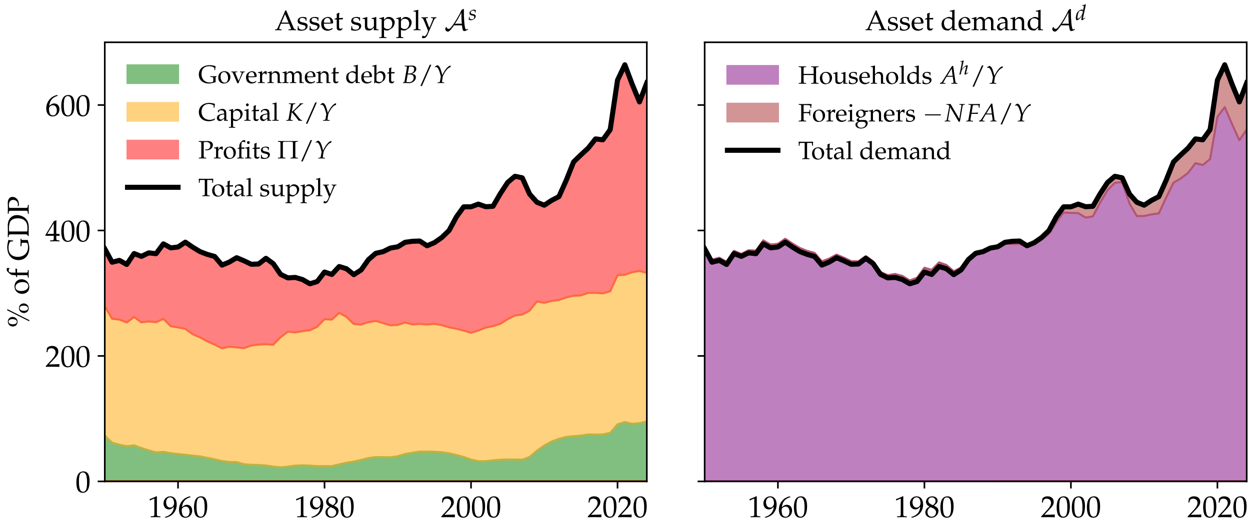


Figure 3: Realized components of asset supply and demand, 1950–2024

equilibrium interest rate r^*. The latter is inconsistent with the decline in returns observed in figure 2; hence, even though asset supply may have shifted upwards, this cannot be the entire story.

Asset demand, on the other hand, is shifted up by forces that increase desired aggregate asset holdings relative to GDP at given interest rates, such as changes in income distribution or in the age distribution of the population that raise the share of the population with high asset holdings. The right panel of figure 1 shows that these types of shifts raise assets-to-GDP and lower the equilibrium interest rate at the same time, consistent with the patterns observed in figure 2.

In summary, historical trends in A/Y and r suggest that, so far, asset demand has won the race with asset supply—while both may have increased, the increase in asset demand has outpaced the increase in asset supply. In the rest of the paper, we use economic theory to quantify these historical shifts in asset supply and demand as well as their individual components. We then use projections of each of these components to consider what might happen to interest rates and the sustainability of government debt in the future.

Formally, we represent the shifters of asset supply and demand with a parameter η that augments both curves $\mathcal{A}^s(r,\eta)$ and $\mathcal{A}^d(r,\eta)$. Taking logs of (2) and differentiating with respect to η, we find that the first-order change in r^* from a movement in η is given by

$$dr^* = \frac{\partial \log \mathcal{A}^s - \partial \log \mathcal{A}^d}{\epsilon^s + \epsilon^d}, \tag{5}$$

where

$$\epsilon^s \equiv -\frac{\partial \log \mathcal{A}^s(r,\eta)}{\partial r} \qquad \text{and} \qquad \epsilon^d \equiv \frac{\partial \log \mathcal{A}^d(r,\eta)}{\partial r}$$

are the (semi-)elasticities of asset supply and demand with respect to r (normalized so that they are both usually positive), and where we use $\partial \log \mathcal{A}^s$ as a shorthand for the "partial equilibrium" shift $(\partial \log \mathcal{A}^s(r,\eta)/\partial\eta) \cdot d\eta$ in asset supply, and likewise for $\partial \log \mathcal{A}^d$.

Equation (5) shows that the equilibrium movement in r^* is determined by the *difference* in supply and demand shifts. Since the denominator $\epsilon^s + \epsilon^d$ is generally positive, a decline in r^* requires a relatively larger shift in asset demand, i.e. $\partial \log \mathcal{A}^d > \partial \log \mathcal{A}^s$, formalizing the intuition described above.

We can also solve for the first-order equilibrium asset change induced by η, which is

$$d \log \left(\frac{A}{Y}\right) = \partial \log \mathcal{A}^s - \epsilon^s dr^* = \partial \log \mathcal{A}^d + \epsilon^d dr^* \tag{6}$$

$$= \frac{\epsilon^d \cdot \partial \log \mathcal{A}^s + \epsilon^s \cdot \partial \log \mathcal{A}^d}{\epsilon^s + \epsilon^d}. \tag{7}$$

Equation (7) shows that the equilibrium movement in assets-to-GDP is a weighted average of the shifts in asset supply and demand. Asset supply shifts have a larger effect on equilibrium A/Y when demand is elastic, and vice versa.

Given observed equilibrium changes in r^* and A/Y, we can use equations (5) and (6) to back out the shifts $\partial \log \mathcal{A}^s$ and $\partial \log \mathcal{A}^d$ in the asset supply and demand curves, provided we have values for the two elasticities ϵ^d, ϵ^s, as

$$\partial \log \mathcal{A}^s = d \log(A/Y) + \epsilon^s dr^* \tag{8}$$

and

$$\partial \log \mathcal{A}^d = d \log(A/Y) - \epsilon^d dr^*. \tag{9}$$

2.3 Recovering historical shifts in asset supply and demand

We now implement this simple framework using the time series of U.S. interest rates and assets-to-GDP from figure 2 as our measures of r_t^* and A_t/Y_t at each time t. This assumes, in effect, that long-run equilibrium is achieved at each t, an assumption we maintain throughout the paper.[5] We use the values $\epsilon^s = 19.8$ and $\epsilon^d = 8.9$ that we obtain in section

[5] In principle, shocks have transitional effects before moving the system back to steady state, and nominal rigidities can imply short-run departures of observed r and A/Y from their flexible-price counterparts. We abstract from these effects for the purpose of this paper, noting that they should cause short-term departures

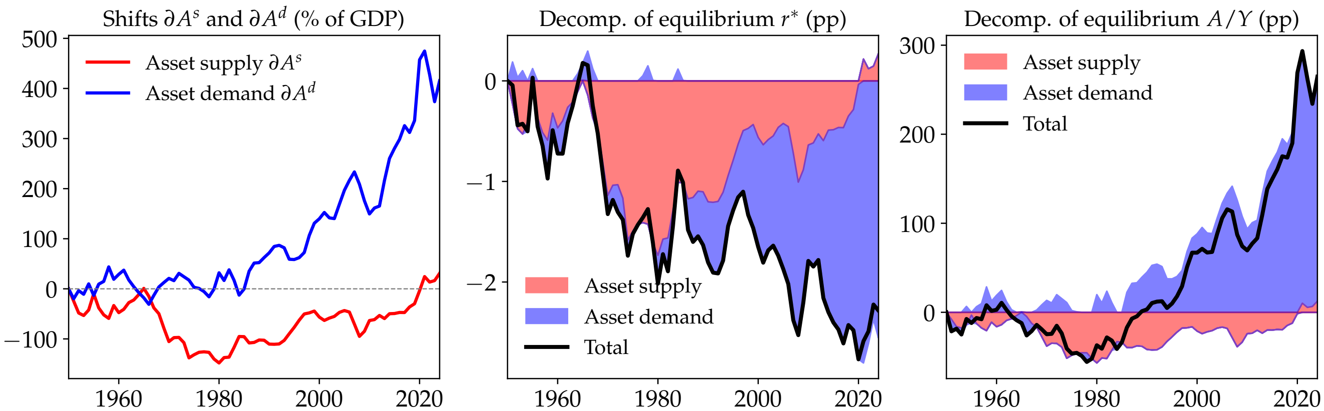


Figure 4: Decomposing asset market equilibrium into demand and supply shifts

4, and then back out the shifts $\partial \log \mathcal{A}^s$ and $\partial \log \mathcal{A}^d$ from equations (6)–(7).

Figure 4(a) shows the results. In the left panel, we display shifts as shares of initial GDP by taking the exponential $\partial \mathcal{A} \equiv \exp(\partial \log \mathcal{A}) - 1$. We see that asset supply and demand shifts remain small until the late 1960s; then, asset supply declines through the early 1980s. Since then, both shifts have risen dramatically, though asset demand has outpaced asset supply, rising by around 400% of GDP relative to a cumulative asset supply increase of only around 100% of GDP.

The other two panels of figure 4 use our recovered supply and demand shifts to decompose equilibrium movements in r^* and A/Y by implementing equations (5) and (7). Falling asset supply is mostly responsible for the reductions in r^* and A/Y early on. Starting in the 1980s, rising asset demand has been the leading contributor to the decline in interest rates and the rise in A/Y.

3 A model of asset demand and supply

We now turn to the model underlying the framework of the previous section. This model will ultimately allow us, in section 4, to decompose our identified asset supply and demand shifts into their underlying causes.

Section 3.1 provides a high-level overview; sections 3.2–3.5 cover the details and results.

from the paths our model recovers but not alter the broad magnitudes over the medium-to-long run.

3.1 Model overview

The model is of the long run: we compare balanced growth paths and abstract away from transition dynamics. On the household side, there are overlapping generations, and also heterogeneity by permanent type θ. Households live from age $j = 0$ to a maximum of $j = J$, with a population share of π_j of each age and $p(\theta)$ (independent of age) of each type.

Households earn wages on their exogenous labor endowment, pay taxes, and receive social security payments from the government. Given this income, they solve a life-cycle consumption-savings problem, investing in private assets and government bonds. There is no aggregate risk, but we allow the return on government bonds r^f to be lower than that on private assets r^r, driven by reduced-form household preferences for "safe" vs. "risky" assets. We say that the *risk premium* is the spread in rates, $rp \equiv r^r - r^f$, and the overall return $r \equiv \bar{s}r^f + (1-\bar{s})r^r$ is the average of the two returns at a baseline safe share $\bar{s}$.

Firms produce output Y using capital K and effective labor L, charging a markup over costs and paying taxes on net revenue. The firm chooses K based on a required after-tax rate of return r^r, and this return is also used to capitalize profits from markups into an asset value Π. The government taxes all net income in the economy at rate τ, and spends on direct government consumption G, social security payments T, and interest $r^f B$ on its debt B.

On a balanced-growth path, the economy grows at the same rate g as the stock of effective labor L, which is given by $g = (1+n)(1+\gamma) - 1$, where n is the growth rate of effective labor supplied by households, and γ is labor-augmenting technology growth.

In the remainder of this section, we narrate the model in more detail, discussing how the asset supply and demand curves respond to shifts, and characterizing their slopes ϵ^s and ϵ^d. The asset supply side, which concerns the production and government sectors, is relatively straightforward and covered in section 3.2. The asset demand side, which primarily involves life-cycle household assets, is split into three parts: section 3.3 describes and aggregates the core model, section 3.4 shows how several shifters can be captured using "compositional" shift-shares, and section 3.5 provides sufficient statistics for other shifters, as well as the slope of the asset demand curve ϵ^d.

3.2 Asset supply: production and government

Asset supply comes from the production side and the government, which we now narrate in more depth. Additional details are in appendix B.1.

Production and profits. We assume that output is produced by a continuum of identical, monopolistically competitive firms, following a Cobb-Douglas production function $Y = K^{\alpha}L^{1-\alpha}$, where K is private capital and L is effective labor supplied by households. All firms set a markup of μ over marginal cost and pay a tax of τ on earnings net of depreciation. Capital depreciates at a constant rate δ, and cash flows from the firm are capitalized at the "risky" rate r^r.

Together, these assumptions imply that the user cost of capital is $r^r/(1-\tau)+\delta$, the cost share of output is $1/\mu$, and the capital share of costs is α. Solving for the implied capital-output ratio gives us

$$\frac{K}{Y} = \frac{\alpha}{\mu}\frac{1}{\frac{r^r}{1-\tau}+\delta}. \tag{10}$$

Profits are a constant share $1-1/\mu$ of total output, and thus grow at the same rate g as the rest of the economy. Their after-tax capitalized value can be calculated to be

$$\frac{\Pi}{Y} = (1-\tau)(1+g)\frac{1-\frac{1}{\mu}}{r^r-g}. \tag{11}$$

The labor share of costs is $1-\alpha$, implying that the pretax real wage is

$$w = \frac{1-\alpha}{\mu}\frac{Y}{L}. \tag{12}$$

Note that the labor share of GDP is $s_L \equiv wL/Y = (1-\alpha)/\mu$.

Government. The government issues debt B, spends G directly on government consumption, pays social security benefits T, and levies a tax of τ on all net income $Y-\delta K$ in the economy, as well as on social security benefits.[6] This implies a budget constraint on the balanced-growth path of

$$\left(\frac{r^f-g}{1+g}\right)B + G + T = \tau(Y+T-\delta K). \tag{13}$$

For now, we will assume that, in the face of shocks to the government budget constraint, G is determined residually to satisfy (13).

[6] With these assumptions, an increase in τ proportionally decreases the entire profile of after-tax-and-transfer labor income.

Aggregate asset supply: shifters and elasticity. As described in (3), which we reproduce here, aggregate asset supply is the sum of K/Y, Π/Y, and B/Y:

$$\mathcal{A}^s = \frac{K}{Y} + \frac{\Pi}{Y} + \frac{B}{Y}.$$

Substituting in (10) and (11) for K/Y and Π/Y in the expression for asset supply, we have

$$\mathcal{A}^s = \frac{\alpha}{\mu}\frac{1}{\frac{r^r}{1-\tau}+\delta} + (1-\tau)(1+g)\frac{1-\frac{1}{\mu}}{r^r-g} + \frac{B}{Y}. \tag{14}$$

Further, writing $r^r = r + \bar{s}\cdot rp$ and $1+g = (1+\gamma)(1+n)$, we have a full characterization of asset supply $\mathcal{A}^s$ as a function of the average rate of return r and nine parameters: the capital share and depreciation rate α and δ, markups μ, taxes τ, the risk premium rp and baseline safe share $\bar{s}$, productivity and population growth γ and n, and debt-to-GDP B/Y. In section 4, we will consider how shifts in these parameters affect the asset supply curve.

Differentiating (14) with respect to r, using the fact that r^r moves one-for-one with r, we obtain the semielasticity of asset supply:

$$\epsilon^s = \frac{K}{A}\frac{1}{r^r+(1-\tau)\delta} + \frac{\Pi}{A}\frac{1}{r^r-g}. \tag{15}$$

3.3 Asset demand: household life-cycle model and aggregation

The main component of aggregate asset demand is household asset demand A^h, which we obtain by aggregating the solution to a life-cycle household problem across ages and types.

Household life-cycle problem. Households live from age $j=0$ to a maximum of age $j=J$. They face a survival schedule ϕ_j giving the probability of surviving from age j to age $j+1$, and we write $\Phi_j = \prod_{j=0}^{j-1}\phi_j$ for the overall probability of surviving to age j.

Households solve the following life-cycle problem:

$$\max_{\{c_j,x_j,b_j,a_j,s_j\}} \sum_{j=0}^{J} \Phi_j \left(\beta_j u(c_j,a_j,s_j) + (1-\phi_j)\kappa_j v(b_{j+1})\right) \tag{16}$$

$$c_j + \phi_j x_{j+1} + b_{j+1} = y_j + (1+s_j r^f + (1-s_j)r^r)a_j \tag{17}$$

$$a_j = x_j + b_j, \qquad a_0 = 0,$$

where net income y_j, which we will later specify in terms of labor and transfer income, is taken as exogenous. For simplicity of notation, we leave the permanent type θ implicit.

At age j, households can allocate wealth going into period $j+1$ between two assets, an actuarially fair annuitized asset x_{j+1} and a non-annuitized asset b_{j+1}, which sum to a total asset position a_{j+1}. The annuitized asset costs $\phi_j \leq 1$, since it is only paid out in the event of survival. The non-annuitized asset is consumed in the event of death, yielding utility $\kappa_j(1-\phi_j)v(b_{j+1})$; this can be interpreted as representing utility from end-of-life medical expenses or bequests.[7] β_j and κ_j are arbitrary age-dependent utility shifters, which will allow us to match data on c_j and b_{j+1}.

In addition to assets and consumption c_j, households choose the share s_j of their overall asset position a_j that is held in safe vs. risky assets. This share affects utility, which we assume takes the form $u(c_j, a_j, s_j) \equiv \hat{u}(c_j + \overline{rp}(s_j - \bar{s})a_j)$—with a consumption-equivalent benefit from exceeding the baseline safe share $\bar{s}$, which can be interpreted as a convenience yield, and a cost from going below $\bar{s}$, which can be interpreted as a cost of excessive risk. Since there is a linear cost of risk exposure, the demand for such exposure is infinitely elastic, so the existence of an optimum for s_j requires that $r^r - r^f = \overline{rp}$.[8]

We further assume that $\hat{u}$ and v are both power utility functions with a common elasticity of intertemporal substitution σ. In appendix B.2, we show that with this assumption, (16)–(17) can be reduced to a simpler one-asset problem, where households choose a path of overall assets a_j subject to the average return $r = \bar{s}r^f + (1-\bar{s})r^r$. This will be useful for deriving sufficient statistic formulas in section 3.5.

For now, we observe that the solution to (16)–(17), holding preference parameters β_j, κ_j, and σ fixed, implies a mapping from r and the paths of y_j and ϕ_j to the path of assets a_j. Importantly, since $a_0 = 0$, this mapping is *homogeneous of degree 1* in y_j: if we scale incomes by a constant factor, then assets will scale by the same factor.

Growth and aggregation. We now aggregate the household problem, writing $N_{jt}(\theta)$ for the number of households of age j and type θ at any time t, and $a_{jt}(\theta)$ and $e_{jt}(\theta)$ for the assets and labor endowments of these households. The aggregate household asset demand A_t^h and effective labor supply L_t at time t are then given by

$$A_t^h \equiv \sum_{\theta,j} N_{jt}(\theta) a_{jt}(\theta) \qquad \text{and} \qquad L_t \equiv \sum_{\theta,j} N_{jt}(\theta) e_{jt}(\theta). \tag{18}$$

[7]While $v(b_{j+1})$ can be interpreted as utility from bequests, we simplify the model by not including b_{j+1} in the income of children, instead assuming that non-annuitized assets disappear or are consumed immediately upon death.

[8]If the utility cost of risk is convex, it is possible to have the liquidity premium depend on government bond supply as in Krishnamurthy and Vissing-Jorgensen (2012) (see Auclert et al., 2024). In section 6, we consider an extension of our model that allows for such endogenous liquidity premia.

We assume social security is indexed by the real wage w, a generosity parameter $\mathcal{T}$, and an age schedule $t_{jt}(\theta)$, so that aggregate transfers are $T_t = \sum_{\theta,j} N_{jt}(\theta) w \mathcal{T} t_{jt}(\theta)$.

To evaluate a_{jt} and e_{jt} in (18), we must take into account the fact that economic growth implies that households born at different times have different levels of income. We assume that labor endowments and social security income grow across cohorts at a constant rate γ, so that $e_{jt}(\theta) = (1+\gamma)^{t-j} e_j(\theta)$ and $t_{jt}(\theta) = (1+\gamma)^{t-j} t_j(\theta)$.[9] Net income $y_{jt}(\theta)$ is the sum of after-tax income and transfers:

$$y_{jt}(\theta) = (1-\tau) w (e_{jt}(\theta) + \mathcal{T} t_{jt}(\theta)).$$

Hence, we have $y_{jt}(\theta) = (1+\gamma)^{t-j} y_j(\theta)$, with $y_j(\theta) \equiv (1-\tau) w (e_j(\theta) + \mathcal{T} t_j(\theta))$. Homogeneity of the life-cycle problem implies that household asset holdings also grow across cohorts at rate γ, so that $a_{jt}(\theta) = (1+\gamma)^{t-j} a_j(\theta)$, where $a_j(\theta)$ is the solution to the household problem for a cohort born at time 0.

To model inequality, we allow for type-dependent shifters $\lambda(\theta)$ that scale both labor endowments and social security income proportionately, writing for instance $e_j(\theta) \equiv \lambda(\theta) \bar{e}_j(\theta)$. For a given θ, the entire path of income, and therefore assets by homogeneity, is scaled by both $\lambda(\theta)$ and $(1-\tau)w$. We use bars to denote assets when these factors are normalized to 1, so that $a_j(\theta) = (1-\tau) w \lambda(\theta) \bar{a}_j(\theta)$.

Plugging the above expressions into (18), we obtain a time-invariant ratio A^h/Y of household assets to GDP,

$$\frac{A^h}{Y} = s_L \frac{A^h}{wL} = (1-\tau) s_L \frac{\sum_{\theta,j} p(\theta) \pi_j (1+\gamma)^{-j} \lambda(\theta) \bar{a}_j(\theta)}{\sum_{\theta,j} p(\theta) \pi_j (1+\gamma)^{-j} \lambda(\theta) \bar{e}_j(\theta)}, \tag{19}$$

where, as above, $s_L = wL/Y = (1-\alpha)/\mu$ is the labor share, and $p(\theta)\pi_j$ is the fraction of households of age j and type θ.

Asset demand. Total asset demand $\mathcal{A}^d$, in (4), is the sum of household assets A^h/Y and foreign assets $-NFA/Y$. We take the latter as a parameter, and then observe from (19) that changes in taxes τ and the labor share s_L simply scale household assets A^h/Y.

What remains is to characterize the ratio in (19) in response to various shifters. We separate these into two broad classes. First, we consider *compositional* shifters—demographics,

[9] On a given balanced growth path, it is irrelevant whether productivity growth loads on cohorts or time, since the two are equivalent under a reparametrization of e_j. For counterfactuals where growth changes, however, the distinction matters, since higher growth rates loading on time steepen income profiles, which reduces savings through an anticipatory effect. We opt for the cohort formulation since it is more tractable analytically, requires fewer assumptions and data sources, and because it is unclear whether time-varying anticipatory effects are realistic. See appendix B.7 for more discussion.

productivity growth (given our cohort effect assumption), and inequality. These do not change the household decisions $\bar{a}_j(\theta)$; instead, they change the scaling factors π_j, $(1+\gamma)^{-j}$, and $\lambda(\theta)$ in (19), thereby changing the relative mix of assets $\bar{a}_j(\theta)$ and labor $e_j(\theta)$ in the numerator and denominator across ages and types. We cover these types of shifts in section 3.4. Second, we consider *life-cycle* shifters—social security and longevity—which directly change the solution $\bar{a}_j(\theta)$ to the normalized life-cycle problem. We cover these, along with the closely related semielasticity of asset demand ϵ^d, in section 3.5.

3.4 Asset demand: compositional effects

Demographics. At any time t along a balanced growth path, the average asset holdings and gross labor earnings at each age are given by

$$a_{jt} = (1-\tau)w \sum_{\theta} p(\theta)(1+\gamma)^{t-j}\lambda(\theta)\bar{a}_j(\theta); \quad we_{jt} = w \sum_{\theta} p(\theta)(1+\gamma)^{t-j}\lambda(\theta)\bar{e}_j(\theta). \tag{20}$$

Using these expressions, we can rewrite (19) as s_L times the ratio of $\sum_j \pi_j a_{jt}$ and $\sum_j \pi_j we_{jt}$. This implies, as in Auclert et al. (2024), that the shift in log household asset demand associated with shifting the population distribution from some π_j^0 to some π_j^1 is given by:

$$\Delta \log \frac{A^h}{Y} = \log\left(\frac{\sum_j \pi_j^1 a_{jt}}{\sum_j \pi_j^0 a_{jt}}\right) - \log\left(\frac{\sum_j \pi_j^1 we_{jt}}{\sum_j \pi_j^0 we_{jt}}\right). \tag{21}$$

where average assets a_{jt} and gross labor incomes we_{jt} can be taken from the data in any reference year t. We see that A^h/Y increases if the composition π_j of the population shifts toward ages with high average assets, and also if it shifts toward ages with low average labor income.

Figure 5 illustrates equation (21) by plotting profiles of asset holdings and gross labor earnings from a 2016 reference year, together with historical and projected age distributions from 1950 and 2100.[10] The asset profile increases rapidly during working life and features limited decumulation at old age, while the labor income profile follows the classic hump shape. Through the lens of (21), an older population increases asset demand because there is a higher share π_j^1 in older ages j with high asset holdings and limited labor supply. Given these profiles, a shift from the 1950 population to the 2100 population implies a 63

[10]As we explain in section 4, we choose 2016 as a base year since it has a rich set of available age profiles. Auclert et al. (2024) show that broadly similar results are obtained even when alternative base years are chosen for the profiles.

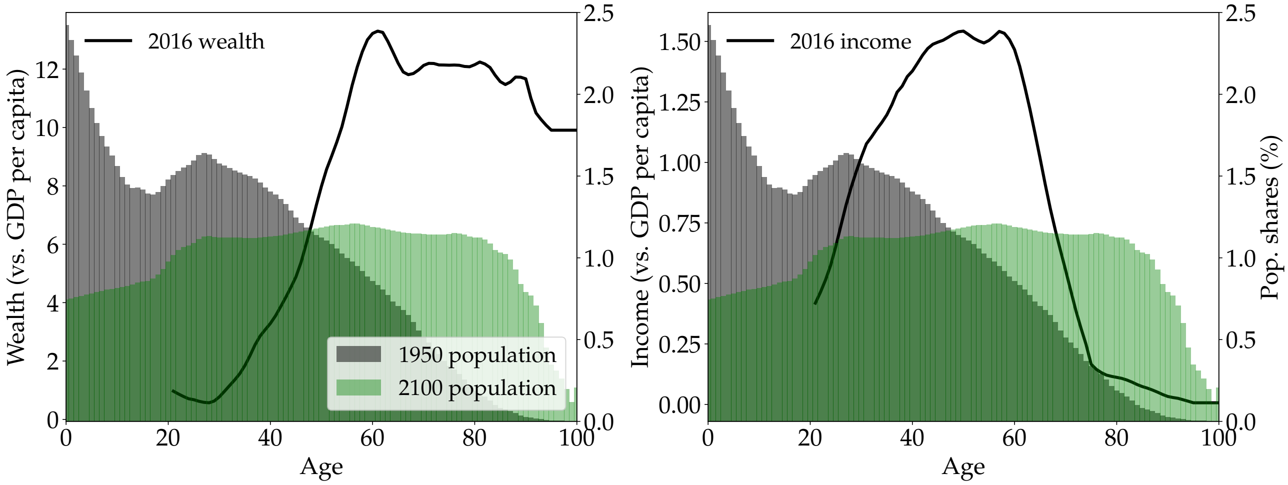


Figure 5: Compositional effect from population aging

log point increase in log household asset demand, equivalent to 327% of GDP increase starting from the 1950 level of A^h/Y. Of this, nearly all comes from higher asset holdings (left panel of figure 5), since over the entire horizon aggregate labor supply is roughly unchanged, as the decline in the young population is offset by an increase in the elderly population (right panel of figure 5).

Productivity growth. For changes in the productivity growth rate across cohorts, we note from (19) that the effect on household asset demand from changing the cohort growth rate from γ^0 to γ^1 is equivalent to changing π_j^0 to $\pi_j^1 = \left(\frac{1+\gamma^1}{1+\gamma^0}\right)^{-j} \pi_j^0$, so we can apply (21) in this case as well.[11] In particular, the effect of a slowdown in productivity on asset demand is identical to the effect of a lower population growth rate. In appendix figure B.1, we use this equivalence to visualize the effects of a decline in productivity growth in terms of an effective aging of the population. Given the 2016 age profiles, a 1% decline in productivity growth implies a 14 log point increase in aggregate household demand, or 56% of GDP in levels.[12]

[11] Note that, while π_j^1 does not sum to one, it can be transformed into a population distribution by uniform scaling, and that scaling π^1 does not affect the formulas (19) and (21) regardless.

[12] In appendix B.7, we discuss an additional life-cycle saving effect that emerges if productivity growth changes load on time rather than cohorts, and calculate that this roughly doubles the asset demand effect.

Inequality. We model changes in inequality as shifts in the uniform scaling factors $\lambda(\theta)$.[13] We can write average asset holdings and labor income for a type θ at any time t as

$$a_t^h(\theta) = (1-\tau)w\lambda(\theta)\sum_j \pi_j(1+\gamma)^{t-j}\bar{a}_j(\theta); \quad we_t(\theta) = w\lambda(\theta)\sum_j \pi_j(1+\gamma)^{t-j}\bar{e}_j(\theta),$$

which is similar to (20), except that here we aggregate across j instead of θ.

Using this, we can rewrite the fraction in (19) as s_L times the ratio of $\sum_\theta p(\theta)a_t^h(\theta)$ and $\sum_\theta p(\theta)we_t(\theta)$. In appendix B.3, we show that the change in A^h/Y associated with a shift from a level of permanent income inequality $\lambda^0(\theta)$ to another level $\lambda^1(\theta)$ is then given by

$$\Delta\frac{A^h}{Y} = s_L\sum_\theta\left[\rho^1(\theta)-\rho^0(\theta)\right]\frac{a_t^h(\theta)}{we_t(\theta)}. \tag{22}$$

where $\rho^i(\theta) = \frac{p(\theta)\lambda^i(\theta)e_t(\theta)}{\sum_{\theta'} p(\theta')\lambda^i(\theta')e_t(\theta')}$ is the share of labor income accruing to type θ, and $a_t^h(\theta)/we_t^h(\theta)$ is the asset-to-income ratio for type θ.[14] In the data, we will find that higher permanent income types θ also have higher asset-to-income ratios, consistent with a large literature on this topic (see Dynan et al. 2004 for seminal work on how rich households save more than poor ones.) Changes in income inequality then increase aggregate asset demand, because they raise the share of income accruing to groups with high ratios of assets to income.

Figure 6 illustrates the logic of the shifter, based on our application in the next section. It splits the population into 100 permanent types θ, each with mass 1, corresponding to percentiles of the permanent income distribution. It then plots the implied ratio of overall assets to income a^h/we in each group, based on the estimated relationship between permanent income and assets in microdata, and shows how the mass ρ of gross labor income shifted across types from 1970 to 2016. With higher inequality, ρ shifted toward types with higher asset-to-income ratios. Per equation (22), this increased household asset demand by 17 log points.

[13] This can be thought of as capturing shifts in permanent income inequality. Kopczuk, Saez and Song (2010) and Guvenen, Kaplan, Song and Weidner (2017) provide evidence that this has been a key source of rising wage inequality in the U.S.

[14] In appendix B.3, we also derive a more direct analog of (21) for the inequality case, but (22) better captures the intuition.

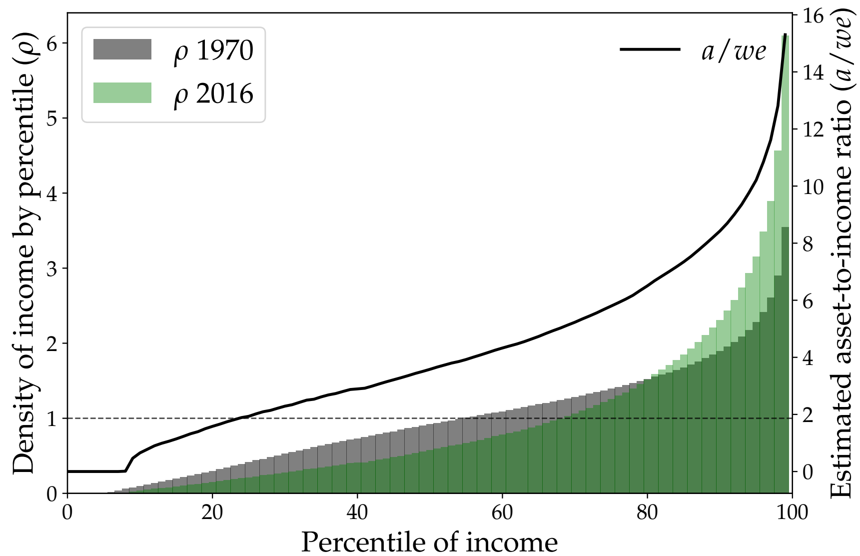


Figure 6: Compositional effect from rising inequality

3.5 Asset demand: life-cycle effects

Now, we consider shifters that affect normalized asset demand by age $\bar{a}_j(\theta)$ in (20). This requires us to re-solve the life-cycle problem, rather than just shifting weights on the normalized problem. Here, for simplicity, we suppress permanent types θ in our notation; each expression can be interpreted as holding for a given θ.[15]

We start by stating a result from appendix B.2, which is that the full life-cycle model in (16)–(17) can be reduced to the following simpler problem:

$$\max_{\tilde{c}_j, a_{j+1}} \sum_j \Phi_j \tilde{\beta}_j \tilde{c}_j^{1-\sigma}$$
$$s.t. \quad \tilde{c}_j + \phi_j a_{j+1} = y_j + (1+r)a_j,$$

where $\tilde{c}_j \equiv c_j + (1-\phi_j)b_{j+1} + rp(s_{j+1} - \bar{s})a_{j+1}$ is "effective" consumption, a composite that includes end-of-life spending and any excess safe assets held, in addition to ordinary consumption.

Shifts to income profile. We start by considering shifts to the net income profile y_j. For intuition, we state the result for the special case where $r = g$ on the balanced-growth path, in which case life-cycle and cross-sectional shares are identical. In practice, this case

[15]In our implementation, we will generally apply the sufficient statistics directly for the entire population, since we lack the data necessary for disaggregation by θ.

delivers a close approximation to the general result, stated in appendix B.4.

Proposition 1. *Suppose there is a shift $\{dy_{j0}\}$ in net income by age and that $r = g$. To first order, the resulting change in aggregate household asset holdings A^h is given by*

$$dA^h = \frac{1}{1+r} \sum_{j=0}^{J} \pi_j dy_{j0} (\mathbb{E}Age_{\tilde{c}} - j) \tag{23}$$

where $\mathbb{E}Age_{\tilde{c}} \equiv \left(\sum_{j=0}^{J} \pi_j \tilde{c}_{j0} j\right) / \left(\sum_{j=0}^{J} \pi_j \tilde{c}_{j0}\right)$ is the cross-sectional average age of effective consumption $\tilde{c}$.

Proposition 1 shows that the average age of effective consumption and the age distribution of the population are sufficient statistics for the effect of any change in life-cycle income profiles on aggregate asset holdings. The intuition for equation (23) is that an increase in net income at age j, in the absence of shocks to preferences or r, is smoothed over the full life-cycle, causing a proportional increase in effective consumption at every age. This smoothing involves a change in asset holdings, which is positive if the income is received earlier than consumption, and negative if the income is received later than consumption, with the general contribution to assets being $\mathbb{E}Age_{\tilde{c}} - j$, the *average* number of periods that the income predates consumption (for closely related results, see Lee and Mason 2011).[16]

Social security generosity. We can apply this result to any change in the life-cycle pattern of income, including social security generosity.

Corollary 1. *If there is a change in the generosity of social security generating a total change in tax-adjusted social security spending of $(1-\tau)dT$, then if $r = g$, to first order the resulting change in A^h is*

$$dA^h = \frac{\mathbb{E}Age_{\tilde{c}} - \mathbb{E}Age_t}{1+r} \cdot (1-\tau)dT \tag{24}$$

and $\mathbb{E}Age_t \equiv \left(\sum_{j=0}^{J} \pi_j t_j j\right) / \left(\sum_{j=0}^{J} \pi_j t_j\right)$ is the cross-sectional average age of social security receipts.

This corollary provides a sufficient statistic for the savings effect of social security: the effect is proportional to the difference $\mathbb{E}Age_{\tilde{c}} - \mathbb{E}Age_t$ between the average age of

[16] The $1/(1+r)$ is a small adjustment for timing. Note that we write the shock in terms of the cross-sectional change in income at each age, dy_{j0}, which is $(1+g)^{-j}$ times the life-cycle change dy_j facing an individual.

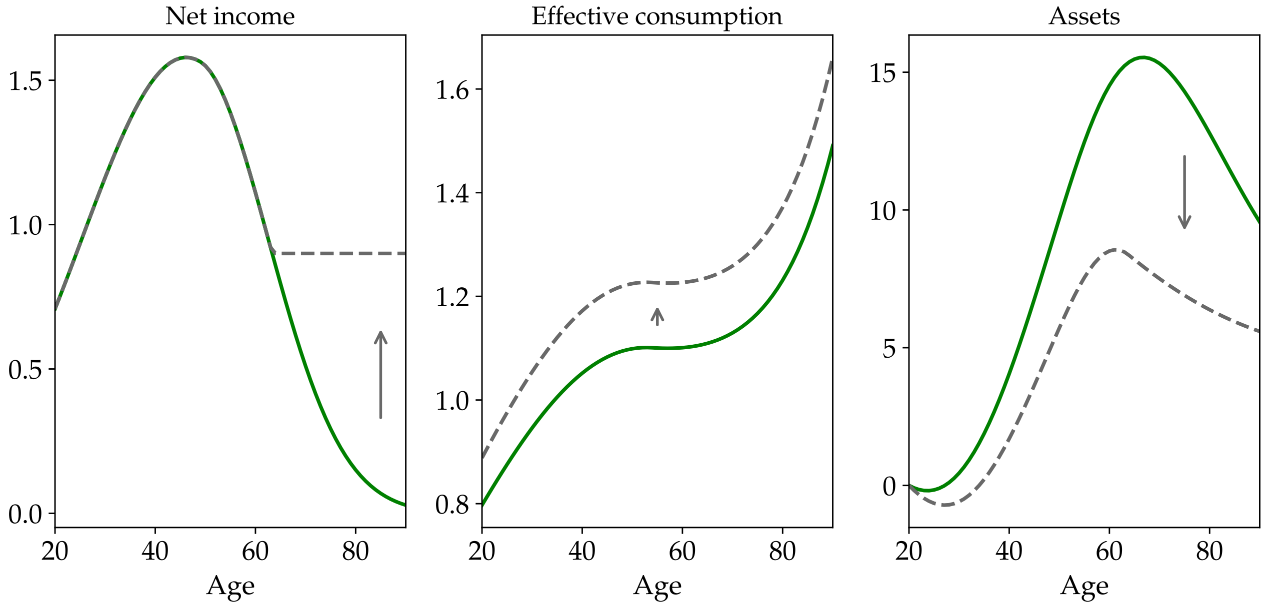


Figure 7: Illustration of life-cycle effect of social security

consumption and of social security receipts. Since social security is received later in life, this effect will generally be negative.

The mechanics of corollary 1 are illustrated for a hypothetical life-cycle problem in figure 7. This shows how a rise in social security generosity increases net income late in life, leading to a uniform shift up in the effective consumption schedule across the life-cycle, and thus to a fall in assets through reduced accumulation.

Shifts to survival probabilities. The income effect formula in proposition 1 can also be used to analyze the effect of shifts in longevity ϕ_j, beyond the compositional effects working through π_j that we have already characterized. As explained in appendix B.5, the reason is that ϕ_j drops out of the first-order conditions of the household problem (16)–(17), since it shifts prices and utilities in offsetting ways. The shock $d\phi_j$ therefore only matters through the budget constraint (17). In particular, by increasing consumption needs late in life, it is equivalent to a negative income shock, giving us the following corollary to proposition 1.

Corollary 2. *Given a change in longevity* $\{d\phi_j\}$, *the resulting change in aggregate assets coincides with the change resulting from an income shock* $dy_{j0} = -d\phi_j x_{j+1,0}$, *where* $x_{j+1,0} = a_{j+1,0} - b_{j+1,0}$ *is the cross-sectional profile of annuitized assets.*

This corollary suggests that changes in longevity would be very powerful if all assets were annuitized, $x_j = a_j$, rather than being held for bequests or other end-of-life consumption b_j. Intuitively, the extent to which longevity is a negative income shock can be measured by looking at how much households use annuitization to insure against it.

In practice—aside from Social Security, which we treat separately—the vast majority of household assets are not annuitized, especially at older ages where changes in ϕ_j are most likely to occur.[17] Low annuitization in old age is consistent with the finding in the life-cycle literature that observed asset holdings late in life are hard to justify through consumption motives alone, and instead are held for bequests or other end-of-life motives captured in our b_j (see, e.g., De Nardi, French and Jones 2016a). For similar reasons, Auclert et al. (2024) find little direct effect of longevity in a structural model once bequest motives are included. Given this, our quantitative analysis starting in section 4 will not include life-cycle effects of longevity changes, but only the compositional effects working through the age distribution.

Semielasticity to r. Last, we consider the semielasticity of asset demand with respect to changes in the interest rate. This effect also works through the life-cycle decision and can be obtained by aggregating the derivative of the life-cycle problem with respect to r. We obtain the following proposition. As in the case with income shocks, we focus on the case with $r = g$, with the general formula provided in the appendix B.6.

Proposition 2. *Starting from a balanced growth path with $r = g$, the semielasticity of asset demand $\epsilon^d \equiv \partial \log(A^h/Y)/\partial r$ is given by*

$$\epsilon^d = \frac{1}{1+r}\left(\sigma \frac{\tilde{C}}{(1+g)A^h} VarAge_{\tilde{c}} + \mathbb{E}Age_{\tilde{c}} - \mathbb{E}Age_a\right). \tag{25}$$

The first term captures the substitution effect, coming from steeper consumption paths in response to higher interest rates. This effect scales in the intertemporal elasticity of substitution σ, as well as the cross-sectional *variance* of the age of consumption, reflecting the fact that when consumption is more dispersed over the life-cycle, there is greater scope to intertemporally substitute. The other terms $\mathbb{E}Age_{\tilde{c}} - \mathbb{E}Age_a$ reflect the income effect from earning higher r on assets; this can be regarded as a special case of proposition 1.

The substitution effect is illustrated in Figure 8, which shows how a higher interest rate

[17] Other than Social Security, almost all annuitized assets in the United States are held in defined-benefit pensions. Mechanically, however, the value of these pensions peaks at retirement, and declines in old age as fewer expected years remain to be paid out. Although there may be frictions to additional private annuitization, most Americans have the option to cost-effectively annuitize at the margin by delaying Social Security, yet on average choose not to do so.

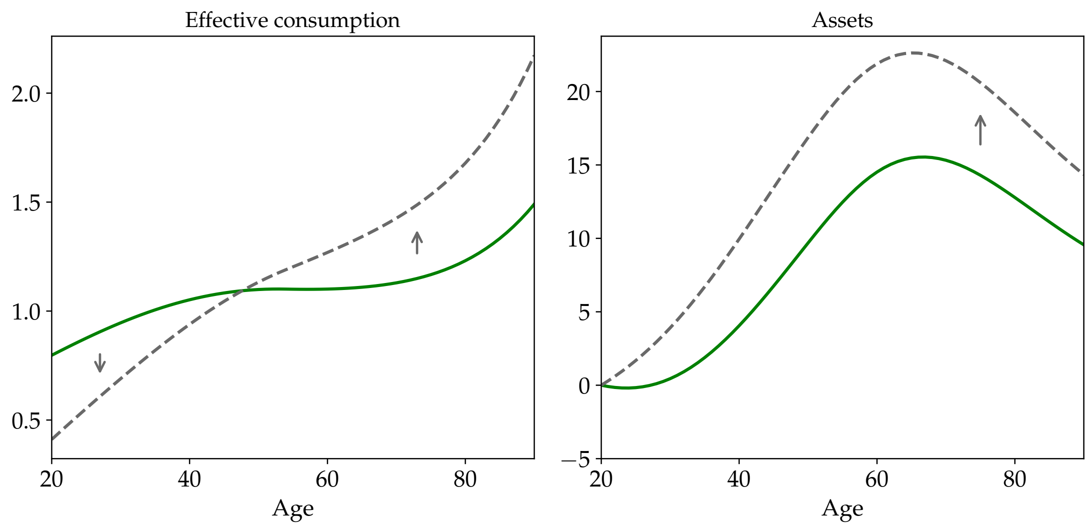


Figure 8: Illustration of life-cycle substitution effect of r

rotates the life-cycle consumption path to become steeper, leading to higher assets through increased accumulation early in life.

4 The race behind us: drivers of r^* and W/Y, 1950–2025

We now use the expressions for the components of asset supply and demand from the previous section to shed light on the movements in the asset supply and demand shifters we recovered in section 2. Which drivers are responsible for the large increase in asset demand? Which explain the U-shape in asset supply? And which explain the secular decline in r^*?

4.1 Asset supply

To construct the components of asset supply, we start from the steady-state equation (14). Substituting in the expression for the risky rate $r^r = r + \bar{s} \cdot rp$, we obtain:

$$\mathcal{A}^s(r, \boldsymbol{\eta}) = \frac{\alpha}{\mu} \frac{1}{\frac{r+\bar{s}\cdot rp}{1-\tau} + \delta} + (1-\tau) \frac{1 - \frac{1}{\mu}}{r + \bar{s} \cdot rp - g} + \frac{B}{Y}, \tag{26}$$

Equation (26) expresses long-run asset supply $\mathcal{A}^s$ as a function of r and shifter parameters $\boldsymbol{\eta} \equiv (\alpha, \mu, \delta, \tau, g, B/Y, rp, \bar{s})$.

Like in section 2, we treat this steady-state equation as if it holds at each point in time, with all parameters allowed to vary over time except for $\bar{s}$. We calibrate $\bar{s}$ to the ratio of government bonds to total assets B/A, averaged over the whole period 1950–2024, and treat the remaining parameters as time-varying shifters of asset supply. We use aggregate data to recover these shifters: we take the depreciation rate δ_t, the tax rate τ_t, the growth rate $g_t = (1+\gamma_t)(1+n_t) - 1$ and the debt-to-GDP ratio B_t/Y_t in each year directly from the data, and use our earlier estimates of r_t^r and r_t^f to construct a time series for the risk premium rp_t. We then use data on the labor share and the capital share to recover both α_t and μ_t.

Given our recovered $\{\boldsymbol{\eta}_t\}$, we can construct the asset supply function $\mathcal{A}_t^s$ at any point t by keeping r fixed and letting $\{\boldsymbol{\eta}_t\}$ vary. We can further use equation (26) to decompose changes in asset supply relative to any date t_0 as the sum of the contributions from each individual component of $\{\boldsymbol{\eta}_t\}$.[18] (See appendices A and C for more details on the aggregate data and decomposition, respectively.)

Results. Figure 9 shows the results of this procedure, tracing out the evolution of asset supply as well as its components since 1950s. The solid line shows the overall effect of all drivers $\Delta\mathcal{A}^s(r_{1950}, \boldsymbol{\eta}_t)$ obtained from equation (26).

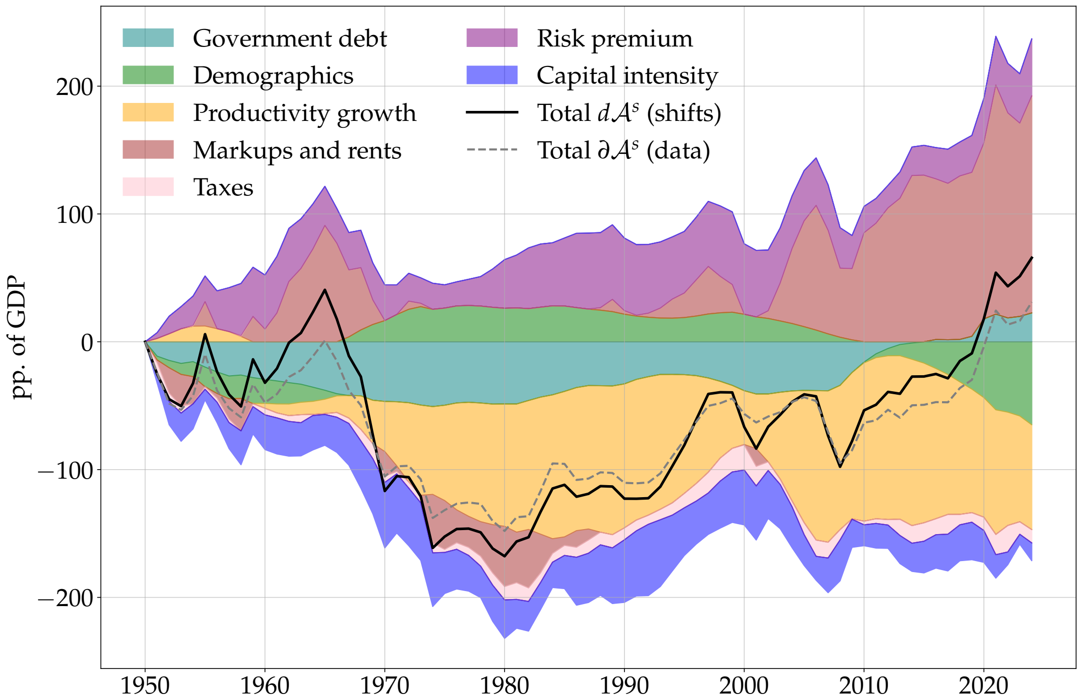
For comparison with section 2, the dotted line plots the asset supply backed out from the relation $d\log\mathcal{A}^s = d\log(A/Y) + \epsilon^s dr$, implying an estimated asset supply shifter $\hat{\mathcal{A}}_t^s = (A/Y)_t \times \exp([r_t - r_{1950}]\epsilon^s)$, where $(A/Y)_t$ is the observed ratio of assets to GDP at time t. We obtain ϵ^s by using the time series for our shifters together with the time series for r to form the average of $\epsilon^s = \frac{K}{A}\frac{1}{r^r+(1-\tau)\delta} + \frac{\Pi}{A}\frac{1}{r^r-g}$ over the time period. This delivers $\epsilon^s = 20.1$. The two curves are very close, since our calibration ensures $\mathcal{A}^s(r_t, \boldsymbol{\eta}_t) = (A/Y)_t$, meaning that the curves only differ due to $\hat{\mathcal{A}}_t^s$ being a log-linear approximation.

The shaded colors show the contribution of each component of asset supply to the overall shift in the solid black line. After being stable for the first fifteen years, asset supply fell in the 1970s due to lower productivity growth, which reduced the value of capitalized rents by shrinking the capitalization factor $1/(r^r - g)$. Then, beginning in the 1980s, asset supply turned sharply upward, driven in particular by a return of productivity growth in

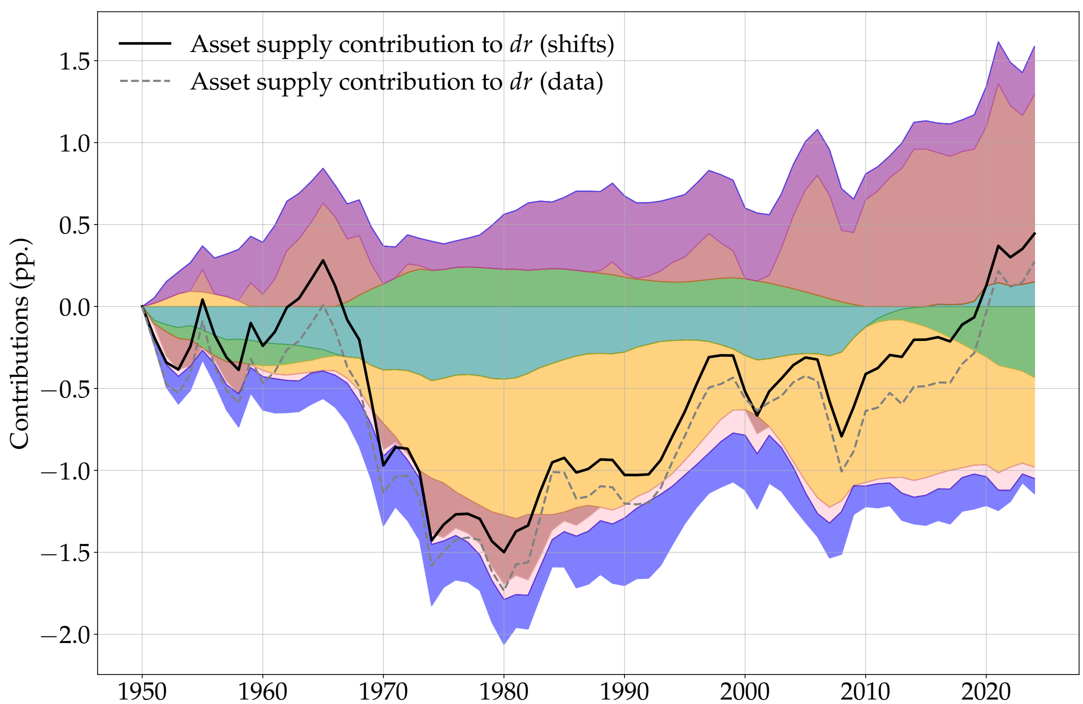
[18] Note that since $\mathcal{A}^s$ is nonlinear in the parameter vector $\boldsymbol{\eta}$, there is no unambiguous decomposition of changes into effects from individual components of η. Our decomposition uses that changes in asset supply between t_0 and t can be expressed as an integral along a line in parameter space $\boldsymbol{\eta}_s = (1-s)\boldsymbol{\eta}_{t_0} + s\boldsymbol{\eta}_t$ for $s \in [0,1]$. This line goes from $\boldsymbol{\eta}_{t_0}$ to $\boldsymbol{\eta}_t$, yielding

$$\mathcal{A}^s(\boldsymbol{\eta}_t, r_{t_0}) - \mathcal{A}^s(\boldsymbol{\eta}_{t_0}, r_{t_0}) = \sum_i \int_0^1 \frac{\partial\mathcal{A}^s(\boldsymbol{\eta}_s, r_{t_0})}{\partial\eta_i} \times (\eta_{i,t} - \eta_{i,t_0})ds.$$

which linearly decomposes changes in $\mathcal{A}_t^s - \mathcal{A}_{t_0}^s$ into individual shifters $\eta_{i,t} - \eta_{i,t_0}$.



(a) Asset supply



(b) Interest rate

Figure 9: Asset supply shifts and effects on interest rate

Driver	**Contribution to asset supply shift** *(% of GDP)*	**Contribution to interest rate** *(bp)*
Government debt	23	15
Demographics	-65	-43
Productivity growth	-82	-55
Markups and rents	170	114
Taxes	-10	-7
Risk premium	45	30
Capital intensity	-14	-9
Total shift explained	*67*	*45*
Total shift data	*31*	*27*

Table 1: Drivers of asset supply, 1950 – 2024

the 1990s, and then by a surge in markups and rents, as well as a reversal of the post-WWII fall in government debt.

Figure 9(b) shows how each of these drivers contributed to the evolution of interest rates. In table 1, we summarize the contributions to both the total asset supply shift and interest rates for the overall period 1950–2024. Strikingly, the increase in markups and rents alone, by pushing up the value of capitalized rents, prevented over a full percentage point reduction in interest rates.

4.2 Asset demand

To construct the components of asset demand, we first note that changes in asset demand $\mathcal{A}^d = \mathcal{A}^h - \frac{NFA}{Y}$ between any two dates can be expressed as

$$\Delta \mathcal{A}^d = \mathcal{A}^h_0 \times \left(\exp(\Delta \log \mathcal{A}^h) - 1 \right) - \Delta \frac{NFA}{Y}.$$

To estimate $\Delta \mathcal{A}^d$, we first treat the net foreign asset term as an exogenous shifter, which we measure in appendix A. For changes in household asset demand, we use the long-run relationship (19) to write

$$\Delta \log \mathcal{A}^h = \Delta \log(1-\tau) + \Delta \log s_L + \Delta \log \left(\frac{\sum_{\theta,j} p(\theta)\pi_j(1+\gamma)^{-j}\lambda(\theta)\bar{a}_j(\theta)}{\sum_{\theta,j} p(\theta)\pi_j(1+\gamma)^{-j}\lambda(\theta)e_j(\theta)} \right)$$

Again, we treat this long-run relationship as if it applies period-by-period. For τ_t and s_{Lt}, we use the same national accounts data that we used in the previous section. For the last term, we do not have closed form expressions for all of the effects, but we can use the sufficient statistics from section 3 to calculate the effect on log household demand from each individual shifter. We will calculate these log effects separately, disregarding any interaction effects, and then aggregate log-linearly.

We first consider compositional effects. Using national accounts data, the UN World Population prospects, and the Distributional National Accounts (DINA), we obtain time series for the population distribution π_{jt}, the productivity growth rate γ_t, and the inequality shifters $\lambda_t(\theta)$. We then combine these with micro data on asset and labor income by age and income group[19] to calculate the implied change in log household asset demand using the shift-share formulas (21) and (22) in section 3.4.

Beyond compositional effects, in our main analysis the only shifter with a life-cycle effect is the generosity of social security $\mathcal{T}$. We back out $\mathcal{T}_t$ in each year using the share of GDP going to social security, the age distribution of the population, and the profile of social security payments by age. We then calculate the implied changes in household asset demand from the variation in $\mathcal{T}_t$ using the result in Corollary 1.[20]

Results. Figure 10(a) shows the contributions of our seven drivers to the overall shift in asset demand from 1950 until 2024. Table 2 summarizes the effect over the whole time period.

For reference, as in the previous figure, we include the shift inferred in section 2 using

[19] For the asset and income profiles a_{jt} and we_{jt} in (21), we use 2016 profiles from the SCF and the PSID. For the inequality shift-share (22), we estimate the time series of shifters $\lambda_t(\theta)$ (and thus of income shares by group $\rho_t(\theta)$) from the DINA, and the the ratio of assets to labor income $\frac{a_t^h(\theta)}{we_t(\theta)}$ by type θ from the cross-sectional relationship between income and asset holdings in the PSID. For inequality, we only consider changes starting in 1970 due to data availability. Also, for shifts in productivity growth and inequality, we conduct a gradual phase-in that takes into account that these shifters occur at the cohort level. For a more complete description of the data construction and estimation equations, see appendix C.

[20] Formally, we define the effect of changes in generosity as $\Delta\mathcal{T} \times \frac{d \log \mathcal{A}^h}{d\mathcal{T}}$, where $\frac{d \log \mathcal{A}^h}{d\mathcal{T}}$ is a time-invariant sensitivity of log household asset demand to $\mathcal{T}$, which is an average of sensitivites across each year 1950 to 2024. For each year, we define the sensitivity of log assets to $\mathcal{T}$ by converting a change in $\mathcal{T}$ to a change in total payments $(1-\tau_t)\Delta T$ and using the $r \neq g$ version of the formula in Corollary 1, normalized by household asset holdings in that year.

the equation $\partial \log(\mathcal{A}^d) = d\log(A/Y) - \epsilon^d dr^*$. To estimate ϵ^d, we apply the sufficient statistic from proposition 2 using an intertemporal elasticity of substitution $\sigma = 1/3$, yielding a value $\epsilon^d = 9.3$. The elasticity can be expressed as a sum of an income and substitution effect as $\epsilon^d = -7.4 + \sigma \times 50.2$, where the negative income effect reflects a decreasing need to save when retirement assets pay a higher return. Our choice of $\sigma = 1/3$ is intermediate between the common $\sigma = 1$ benchmark and macro estimates, which are close to $\sigma = 0$ when corrected for reporting bias, and is consistent with a 0.3–0.4 meta-estimate for asset holders.[21] (See Havránek 2015 for both estimates.)

Overall, our drivers jointly explain a 316% of GDP increase in asset demand over the period, which is 75% of the asset demand shift backed out from the data. The main positive drivers are demographic change, rising inequality, falling productivity growth, and increasing inflow of foreign funds.

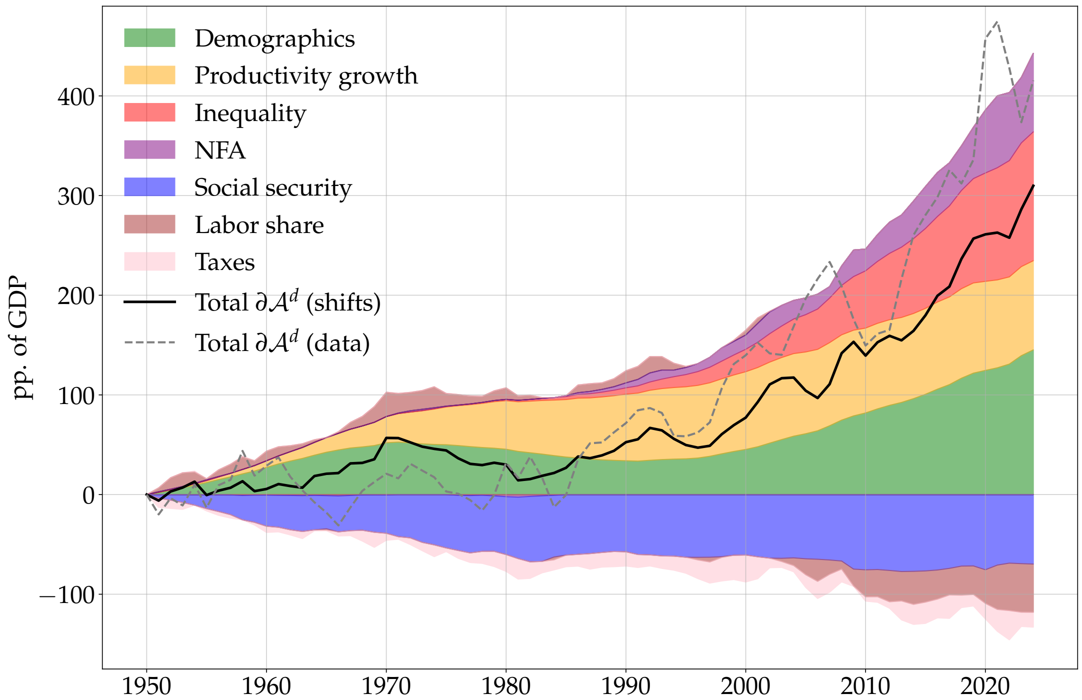
The demographic effect (in green) is large, at 145% of GDP over the whole 1950-2024 period, and comes in two waves. First, demography raises asset demand between 1950 and 1970, as the small cohorts born during the Great Depression and the Second World War enter the labor force, generating an adult population that skews old. Subsequently, asset demand is shaped by the movement of baby boomers through the age structure; asset demand falls when these large cohorts enter the asset-poor young ages, and rises again as they move into the high-asset periods of life. The magnitude of this effect is close to that reported in Auclert et al. (2024).

Beyond demographics, falling productivity growth (in red) pushed up asset demand a further 90% of GDP through a similar mechanism, that is, by raising the economic weight of old households with high asset levels and low labor supply, relative to that of young households with low assets and high labor supply, as illustrated in appendix figure B.1.

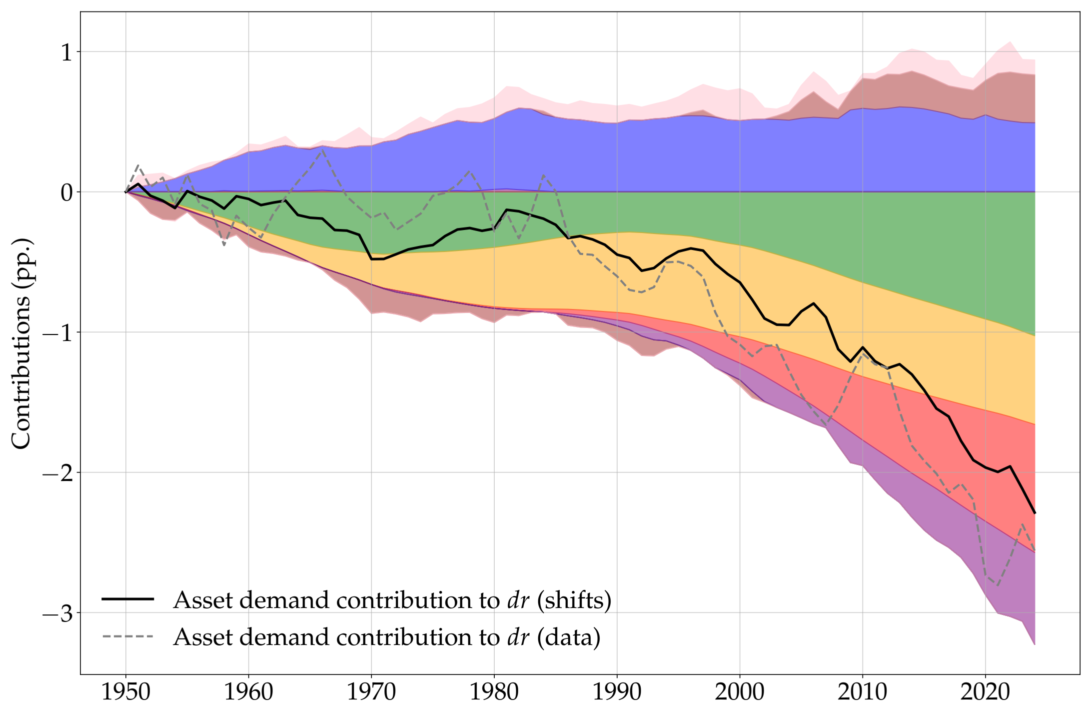
Inequality (in orange) began to rise in the 1970s, shifting a greater share of income toward high-income households, who tend to have larger asset holdings relative to their labor income level. While this redistribution gradually increased asset demand, the effect only became pronounced in the 1990s, as wealthy individuals reached middle age and began to accumulate more assets. This momentum effect explains why the asset demand effect of inequality continues to increase in 2025 despite early-in-life inequality being relatively stable since the early 2000s. Over the 1950-2024 period, inequality pushed up asset demand by 130% of GDP. Mian et al. (2020) have called this the "saving glut of the rich".

A fourth driver of rising asset demand is the inflow of foreign capital, captured by

[21] Note that a higher σ, implying a higher ϵ^d, would imply an even larger asset demand shift in section 2.



(a) Asset demand



(b) Interest rate

Figure 10: Asset demand shifts and effects on interest rate

the negative net foreign asset (NFA) position (in purple). Beginning in the 1990s and accelerating after the Great Recession, these inflows contributed to domestic asset demand by increasing the supply of foreign savings available to U.S. financial markets. This driver can be viewed as a reduced-form way of capturing the "global savings glut" (Bernanke 2005). From 1950 until 2024, it accounted for an increase in asset demand of 79% of GDP.

In contrast, three other forces have reduced asset demand. The main such force is the expansion of social security (in blue), which significantly reduced the need for private savings. Social security expenditures as a share of GDP rose by a factor of almost 20 between 1950 and 2024. While some of this increase reflects an aging population and a lower labor share, the implied increase in generosity parameter $\mathcal{T}$ is still a factor of 10; this reduced household dependence on self-financed retirement assets.[22] This, all else equal, would have reduced asset demand by 70% of GDP. The decline in the labor share (in brown), particularly since 2000, has further lowered asset demand. This effect follows from the fact that household asset demand ultimately scales with labor income. Through a similar scaling mechanism, increases in taxes (in pink) have had a modest negative impact on asset demand.

For all seven drivers of asset demand, figure 10(b) shows how each caused opposite movements in the interest rate. Demographics and inequality drove down interest rates by around 1 pp each. The slowdown in productivity growth and foreign capital inflows each contributed about a 70 bp decline. Without the expansion of social security, the decline of the labor share and the increase in taxes, interest rates would have declined by almost a full percentage point more.

4.3 Comparison with the literature

We now discuss how our historical results compare to those from other papers in the literature that, like us, attempt to account systematically for the evolution of r^* through various drivers. Three papers, in particular, are close to us in terms of the forces and the type of structural model they consider: Rachel and Summers (2019), Eggertsson et al. (2019) and, more recently, Peruffo and Platzer (2024).

Table 3 reports the effect of individual forces on r^* in each of these papers, compared to those from our paper in the last column. Broadly speaking, all papers find that demographics, inequality, productivity and the net foreign asset position all pushed down r^*,

[22] See Martin and Weaver (2005) for a discussion of different expansions of social security during the postwar era, including the 1950 amendment that extended eligibility to 9 million more workers and increased average benefit levels by 77%.

Driver	**Contribution to asset demand shift** ***(% of GDP)***	**Contribution to interest rate** ***(bp)***
Demographics	145	-103
Productivity growth	90	-63
Inequality	129	-91
NFA	79	-66
Social security	-70	49
Labor share	-48	34
Taxes	-15	11
Total shift explained	*310*	*-229*
Total shift data	*415*	*-256*

Table 2: Drivers of asset demand, 1950 – 2024

and that government debt pushed r^* up.

Regarding the effect of demographics, our paper falls in the middle of the range of the literature, with the particularly large effects in Eggertsson et al. (2019) (EMR) explained in part by the fact that the compositional effect of demographics is much larger in their model than in the data (see Auclert et al. 2024, appendix G). For changes in productivity growth, our paper finds smaller effects than the rest of the literature, because in our model growth loads on cohort rather than time effects, suppressing the strong life-cycle effect of productivity growth on individual savings present in these papers and the majority of the literature (see appendix B.7 for discussion). Regarding the effects of government debt, our paper finds smaller effects than the literature, though this is largely accounted by the fact that our starting period is right after WWII, when the debt-to-GDP level was still very elevated.[23]

The main difference concerns the effect of changes in market structure and the labor share. This term includes the effect of changes in markups and capital intensity, where capital intensity changes can be driven by changes in the production technology or falling investment prices and a non-unitary elasticity of substitution. Both EMR and Peruffo and Platzer (2024) (PP) find this effect to push down on interest rates, while we find it to be a strong force in the opposite direction. The main reason is that our model includes the

[23]The other driver of differences is assumptions that ultimately impact the elasticities of asset supply and demand, with EMR having $\epsilon_s + \epsilon_d$ of around 20, where ours is nearly 30.

Paper	RS2019	EMR2019	PP2024	Our paper
Period	1971-2019	1970-2015	1965-2015	1950-2024
Demographics	-190	-366	-72	-146
Inequality	-54	–	-96	-91
Productivity	-171	-190	-144	-118
NFA	–	–	-36	-66
Old age healthcare	100	–	21	–
Social security	127	–	–	49
Government debt	86	211	29	15
Labor share/markups	–	*-96*	-60	139
Risk premium	–	–	–	30

RS2019 denotes Rachel and Summers (2019), EMR2019 denotes Eggertsson et al. (2019), PP2024 denotes Peruffo and Platzer (2024). The labor share/markups line includes forces from changes in capital intensity, including those driven by declines in the relative price of investment in EMR2019.

Table 3: Contributions to change in r^* in alternative papers (basis points)

value of capitalized profits as a part of asset supply. When this effect is included, higher markups raise interest rates, since households need to be enticed to hold more assets (see also Auclert and Rognlie 2018).

5 The race ahead and the role of fiscal policy

Our retrospective analysis identified an important role for the compositional forces of demographics, inequality, and productivity in increasing asset demand. We found that these forces were large enough to swamp the increase in asset supply, leading to lower equilibrium interest rates. In short, asset demand won the race in the last 75 years. We now turn our attention to the race ahead. Is asset supply or demand more likely to win the race in the next 75 years?

That race will likely be shaped by two big drivers. On the asset demand side, population aging will continue, with an increasing number of old, wealthy individuals increasing the aggregate demand for assets. Moreover, even if income inequality itself stops increasing, the effect of past increases in inequality will also continue to increase asset demand, as more unequal young cohorts enter their prime asset accumulation years. On the asset supply side, our focus will be on government debt, which is on an increasing trend and will itself be strongly affected by demographic shifts.

5.1 Continued rise of asset demand

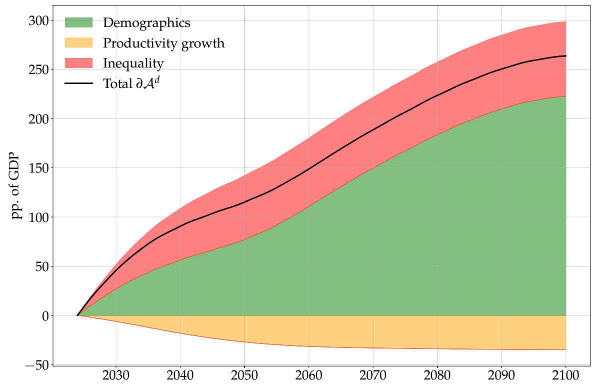
Our projection of asset demand for the next 75 years follows the same methodology that we used to construct our retrospective exercise in figure 10. To do this, we need to make assumptions about the evolution of the underlying drivers of asset demand. Since we do not have reliable forecasts of the future evolution of income inequality, productivity growth, the labor share, the foreign demand for assets, taxes, or the generosity of social security generosity, we consider a benchmark where all these factors are held constant at their current level. Since we do, on the other hand, have informative forecasts of the future demographic evolution of the the U.S. population, we make use of these projections. Specifically, our benchmark population projection is the (geometric) average of the United Nation's low- and medium-fertility population projections. As we explain in appendix D.1, we pick this average because the U.N.'s medium fertility scenario appears overly optimistic (assuming a long-term total fertility rate of 1.65, even though fertility is trending downward and has already fallen below this level) while the low fertility scenario appears pessimistic (assuming that the fertility rate collapses to below 1.2 over the next 10 years).[24] We consider robustness to using either of the U.N.'s scenarios in section 6.

Our new starting year is 2024, with a level of wealth-to-GDP of 635%. For interest rate projections, we also recalculate the semielasticities of asset demand and supply using our 2024–2100 demographic assumptions. This gives us $\epsilon_d = 12$, as increasing longevity gives more scope for substitution, while $\epsilon_s = 20$ remains almost the same.

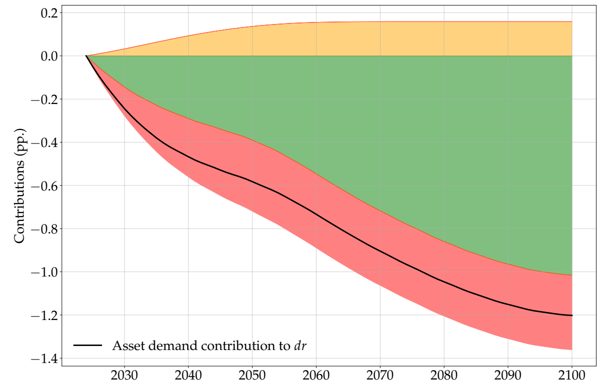
Figure 11 shows the result of this exercise. Asset demand is projected to further increase by around 300% of GDP, similar to the magnitude of the overall asset demand increase observed between 1950 and 2025 in figure 10. This large effect stems mostly from the continuing aging of the population, working through the same compositional effect of old individuals having more assets and a lower labor supply. There is also an inertial effect of inequality, as cohorts that started out more unequal reach their prime asset accumulation ages, as well as a partially offsetting effect from productivity growth, stemming from our assumption that new cohorts benefit from a higher rate of labor-augmenting productivity growth (about 1.8%) than those that were born between 2000 and 2025. Overall, these numbers highlight the quantitative bite of likely changes in the population age distribution.

Increasing asset demand causes continued downward pressure on interest rates. As the

[24] See, for instance, Fernández-Villaverde (2025) for discussion of how the medium U.N. projections have repeatedly overestimated future fertility.



(a) Asset demand



(b) Interest rate

Figure 11: The long view on asset demand shifts and effects on interest rate

right panel of figure 11 shows, with no changes to the asset supply curve, our calculated elasticities suggest that the interest rate would decline by around 120 bps.

5.2 Asset supply and the demographic drag on fiscal policy

To project asset supply forward, we similarly assume that the non-demographic drivers of asset supply are held constant. Demographics, however, has significant effects on the evolution of asset supply. First, there is relatively small effect whereby a small population growth reduces the growth rate of rents, and thus their capitalized value. Second, and more importantly, demographics has fiscal effects that affect primary deficits and thereby the trajectory of government debt.

Government debt relative to GDP follows the law of motion

$$\frac{B_{t+1}}{Y_{t+1}} = \frac{1+r_t^f}{1+g_t}\frac{B_t}{Y_t} + \frac{PD_t}{Y_t} \tag{27}$$

where PD_t is the primary deficit, r_t^f is the risk-free rate and g_t the growth rate. The primary deficit can be obtained from equation (13) as:

$$PD_t = G_t + (1-\tau)T_t - \tau\left(Y_t - \delta K_t\right), \tag{28}$$

which consists of total outlays, government consumption plus transfers net of taxes, minus income taxes, given by the tax rate τ times GDP net of depreciation (since depreciation is untaxed).

An aging population affects primary deficits by placing upwards pressure on government consumption and transfers, as older people consume more healthcare and receive larger social security payments. To asses this effect for government consumption, we project total spending on Medicare and Medicaid by holding fixed the relative amount of expenditure on these programs by age and letting the age population of the distribution evolve. We further assume that the rest of outlays remains constant relative to GDP. Given a base year of t_0, which we assume to be 2024, this gives us the forecast:

$$\frac{G_t}{Y_t} = \frac{G_{t_0}}{Y_{t_0}} + \frac{G_{t_0}^{\text{health}}}{Y_{t_0}} \left(\frac{\sum_j \pi_{jt} g_j^{\text{health}} / \sum_j \pi_{j,t} we_{jt_0}}{\sum_j \pi_{jt_0} g_j^{\text{health}} / \sum_j \pi_{j,t_0} we_{jt_0}} - 1 \right), \tag{29}$$

where $\frac{G_{t_0}^{\text{health}}}{Y_{t_0}}$ is total spending on Medicare and Medicaid relative to GDP in 2024, and we use g_j^{health} as the profile of benefits per capita from Medicaid and Medicare (measured in 2016 using data from De Nardi, French, Jones and McCauley 2016b).[25]

Similarly, we project total transfers relative to GDP by assuming that social security payments grow with demographics at unchanged generosity, and the remainder of transfers stays constant as a fraction of GDP. This gives us the formula:

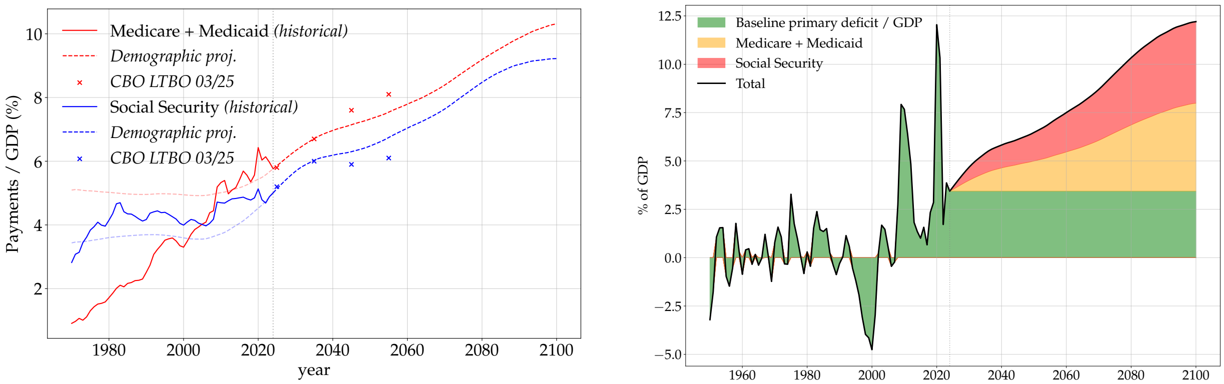
$$\frac{T_t}{Y_t} = \frac{T_{t_0}}{Y_{t_0}} + \frac{T_{t_0}^{SSA}}{Y_{t_0}} \left(\frac{\sum_j \pi_{jt} t_j / \sum_j \pi_{jt} we_{jt_0}}{\sum_j \pi_{jt_0} t_j / \sum_j \pi_{j,t_0} we_{jt_0}} - 1 \right) \tag{30}$$

where $\frac{T_{t_0}^{SSA}}{Y_{t_0}}$ is total social security spending relative to GDP and t_j is average social security received at age j.

Appendix D.2 shows the age profiles of g_j^{health} and t_j, together with the forecasted evolution of the age distribution until 2100. Healthcare spending and social security payments increase dramatically around age 65, and the fraction of the population aged 65 or above almost doubles until the end of the century. Equations (29) and (30) therefore imply a near doubling of health expenditure and social security payments relative to GDP.

Figure 12(a) displays this projected evolution of Medicare / Medicaid spending and social security payments. The solid line shows the historical evolution of these programs since 1970, as together they rose from 4% of GDP to about 11% of GDP in 2024. The dashed line then shows the projected expansion from demographics at fixed generosity to a combined 14% of GDP in 2050 and 20% of GDP in 2100. As the red and blue crosses show, our projections through 2055 are broadly similar to those from recent CBO forecasts; our purely demographic approach overshoots the CBO on Social Security, likely because it

[25] We thank the authors for sharing this data with us.



(a) Social security and Medicare/Medicaid

(b) Primary deficit

Figure 12: Projecting government outlays and the primary deficit

does not take into account the continued phase-in of the 67 retirement age, but undershoots on Medicare and Medicaid.[26]

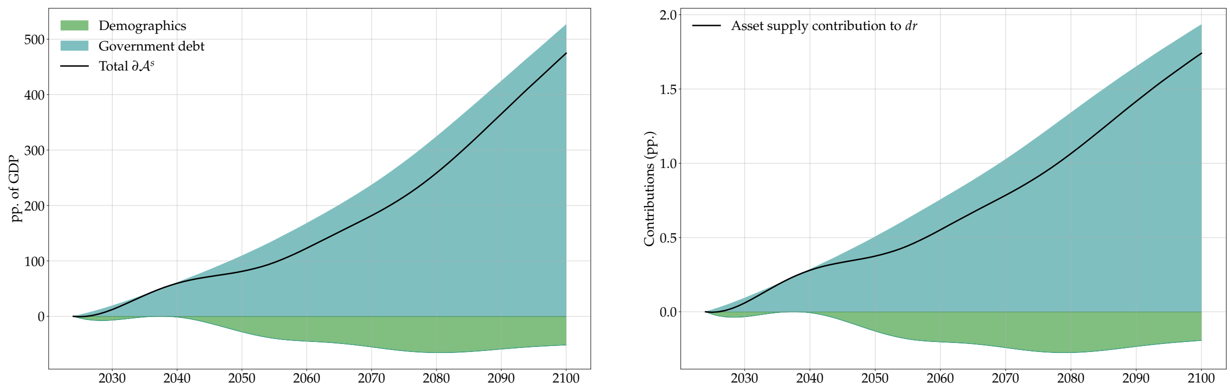
Combining our projections for government spending (29) and transfers (30), we construct a path for the primary deficit going forward using equation (28) at a constant tax rate τ. Figure 12(b) shows that in this scenario, the primary deficit grows from an initial level of 3.4% of GDP in 2024 to 7.0% of GDP in 2055 and 12.2% of GDP in 2100.[27] These projections are higher than those made by the CBO in their latest Long-Term Budget Outlook (Congressional Budget Office, 2025), which projects a primary deficit of only 1.9% in 2055, despite a similar increase in health spending and social security. This is because CBO forecasts a reduction of "other mandatory spending" by 1.5% of GDP, a decline in discretionary spending by 1.2%, and an increase in tax revenue by 2.2% of GDP, and also takes the endogenous response of interest rates into account. Our exercise here is different in nature: we hold interest rates and taxes constant and focus solely on the role of demographic shifts rather than the effect of additional predictable changes in tax revenue.[28] We return to implications for interest rates in the next section.

Given the path for primary deficits PD_t/Y_t constructed in this way, we use equation

[26]The figure also displays the result of equations (29) and (30) applied backwards to past age distributions. Our assumption explains fairly well the evolution of social security payments, but it understates the rise of Medicare and Medicaid because our procedure does not account for the increase in the generosity of these programs over time, as Newhouse (1992) already pointed out.

[27]These projections do not include any recent adverse fiscal developments, such as the Big Beautiful Bill passed in July 2025.

[28]CBO's projected decline in discretionary spending is in part due to projected reductions in defense spending relative to GDP. Their increase in tax revenue is in part due to assumed expiration of the Tax Cuts and Jobs Act (which was made permanent in the Big Beautiful Bill in July 2025). On the other hand, it also accounts for the effect of real bracket creep on tax revenue, which our simple approach does not capture.



(a) Asset supply **(b)** Interest rate

Figure 13: The long view on asset supply shifts and effects on interest rate absent fiscal adjustment

(27) to project the evolution of debt-to-GDP B_t/Y_t, assuming a constant interest rate r^f and our demographic projection for the output growth rate g_t. This exercise implies a surge in government debt of over 500% of GDP by 2100. Our overall forecast for aggregate asset supply is lower than this by about 50% of GDP, because declining population growth, by increasing $r - g$, also reduces the capitalized value of assets.

5.3 Debt sustainability and the fiscal policy race

Comparing the magnitudes of the asset supply and demand shifts in figures 11 and 13, it is immediately obvious that, absent a fiscal consolidation, asset supply will ultimately overwhelm asset demand. Indeed, absent such a consolidation, government debt and hence asset supply diverges, while asset demand eventually stops increasing as the population reaches a demographic steady state. Equilibrium in the asset market requires fiscal consolidation to eventually occur, with the timing of this consolidation determining whether and by how much interest rates ultimately increase.

Here, we assume that fiscal consolidation is achieved by increasing the tax rate on labor, with alternative consolidation scenarios discussed in section 6. We first allow for the possibility that, going forward, the labor tax rate τ^l and the capital tax rate τ^k might differ from their common value of τ in our benchmark. With this distinction included, equation (27) implies that achieving a stable debt-to-GDP ratio B/Y requires eventually setting τ^l and/or τ^k so that tax receipts match outlays, including the cost of sustaining the debt,

$$\tau^l \left(\frac{wL}{Y} + \frac{T}{Y} \right) + \tau^k \left(1 - \frac{wL}{Y} - \delta \frac{K}{Y} \right) = \frac{G}{Y} + \frac{T}{Y} + \frac{r^f - g}{1+g} \frac{B}{Y} \tag{31}$$

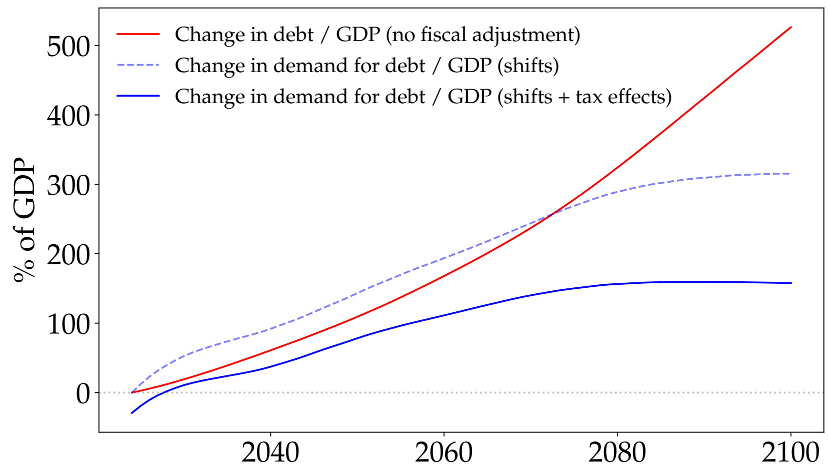


Figure 14: The fiscal race: supply vs demand for government debt

Given that demographic shifts imply much higher levels of G/Y and T/Y, the eventual labor tax rate τ^l required to make this equation hold is much higher than the current tax rate. In turn, per equation (19), a higher labor tax rate depresses asset demand.

Figure 14 shows the change in debt/GDP implied by our forecast without fiscal adjustment, compared to the net demand for debt/GDP, equal to asset demand plus the reduction on asset supply from lower asset values, which gives more space for the government to grow its own debt. The dashed line gives the continued asset demand without a tax adjustment. This shows significant capacity of the economy to absorb more debt, but starting in 2075, this demand is no longer enough to meet the rising supply of government debt. The solid line then takes the required labor tax adjustment into account, giving the answer to the question: what is the level of government debt in each year such that, if the government stabilizes debt at that level by setting the labor tax rate to satisfy equation (31), the interest rate would remain constant? By 2100, we see that debt could grow another 150% of GDP without pushing up interest rates. This is because the exact same demographic force that is pushing up government debt is also creating additional demand for that debt.

We next consider the implications of the eventual fiscal consolidation for interest rates. For this, we look at 2100 as a steady state, and consider the implication of having different levels of stabilized government debt to GDP at that time. The primary surplus required to stabilize the debt, $\frac{B}{Y}\frac{r^f-g}{1+g}$, depends on the level of debt and its interest rate r^f, with the interest rate growing with the size of the debt. Together with expenditure levels, the required primary surplus pins down the required fiscal adjustment by equation (31).

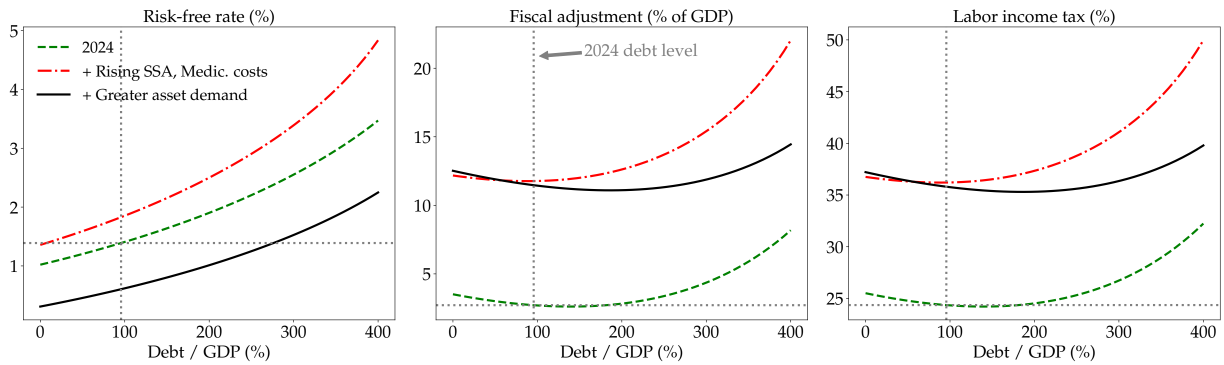


Figure 15: Stabilizing debt in 2100

The black lines in figure 15 shows the outcome of this calculation. The left panel shows interest rates: in line with our findings so far, interest rates would fall almost 100bps if debt/GDP is stabilized at its 2024 level of about 100%, while increasing debt/GDP to about 250% would keep interest rates constant. The middle and right panels shows, however, that any of these options require a very significant fiscal adjustment, of the order of 12% of GDP, equivalent to an increase in the labor tax rate of around 16pp relative to its 2024 level of 20%. Relative to a situation where demographic parameters are stable at their 2024 level (green dotted line)—which requires a much milder adjustment to stabilize debt, of the order of 3% of GDP—this reflects the two effects of demographic change: on the one hand, by putting pressure on the government budget constraint, population aging raises the size of the necessary fiscal adjustment and thus pushes up real interest rates (red dashed line); on the other, aging increases asset demand and therefore allows interest rates to fall.

Taking stock, our analysis suggests that the U.S. economy could look in 75 years a little like Japan does today: much higher debt levels, much higher outlays on old-age programs, and nevertheless similar or lower interest rates. Nevertheless, making debt sustainable requires a fiscal adjustment of nearly 12% of GDP—a dramatic increase given the current tax-to-GDP ratio of around 17%. While these numbers may sound extreme, our international comparisons in section 7 suggest that they may not be so far-fetched.

6 Discussion and robustness

Our prospective analysis required a number of strong assumptions, and we now explore the sensitivity to these assumptions. What if fertility falls faster than we assumed, or instead stabilizes more rapidly? What if the fiscal adjustment takes place in other ways, such as through reducing the generosity of government programs? What if government

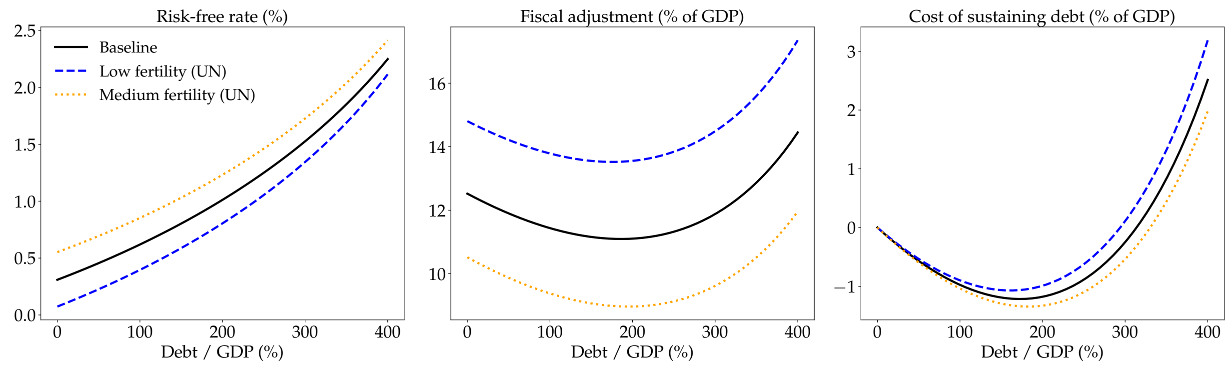


Figure 16: Alternative population projections

debt loses its special convenience benefits as it grows? Throughout this section, we take the black line in figure 15 as a baseline for how high the risk-free interest rate may be in 2100, and how much tax-financed fiscal adjustment there may have to be to balance the government budget.

Varying fertility. Figure 16 compares our baseline (black) to two alternative population projections, the U.N.'s low and medium fertility projections. Since our baseline projection is the average of the two, it lies in between these two alternatives. The U.N. medium fertility scenario assumes that fertility stabilizes immediately, and therefore that the population ages less. This implies a smaller increase in asset demand and a higher risk-free rate. On the other hand, as government-provided old-age benefits are less costly in this scenario, the overall fiscal adjustment needed to balance the budget is about 2% of GDP smaller. The low-fertility scenario has the opposite effects.

The right panel of figure 16 shows the cost of sustaining government debt, equal to $\frac{r^f-g}{1+g}\frac{B}{Y}$ in equation (31). This follows a U shape and is negative at the 2024 debt level, since there $r^f - g < 0$. This gives government room to increase its debt at negative flow cost, though eventually the effect of increasing debt on the risk-free rate overwhelms this benefit, with the cost of sustaining debt bottoming out around 200% of GDP. While a large literature following Blanchard (2019) has focused on this effect given the low $r^f - g$, we see that this term is much smaller than those of increasing health and social security costs from demographics on the government budget, so that stabilizing the debt always requires a large fiscal adjustment irrespective of any $r^f - g$ benefit.

Increasing the retirement age. A policy that is often discussed to keep the cost of public retirement programs down is an increase in the retirement age. In Figure 17 we compare

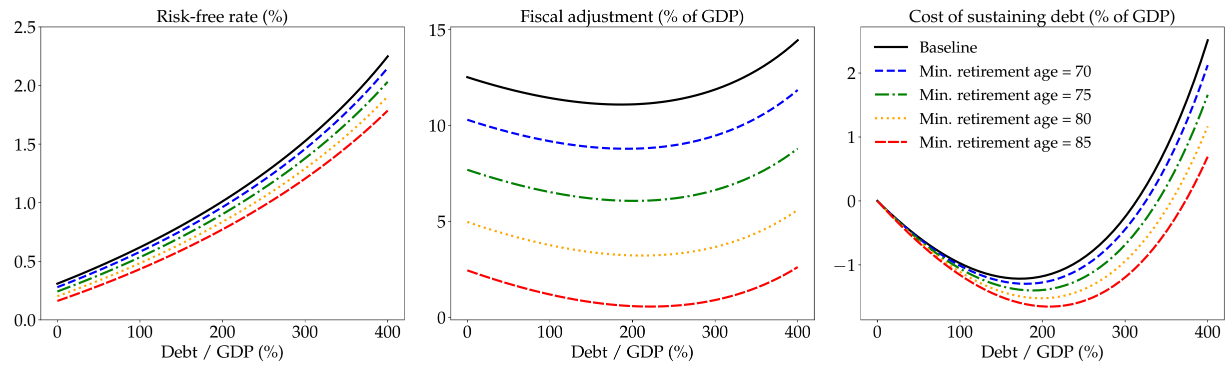


Figure 17: Raising the retirement age

our baseline case of unchanged Social Security and Medicare programs (black) with several alternatives, in which we simultaneously raise the minimum retirement age and the minimum Medicare eligibility age. We implement this as follows. We identify all transfers t_j that are Social Security payments, t_j^{SS} and all payments g_j^{MC} for Medicare. When imposing a minimum retirement age of j^r, we then replace t_j^{SS} and g_j^{MC} by

$$\tilde{t}_j^{SS} = \begin{cases} 0 & j < j^r \\ t_j^{SS} & j \geq j^r \end{cases} \qquad \text{and} \qquad \tilde{g}_j^{MC} = \begin{cases} 0 & j < j^r \\ g_j^{MC} & j \geq j^r \end{cases}$$

We also need to take a stand on the effect of a rising retirement age j^r on labor supply e_j. For simplicity, we assume that the post-transfer income schedule is unchanged, that is,

$$\tilde{e}_j = e_j + t_j^{SS} - \tilde{t}_j^{SS}$$

This assumption implies that households' desired asset holdings A^h are unchanged as the retirement age increases. Asset demand relative to GDP, $\mathcal{A}^d$, however decreases as GDP increases with effective labor supply. This effect, on its own, would imply mildly higher interest rates, but it is more than offset by the effect of a lower tax rate, which increases $\mathcal{A}^d$.

The lines in figure 17 show that an increase in the retirement and Medicare eligibility age is not a panacea. U.S. life expectancy is projected to rise by about 10 years between now and 2100. Even if this is passed through to a retirement age of 75, the need for fiscal adjustment would still be around 8% of GDP, or almost half of current federal tax revenue. If the minimum age for Social Security is increased alone, without an increase in the Medicare eligibility age, or vice versa, the reduction in the fiscal adjustment relative to our baseline is about half.

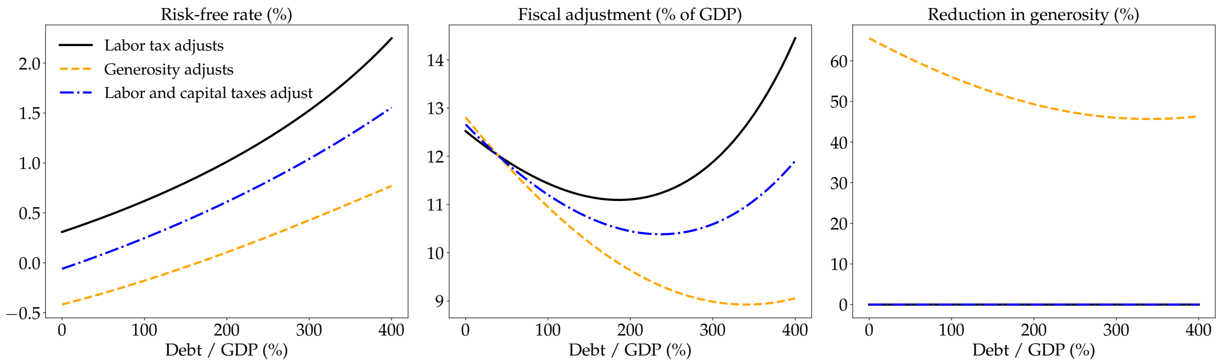


Figure 18: Stabilizing debt in 2100: Labor tax vs. reducing old-age benefit generosity

Alternative financing. Beyond increasing the retirement age, the government can mitigate the projected increases in the primary deficit in figure 12(b) in different ways. Our baseline assumed that labor income taxes were raised. This seems like a reasonable starting point, since currently the payroll taxes dedicated to funding Social Security and Medicare (FICA taxes) are applied to labor income. In figure 18, we compare this with alternative ways to balance the budget. The orange line assumes that the generosity of Social Security and Medicare / Medicaid is scaled down to balance the budget, without an adjustment to the labor tax. This reduces household income later in life, and so it raises asset demand instead of lowering it. As the left panel shows, this lowers the real risk-free rate by over 1pp, helping government finances. In turn, this requires a sizably lower fiscal adjustment. Note, however, that the reduction in program generosity required to balance the budget here is very large, of the order of 50% cuts or more, as shown in the right panel.

The dashed blue line shows the effect of raising both the labor and the capital tax rate by the same amount, setting $\tau^l = \tau^k$ in (31), instead of using labor taxes alone. Raising the capital tax rate increases the user cost of capital and therefore crowds out capital, giving government debt more space to grow at given interest rates–though it also reduces the level of GDP, not visualized in these graphs which are normalized by GDP.

Endogenous risk premia. A large literature estimates how much government bond yields increase when government debt increases (see, for instance, Laubach 2009, Krishnamurthy and Vissing-Jorgensen 2012, or Mian et al. 2022 for a review). This effect, sometimes referred to as the debt sensitivity of interest rates ("DSIR"), is obviously critical for analyses of debt sustainability, including from the CBO (eg. Neveu and Schafer 2024). Typical estimates for the U.S. are between 0 and 4 basis points per 1% of GDP increase in government debt.

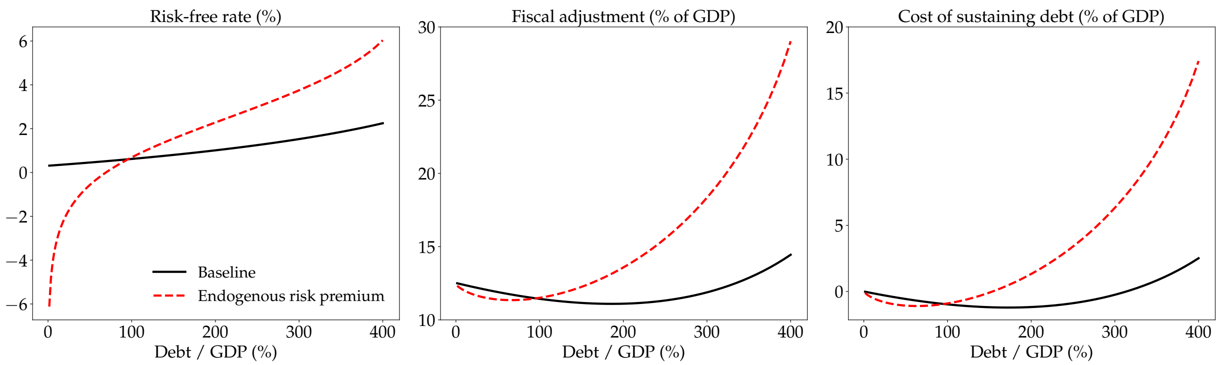


Figure 19: Stabilizing debt in 2100: Endogenous risk premium

By having a fixed risk premium, the DSIR in our model is equal to the sensitivity of the average rate of return to debt issuance, which is given by

$$\frac{\partial r}{\partial \log(B/Y)} = \frac{1}{\epsilon^s + \epsilon^d} \times \frac{B/Y}{A/Y}$$

In our parameterization, and with 2024 values, this sensitivity is around $\frac{1}{12+20} \times \frac{100}{635} = 0.5$ bp, which is inside the range used in the literature, but on the lower end.

To examine what a higher DSIR would imply for our results, we extend our framework to allow the risk premium rp to vary with government debt. Specifically, we let

$$rp = rp_{2024} - v\left(\log\frac{B}{Y} - \log\frac{B_{2024}}{Y_{2024}}\right)$$

where the parameter $v \geq 0$ governs how much the risk premium shrinks as government debt increases. This proxies, in our one-asset framework, for the idea that there is specific demand for government debt that is distinct from that of other assets, so increasing government debt not only raises the average return by increasing total asset supply, but also increases the risk-free rate by more than the average return by increasing the share of government debt in total assets (in the spirit of Krishnamurthy and Vissing-Jorgensen (2012), the convenience yield of safe government debt declines with greater outstanding debt). We calibrate v to 0.016, which gives a DSIR of 2 bps per 1% of government debt.

Figure 19 shows the implication of an endogenous risk premium for our exercise. Now, as debt rises above its 2024 level, the risk-free rate increases rapidly as the risk premium (or convenience yield) declines. With a debt of 250% of GDP, the real interest rate can be up to two percentage points higher than in our scenario with a constant convenience yield. It also implies that the cost of going to very high debt levels is catastrophic: at 400%

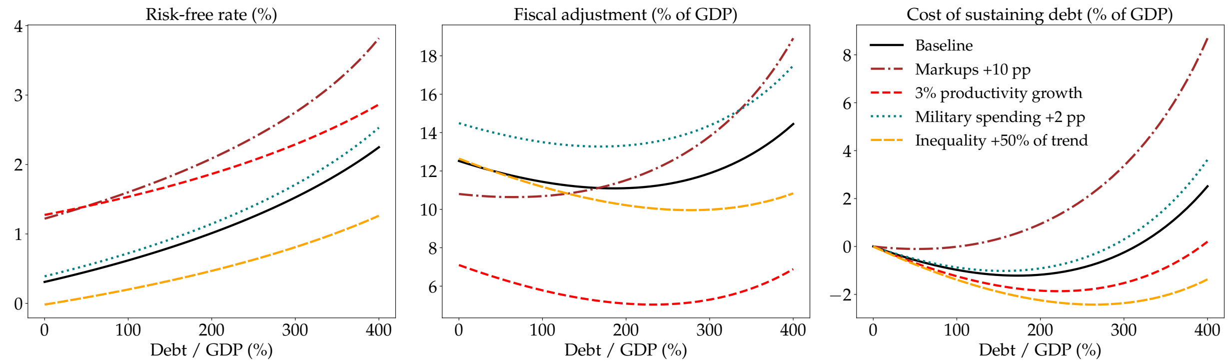


Figure 20: Stabilizing debt in 2100: Various scenarios

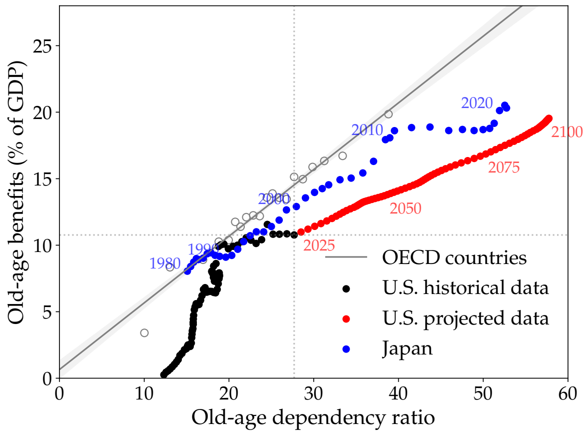
debt-to-GDP, the required fiscal adjustment reaches almost 30% of GDP.

Other secular trends. While our retrospective analysis identified many drivers of interest rates in the past, our prospective analysis so far held the non-demographic drivers constant. In figure 20, we now consider four alternative scenarios for these other variables.

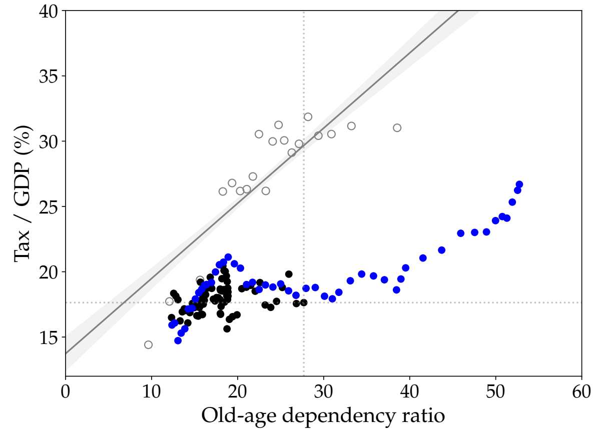
We first consider the implications of continuing increases in markups or productivity. The two red lines respectively consider an increase in markups by 10 percentage points, or a persistent boost in productivity growth (possibly due to artificial intelligence) to 3%. Both of these forces raise asset supply and reduce asset demand, unambiguously pushing interest rates up and making it more expensive to stabilize government debt. This illustrates a *reverse crowd-out effect*: just like government debt can crowd out private asset supply, this shows that rising private asset supply can make it harder for the government to sustain high debt. While an increase productivity growth seems like an unambiguous positive, it can worsen the government's fiscal situation.

The teal dotted line considers an increase in military spending by 2 percentage points of GDP, as may be necessary given heightened geopolitical risks. This adds to the necessary fiscal adjustment, pushing up taxes more than otherwise. As taxes reduce asset demand, interest rates increase. Finally, the dashed orange line increases inequality beyond its 2024 level, by 50% of the 1970–2024 trend. As it concentrates more income in the hands of rich savers, asset demand rises, interest rates fall, and a smaller fiscal adjustment is necessary to stabilize the debt.

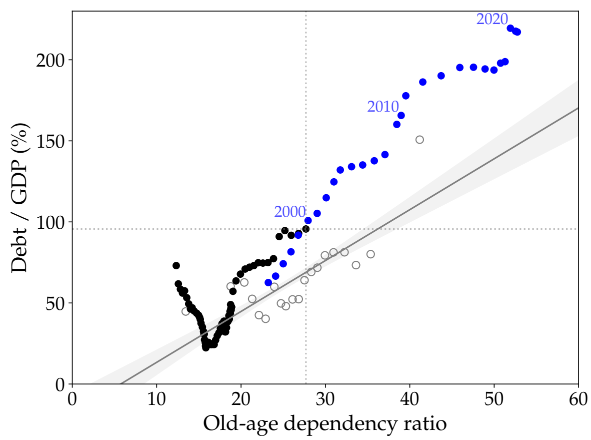
These scenarios show that it is not always intuitive how different secular trends affect the fiscal position of the government. We believe the asset supply and demand framework laid out in this paper provides a useful guide to think through alternative scenarios.



(a) Old-age dependency ratio vs. old-age benefits



(b) Old-age dependency ratio vs. tax revenue



(c) Old-age dependency ratio vs. debt

Figure 21: International comparison

7 International comparison

Our calculations of the last two sections showed a dramatic effect of population aging on government outlays on old-age benefits (Social Security, Medicare, and some of Medicaid). Our baseline assumption was that taxes would not adjust immediately, with government debt financing the additional outlays for a while, and interest rates remaining relatively stable as population aging simultaneously caused an increase in asset demand, absorbing some of the additional debt.

Figure 21 shows that other countries that are further along their demographic transitions appear to be experiencing exactly these types of phenomena. In all panels, we compare the U.S. historical experience with that of OECD countries (gray), shown as a binned scatter with a trend line.[29] We also specifically highlight the historical experience of Japan, in blue.

Panel (a) shows that aging economies across the OECD see strong increases in government old-age spending relative to GDP. The slope relative to old-age dependency is about one half, that is, for a 10 pp. greater old-age dependency ratio, old-age benefits rise 5 pp. of GDP. This is in line with the historical pace at which old-age spending rose in the U.S. as it aged, though the level of old-age benefits has been lower. Going forward, our projections in red forecast a slightly more modest increase in old-age spending than what we could infer from the OECD experience. Our projected trajectory is, in particular, similar to that experienced by Japan between 2000 and 2020.

Panel (b) shows a similar plot, with tax revenue to GDP on the vertical axis. As countries age, they tend to raise tax revenue by less than old age benefits. The Japanese experience, in particular, suggests a significant delay in the adjustment of taxes.

Finally, panel (c) shows debt to GDP on the vertical axis. On average, aging OECD economies increase their debt levels a lot as they age, with Japan a poster child of this phenomenon. If one discounts the early post-world-war-II period of high and rapidly falling U.S. debt levels, the recent rise in U.S. debt is similar to that experienced by Japan before 2000, and the graph suggests that a debt-to-GDP ratio of 200% or more in 2100 is not unreasonable given historical precedents.

8 Conclusion

We propose a new asset supply-and-demand framework to study historical and future trends in aggregate wealth, r^*, and fiscal sustainability. If asset supply wins the race, interest rates rise, and if asset demand wins, they fall.

We use this framework to analyze the implications of demographic change for fiscal sustainability. On the one hand, an aging population increases government outlays on healthcare and social security payments. On the other, an older population demands more government debt. This implies that there is space for the government to finance its additional outlays by increasing its debt. Our calculations suggest that, in 2100, the U.S. could sustain a debt-to-GDP ratio of 250% at the same interest rates as today. However, achieving this requires a fiscal adjustment of 10% of GDP or more in every plausible scenario. The longer this adjustment is delayed, the more government debt supply outstrips its demand, eventually making government debt unsustainable.

[29] The plots here are based on a pooled sample, but all relationships discussed in the following also hold with year or country fixed effects.

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A Aggregate data and parameters

A.1 Calculation of aggregate ratios

First, we describe how the realized components of asset supply and demand in figure 3 are calculated, along with related aggregate series like shares of aggregate income. These are necessary inputs to the calculation of overall r in the next section.

Components of asset supply and demand. We measure capital K as the current-cost stock of private fixed assets (including equipment, structures, and intellectual property) in line 1 of the BEA's fixed asset table 2.1,[A-1] and annual GDP Y from line 1 of NIPA table 1.1.5, dividing to obtain K/Y.[A-2] We measure B as federal debt held by the public, which we take from two sources: first, through 1970, gross federal debt held by the public at the end of the fiscal year (which, during this period, ended on June 30) as reported by the CEA in the Economic Report of the President; then, from 1970 onward, Treasury debt held by the public at the end of the calendar year as reported in the Treasury Bulletin. We measure household net assets A^h as household and nonprofit net worth from the financial accounts (series FL152090005.A, line 40 of table B.101), and the net foreign asset position NFA as minus the net worth of the rest of the world in the Integrated Macroeconomic Accounts (series FL262090095.A, line 153 of table S.9.a). Finally, we measure Π, the capitalized value of profits, residually by taking

$$\frac{\Pi}{Y} = \frac{A^h}{Y} - \frac{NFA}{Y} - \frac{B}{Y} - \frac{K}{Y}$$

so that asset market clearing is satisfied. Note that under this definition of Π/Y, it will include not only the capitalized value of any pure profits (or unmeasured, intangible capital) but also the capitalized value of any land rents.

Income shares. Next, we divide GDP into income shares. We start by splitting GDP into four categories: unambiguous labor income, unambiguous net capital income, depreciation of private capital, and "ambiguous" income (which does not map neatly to any category). We measure unambiguous labor income as the compensation of employees in the corporate, household, nonprofit, and government sectors, summing lines 4, 43, 50, and 57 of NIPA table 1.13. We measure unambiguous net capital income as all profits, rents, net interest, and current transfer payments

[A-1] As of writing this paper, FA table 2.1 only covered up to 2023. To ensure that all series cover 2024, we extend the series to 2024 using table L.4.s of the financial accounts, taking line 8 (all fixed assets) minus lines 12 and 13 (government fixed assets). This agrees with line 1 of FA table 2.1 through 2023.

[A-2] For all nominal stocks that are measured at the end of the year, like K we average end-of-year values to obtain a value that is more comparable to flow GDP; for instance, we measure 2020 K as the average of the 2019 and 2020 end-of-year values in table 2.1. This is useful to avoid distortions during periods of high inflation, which would otherwise exaggerate capital (measured at the beginning of the year) relative to GDP (measured throughout the year).

in the corporate, household, and nonprofit sectors, summing lines 7, 8, 46, 47, 53, and 54 of NIPA table 1.13, as well as line 10 of NIPA table 1.14. We measure depreciation of private capital as line 22 of NIPA table 1.10. Finally, we measure "ambiguous" income as the sum of production taxes net of subsidies (line 7 minus 8 of NIPA table 1.10), depreciation of government capital (line 23 of table 1.10), the statistical discrepancy (line 24 of table 1.10), and all net income of the non-corporate business sector excluding production taxes (line 10 minus 17 of NIPA table 1.13, adding back current transfer payments from line 21 of table 1.12 minus line 10 of table 1.14). (We include the last component because of the well-known difficulty in distinguishing labor from capital income in closely held businesses.)

We verify that these four categories exactly sum to GDP. We assume that depreciation of private capital is well measured, and then split "ambiguous" income between labor and net capital income in the same proportion as unambiguous labor vs. unambiguous net capital income.[A-3] We are left with three components of GDP: labor income $s_L Y$, net "capital" income $(1 - s_L)Y - \delta K$ (which in our model will also include pure profits), and depreciation δK.

Taxes and primary balance. We measure total federal tax revenue as total federal receipts excluding interest (NIPA table 3.2 line 40 minus line 14). We divide this by net income $Y - \delta K$ to obtain the overall federal tax rate τ. We measure the primary balance as total federal receipts excluding interest, minus total federal expenditures excluding interest (table 3.2 line 43 minus line 33).[A-4]

A.2 Calculation of returns r and growth g

Calculation of growth g and components n and γ. To calculate returns in a model-consistent way, it is first necessary to calculate GDP growth g. In the model of section 3, g equals the growth rate of augmented effective labor, given by $(1 + g) = (1 + n)(1 + \gamma)$, where n is the growth rate of effective labor $\sum_j e_j N_{jt}$, and γ is the growth rate of labor-augmenting productivity.[A-5] We measure raw n by using the fact that measured gross labor income we_j at each age is proportional to its effective labor supply e_j, so that year-over-year growth is given by the measured ratio of $\sum_j we_j N_{jt}$ and $\sum_j we_j N_{j,t-1}$, with we_j taken from our base year. We measure raw γ by taking utilization-adjusted TFP from Fernald (2014) and dividing by the labor share to obtain equivalent labor-augmenting productivity growth. Finally, to be closer to our steady-state concept, we extract

[A-3] Since NIPA table 1.13 currently ends in 2023, we extend to 2024 by taking the compensation share of net factor income, line 2 divided by (line 1 minus line 7, plus line 8, and minus line 21) from NIPA table 1.10, and assuming that it varied by the same proportion from 2023 to 2024 as our labor share of GDP excluding depreciation, which has been roughly true historically.

[A-4] Line 43, which equals the sum of lines 44 through line 47 minus line 48, is missing in earlier years because line 47, which is generally very small, is missing; we calculate it in missing years by setting this line to zero.

[A-5] We treat these as averages over fixed types θ, under the assumption that the fraction of each fixed type is constant.

slow-moving trends of both the raw annual n and γ series, using an HP filter with $\lambda = 100$, and use these both for n and γ and $g \equiv (1+n)(1+\gamma) - 1$.[A-6]

Calculation of risky return r^r. In the model of section 3.2, the "risky" return r^r determines both demand for capital K and the capitalization of profits Π. Net after-tax income on capital as a share of GDP equals $r^r \frac{K}{Y}$. Equation (11) implies that after-tax net profit income $(1-\tau)\left(1-\frac{1}{\mu}\right)$ as a share of GDP equals $\frac{r^r - g}{1+g}\frac{\Pi}{Y}$. The sum of these equals the net "capital" share we have already calculated, $(1-s_L) - \delta K/Y$, times $1-\tau$. This allows us to write:

$$r^r \frac{K}{Y} + \frac{r^r - g}{1+g}\frac{\Pi}{Y} = (1-\tau)\left(1 - s_L - \delta\frac{K}{Y}\right)$$

which we can rearrange as

$$r^r \left(\frac{K}{Y} + \frac{1}{1+g}\frac{\Pi}{Y}\right) = (1-\tau)\left(1 - s_L - \delta\frac{K}{Y}\right) + \frac{g}{1+g}\frac{\Pi}{Y} \tag{A.1}$$

We evaluate both the right side of (A.1) and the expression in parentheses on the left, using the aggregate ratios and g that we have calculated above, and divide the two to obtain our risky rate series r^r, which is displayed in the left panel of figure A.1.

Calculation of safe return r^f. To calculate the safe return r^f on government debt, we proceed as follows. First, we take the nominal Treasury 5-year yield i from H.15 selected interest rates (accessed via FRED as `GS5`), which is available starting in 1953-04. We extend this monthly series back to 1950 by using the 3-month Treasury bill rate, also from H.15 (accessed via FRED as `TB3MS`), as a proxy, obtaining fitted values based on a regression from the 1953-04 through 1959-12 period where both series are available.

We obtain monthly 5-year-ahead inflation expectations π^e from 1982 onward from the Cleveland Fed (`https://doi.org/10.26509/frbc-infexp`), and up to 1982 use a centered 5-year average of CPI inflation.[A-7] (In 1982, we take an average of the two series.)

Finally, we take annual averages of both i and π^e, and construct the raw annual safe return r^f as $r^f = (1+i)/(1+\pi^e) - 1$. As with growth, we extract a slow-moving trend from this series using the HP filter with $\lambda = 100$ for our final r^f safe return series, displayed in the right panel of figure A.1.

It is worth noting that although figure A.1 displays the well-known decline in safe returns from the 1980s to the 2010s, this decline is smaller here (about 4.5 percentage points from peak to trough)

[A-6] For n, we apply the HP filter to a series that goes through 2100, using our baseline population projection.

[A-7] Although these series do not perfectly coincide, they do not display any systematic difference during the 1982–2019 period, with a mean difference of less than 15 basis points. The idea behind using a centered average is that inflation expectations at any moment are driven by some mix of rational expectations (captured by actual ex-post inflation over the next 2.5 years) and adaptive expectations (captured by realized inflation over the past 2.5 years).

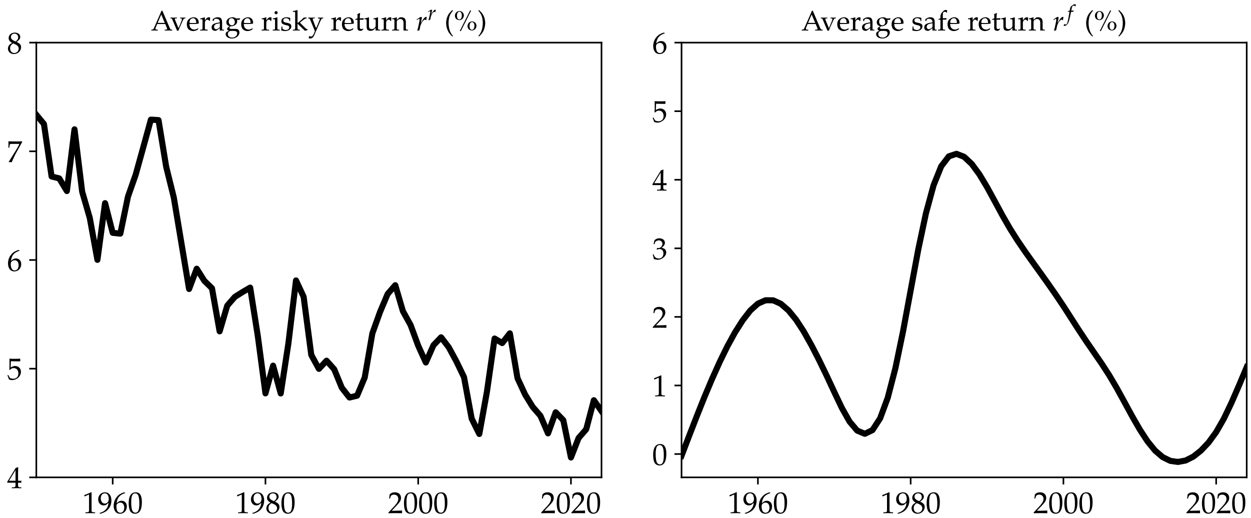

Figure A.1: Estimated risky return (left panel) and safe return (right panel), 1950–2024

than sometimes reported. This is because safe rates were low in the high-inflation 1970s, and our smoothing procedure blends these with the higher rates in the 1980s. There is also no clear long-run trend in safe rates from 1950 to the present. This is because real returns on government debt were quite low in the immediate aftermath of World War II, and also because there has been a recent increase in safe rates in the end of our sample.

Calculation of overall return r. In the model of section 3, the overall return r is a weighted average of r^r and r^f, $r = (1-\bar{s})r^r + \bar{s}r^f$, where $\bar{s}$ is a baseline safe asset share. We obtain $\bar{s}$ by calculating the share of bonds in total assets, B/A, and averaging over the 1950–2024 period, which gives $\bar{s} = 0.11$. The resulting average return r is displayed in figure 2. The risk premium rp is simply $r^r - r^f$, i.e. the left panel minus the right panel of figure A.1.

A.3 Calculation of supply parameters

With r^r, g, τ, and Π/Y from the last two subsections, we can back out the markup μ from $(1-\tau)\left(1-\frac{1}{\mu}\right) = \frac{r^r-g}{1+g}\frac{\Pi}{Y}$, as implied by equation (11). We obtain the depreciation rate δ by dividing the depreciation share $\delta K/Y$ of GDP by K/Y. Finally, we can back out the capital share of costs α from $\alpha = \left(\frac{r^r}{1-\tau} - \delta\right)\mu\frac{K}{Y}$, which follows from equation (10).

B Appendix to section 3

B.1 Production side

Output is produced by a continuum of identical, monopolistically competitive firms with a production function that is Cobb-Douglas in capital and effective labor L_t:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

Firms face a constant-elasticity of demand function with elasticity $\epsilon = \frac{\mu}{\mu-1}$.[A-8] They face a tax τ^k on profits with a depreciation allowance. Assuming they rent out their capital stock at rate R_t and that the wage rate is w_t, so net-of-tax corporate profits are

$$\pi_t = Y_t - w_t L_t - R_t K_t - \tau^k \left(Y_t - w_t L_t - \delta K_t\right)$$

These assumptions imply that the wage per effective unit of labor and the rental rate are given by the first-order conditions

$$w_t = \frac{1-\alpha}{\mu}\frac{Y_t}{L_t} \tag{A.2}$$
$$R_t - \tau^k \delta = \left(1-\tau^k\right)\frac{\alpha}{\mu}\frac{Y_t}{K_t}$$

and profits are $\pi_t = \left(1-\tau^k\right)\left(1-\frac{1}{\mu}\right)Y_t$.

Capital owners are untaxed, they rent the capital at rate R_t and decide how much to invest each period given the steady-state required rate of return of r^r. Their optimal choice of capital then implies that the rental rate is $R_t = r^r + \delta$; capital owners make no profits, and their enterprise value is equal to K_t.

Putting these equations together, we find

$$\frac{K_t}{Y_t} = \frac{\alpha}{\mu}\frac{1}{\frac{r^r}{\left(1-\tau^k\right)} + \delta} \tag{A.3}$$

moreover, the capitalized value of profits satisfies the asset pricing equation $\Pi_t = \frac{\pi_{t+1}+\Pi_{t+1}}{1+r^r}$. In steady state, this implies $\frac{1+r^r}{1+g}\frac{\Pi_t}{Y_t} = \pi_{t+1}/Y_{t+1} + \Pi_{t+1}/Y_{t+1}$, so

$$\frac{\Pi}{Y} = \frac{1+g}{r^r - g}\left(1-\tau^k\right)\left(1-\frac{1}{\mu}\right) \tag{A.4}$$

[A-8] We can microfound this elasticity by assuming that, for household of all ages j, we have $c_{jt} = \left(\int_{k=0}^{1} c_{jkt}^{\frac{\epsilon-1}{\epsilon}} dk\right)^{\frac{\epsilon}{\epsilon-1}}$.

Together, equations (A.2)-(A.3)-(A.4) deliver equations (10)–(12) in the main text after imposing that τ^k is the equal to the overall tax rate τ.

B.2 Life-cycle model and simplification

In the initial formulation (16)–(17) of the problem, households split their assets a_{j+1} on two dimensions: they choose the level of safety s_j, and they also split between annuitized assets x_{j+1} and non-annuitized assets b_{j+1}. We now show that this can be reduced to a simpler one-asset problem.

First, defining $\hat{c}_j \equiv c_j + \overline{rp}(s_j - \bar{s})a_j$, utility u in (16) simplifies to just $\hat{u}(\hat{c})$, and the household problem can be rewritten as

$$\max_{\hat{c}_j, a_{j+1}, b_{j+1}} \sum_j \Phi_j \left(\beta_j \hat{u}(\hat{c}_j) + (1-\phi_j)\kappa_j v(b_{j+1})\right) \tag{A.5}$$

$$s.t. \quad \hat{c}_j + (1-\phi_j)b_{j+1} + \phi_j a_{j+1} = y_j + (1+r)a_j, \tag{A.6}$$

where we have substituted $x_{j+1} = a_{j+1} - b_{j+1}$ on the left of (17), and where $r \equiv \bar{s}r^f + (1-\bar{s})r^r$. Note that this formulation no longer has a choice over s_j.

In (A.5)–(A.6), the split between $\hat{c}_j$ and b_{j+1} is fully intratemporal, allowing for a two-stage budgeting analysis with total effective expenditure in a period given by $\tilde{c}_j = \hat{c}_j + (1-\phi_j)b_{j+1}$. Assuming common power utility with elasticity of intertemporal substitution σ, the choice between $\hat{c}_j$ and b_{j+1} in (A.5)–(A.6) satisfies the first-order condition $\beta_j \hat{c}_j^{-1/\sigma} = \kappa_j b_{j+1}^{-1/\sigma}$, and it follows that the ratio $\hat{c}_j / b_{j+1}$ is fixed at $(\beta_j/\kappa_j)^\sigma$, and therefore that both $\hat{c}_j$ and b_{j+1} are proportional to $\tilde{c}$.

Using this proportionality, we can finally rewrite the problem as

$$\max_{\tilde{c}_j, a_{j+1}} \sum_j \Phi_j \tilde{\beta}_j \tilde{c}_j^{1-\sigma} \tag{A.7}$$

$$s.t. \quad \tilde{c}_j + \phi_j a_{j+1} = y_j + (1+r)a_j, \tag{A.8}$$

where $\tilde{\beta}_j$ is a composite parameter determined by β_j, κ_j, and ϕ. This is a simple, standard life-cycle problem where households at age j choose between effective consumption $\tilde{c}_j$ and annuitized savings a_{j+1}.

B.3 Compositional effect details

Illustrating the cohort productivity shifters. Figure B.1 illustrates the aggregate effect of declining productivity growth on asset demand. As shown in section 3.4, this acts like a shifter of the population distribution akin to a decline of 1% in fertility, tilting the effective population distribution towards older ages, while leaving household asset accumulation decisions unaffected. Through aggregation, this raises aggregate asset demand from composition by 20 log points (left panel). It also raises aggregate labor supply, though by less (7 log points, right panel). The overall

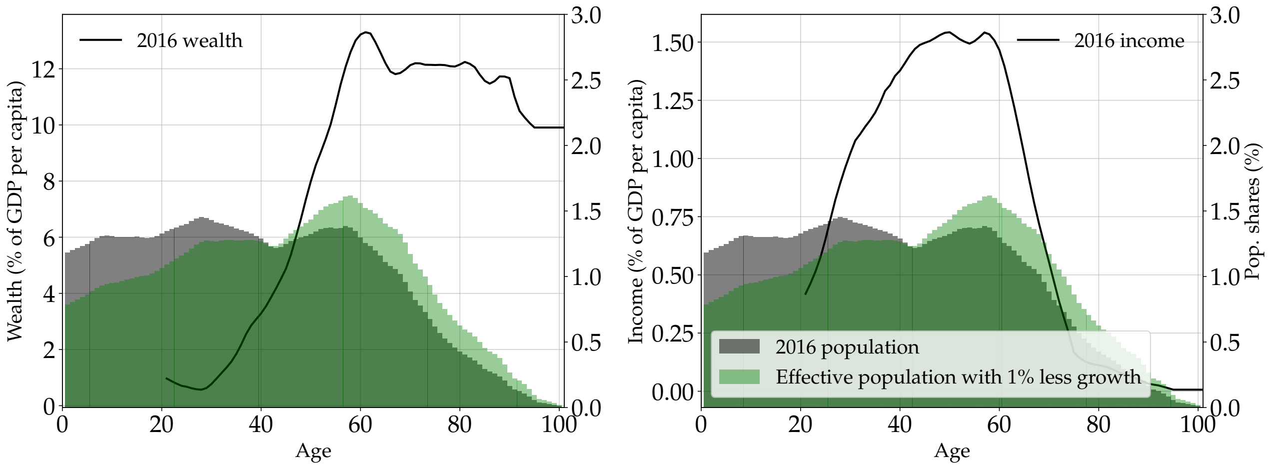

Figure B.1: Cohort productivity shifter

effect is therefore to raise normalized household asset demand A^h/Y by 14 log points.

Effect of inequality. First, we note that substituting in $a_t(\theta) = (1-\tau)w\lambda(\theta)(1+\gamma)^{t-j}\bar{a}_j(\theta)$ and $we_t(\theta) = w\lambda(\theta)\sum_j \pi_j(1+\gamma)^{t-j}\bar{e}_j(\theta)$ into (19), we obtain

$$\begin{aligned}\frac{A^h}{Y} &= s_L \times \frac{\sum_\theta p(\theta)(1+\gamma)^{-t}/w \times a_t(\theta)}{\sum_\theta p(\theta)(1+\gamma)^{-t}/w \times we_t(\theta)} \\ &= s_L \times \frac{\sum_\theta p(\theta)a_t(\theta)}{\sum_\theta p(\theta)we_t(\theta)}.\end{aligned}$$

Further, we note that given a shift of income weights from $\lambda^0(\theta)$ to $\lambda^1(\theta)$, we have that $a_t^{h,1}(\theta) = \frac{\lambda^1(\theta)}{\lambda^0(\theta)}a_t^{h,0}(\theta)$ and $e_t^1(\theta) = \frac{\lambda^1(\theta)}{\lambda^0(\theta)}e_t^0(\theta)$ where $a_t^{h,i}$ and e_t^i is the value of assets and labor endowment associated with the type-specific shifter λ^i. Hence, the shift in aggregate asset demand associated with going from λ^0 to λ^1 is

$$\begin{aligned}\Delta \log \frac{A^h}{Y} &= \log\left(\frac{\sum_\theta p(\theta)a_t^{h,1}(\theta)}{\sum_\theta p(\theta)we^1(\theta)}\right) - \log\left(\frac{\sum_\theta p(\theta)a_t^{h,0}(\theta)}{\sum_\theta p(\theta)we^0(\theta)}\right) \\ &= \log\left(\frac{\sum_\theta p(\theta)\frac{\lambda^1(\theta)}{\lambda^0(\theta)}a_t^{h,0}(\theta)}{\sum_\theta p(\theta)a_t^{h,0}(\theta)}\right) - \log\left(\frac{\sum_\theta p(\theta)\frac{\lambda^1(\theta)}{\lambda^0(\theta)}we_t^0(\theta)}{\sum_\theta p(\theta)we_t^0(\theta)}\right).\end{aligned}$$

To obtain the expression (22) from the main text, we start from (19) and note that

$$\begin{aligned}\frac{A^h}{Y} &= (1-\tau)s_L \frac{\sum_{\theta,j} p(\theta)\pi_j(1+\gamma)^{-j}\lambda(\theta)\bar{a}_j(\theta)}{\sum_{\theta,j} p(\theta)\pi_j(1+\gamma)^{-j}\lambda(\theta)\bar{e}_j(\theta)} \\ &= s_L \times \sum_\theta \frac{\sum_j p(\theta)\pi_j(1+\gamma)^{-j}\lambda(\theta)\bar{e}_j(\theta)}{\sum_{j,\theta} p(\theta)\pi_j(1+\gamma)^{-j}\lambda(\theta)\bar{e}_j(\theta)} \times \frac{\sum_j (1-\tau)p(\theta)\pi_j(1+\gamma)^{-j}\bar{a}_j(\theta)}{\sum_j p(\theta)\pi_j(1+\gamma)^{-j}\bar{e}_j(\theta)} \\ &= s_L \times \sum_\theta \rho(\theta) \times \frac{a_t(\theta)}{we_t(\theta)},\end{aligned}$$

which implies (22).

B.4 Details for effect of income shock

Proposition 1 in general case. Aggregating households in (A.7)–(A.8) cross-sectionally, one can show that at any date t

$$(r-g)A_t^h = \sum_j \pi_j(\tilde{c}_{jt} - y_{jt}). \tag{A.9}$$

Intuitively, the aggregate difference between effective consumption and net labor income must equal aggregate returns on assets, minus net savings (which are gA^h on the balanced growth path). Aggregating (A.8) cross-sectionally, we can also write

$$\sum_j \pi_j(1+\hat{r})^{-j}\tilde{c}_{jt} = \sum_j \pi_j(1+\hat{r})^{-j}y_{jt}, \tag{A.10}$$

where $\hat{r} \equiv (1+r)/(1+g) - 1$. Intuitively, the life-cycle budget constraint is (A.10) with r instead of $\hat{r}$; to convert to cross-sectional units, we multiply by $(1+g)^j$ to reflect growth across generations, which gives (A.10).

Now suppose that there are income shocks $\{dy_{jt}\}$, without any other changes to the problem. Households will shift their consumption schedules proportionately, so we will have $d\tilde{c}_{jt}/\tilde{c}_{jt} \equiv \hat{c}$ for some $\hat{c}$, which we can obtain by solving (A.10):

$$\hat{c} = \frac{\sum_j \pi_j(1+\hat{r})^{-j}dy_{j0}}{\sum_j \pi_j(1+\hat{r})^{-j}\tilde{c}_{j0}}$$

We then plug this into (A.9) to obtain

$$\begin{aligned}dA^h &= \frac{1}{r-g}\left(\hat{c}\sum_j \pi_j\tilde{c}_{j0} - \sum_j \pi_j dy_{j0}\right) \\ &= \frac{1}{r-g}\left(\frac{\tilde{C}}{\tilde{C}^{PV}}dY^{PV} - dY\right)\end{aligned} \tag{A.11}$$

where capitals are interpreted as cross-sectional aggregates, discounted across ages by $\hat{r}$ if there is a

"PV" superscript. (For instance, $\tilde{C} \equiv \sum_j \pi_j \tilde{c}_{j0}$ and $\tilde{C}^{PV} \equiv \sum_j \pi_j (1+\hat{r})^{-j} \tilde{c}_{j0}$.) This gives the effect of an income shock on A^h in the general case where $r \neq g$.

Proposition 1 in special case $r = g$. We can now obtain the special case $r = g$ as a limit of (A.11) as $\hat{r} \to 0$. In the limit, we have $dY^{PV} - dY \sim -\hat{r} \sum_j \pi_j j dy_{j0}$ and $C^{PV} - C \sim -\hat{r} \sum_j \pi_j j c_{j0} = -\hat{r} C \mathbb{E}Age_{\tilde{c}}$, so that the term in parentheses reduces to

$$\hat{r}\left(\mathbb{E}Age_{\tilde{c}} dY - \sum_j \pi_j j dy_{j0}\right) = \hat{r} \sum_j \pi_j (\mathbb{E}Age_{\tilde{c}} - j) dy_{j0}$$

Noting that $r - g = \hat{r}(1+g)$, (A.11) then simplifies in the limit to

$$\frac{1}{1+r} \sum_j \pi_j (\mathbb{E}Age_{\tilde{c}} - j) dy_{j0}$$

where the $\hat{r}$ cancel and $1 + g \to 1 + r$ in the denominator.

B.5 Effects of longevity changes

Proof of corollary 2. Using the formulation of the problem in (A.5)-(A.6), the solution to the household problem is characterized by the following equations

$$\begin{aligned}
\hat{u}'(\hat{c}_j) &= \frac{\beta_{j+1}}{\beta_j} \hat{u}'(\hat{c}_{j+1}), \quad j \in \{0, \ldots, J-1\} \\
\beta_j \hat{u}'(\hat{c}_j) &= \kappa_j v'(b_{j+1}), \quad j \in \{0, \ldots, J\} \\
\hat{c}_j + (1-\phi_j) b_{j+1} + \phi_j a_{j+1} &= y_j + (1+r) a_j \quad j \in \{0, \ldots, J\} \\
a_0 &= a_{J+1} = 0.
\end{aligned}$$

The survival probabilities ϕ_j and the incomes y_j only show up in the budget constraint, so the effect of shocking them is determined by how they affect this. Totally differentiating the budget constraint with respect to the parameters ϕ_j and y_j for a fixed r yields

$$d\hat{c}_j + (1-\phi_j) db_{j+1} + \phi_j da_{j+1} + d\phi_j (a_{j+1} - b_{j+1}) = dy_j + (1+r) da_j.$$

By inspection, the effect of a shock $d\phi_j$ is identical to the effect of $dy_j = -d\phi_j (a_{j+1} - b_{j+1})$, which proves the result.

B.6 Details for the asset demand semi-elasticity ϵ^d

General formula for ϵ^d. To express ϵ^d when $r \neq g$, we write $1 + \hat{r} \equiv \frac{1+r}{1+g}$ for the deviation from $r = g$. Furthermore, we define present value versions of aggregates: $A^{h,PV} \equiv \sum_j \frac{\pi_j a_j}{(1+\hat{r})^j}$ and

$C^{PV} \equiv \sum_j \frac{\pi_j c_j}{(1+\hat{r})^j}$, as well as random variables Age_a^{PV} and Age_c^{PV}, defined by having masses at j proportional to $\frac{\pi_j a_j}{(1+\hat{r})^j}$ and $\frac{\pi_j c_j}{(1+\hat{r})^j}$ respectively.

Given this notation, the general formula for ϵ^d is given by

$$\epsilon^d = \frac{\sigma}{1+r}\frac{C}{A^h}\frac{\mathbb{E}Age_c - \mathbb{E}Age_c^{PV}}{r-g} + \frac{\frac{C/C^{PV}}{A^h/A^{h,PV}} - 1}{r-g},$$

where the first term is a substitution effect, the second term is an income effect, and where the $r = g$ case can be recovered by letting $r \to g$ and applying l'Hôpital's rule. The second term is an income effect that follows from applying the general income shock reuslt (A.11) above with $dY^{PV} = A^{h,PV}dr$ and $dY = Adr$, since the rise in the interest rate relaxes the budget constraint in proportion to asset holdings. For the substitution effect in the first term, see the appendix of Auclert et al. (2024) for a derivation.

B.7 Growth by time vs. cohort

In section 3, we assume that productivity growth γ loads onto cohorts rather than time. This makes no difference when considering a baseline balanced growth path, where either is consistent with identical aggregate outcomes (if the y_j are properly calibrated), but it does matter for the counterfactual effects of growth changes. If cohort-loading growth γ increases, there is no effect on the normalized life-cycle problem (16)–(17), but if growth is instead a time effect, scaling up all incomes at a given time, then higher growth implies a steeper anticipated path of incomes y_j in the life-cycle problem.

This life-cycle steepening effect is the only difference between time and cohort-loading productivity increases. It can be quantified as follows. We feed in an income shock $dy_j = jy_j$ and evaluate it using our general income shock result, (A.11), dividing by A^h to get the log asset effect. Then, we subtract the log change in effective labor from the steepening schedule, $\sum_j \pi_j j e_j \Big/ \sum_j \pi_j e_j$, to get the overall log effect on A^h/Y.[A-9]

Performing this quantification using our base profiles, and demographics from 2024, we find that the extra semielasticity of A^h/Y to time-loading growth, vs. cohort-loading growth, is -47.3. This compares to a long-run semielasticity of -52.3 with respect to cohort-loading growth, which comes from a simple shift-share as visualized in figure B.1. A rough summary of time-loading growth, therefore, is to say that it *doubles* the asset demand effect of cohort-loading growth increases.

It is unclear whether fluctuations in growth in the postwar U.S. time series are better described as being time- or cohort-loading. We opt for the more conservative cohort-loading estimate for the asset demand effect of productivity growth for several reasons. First, it is simpler, as it can be implemented by a shift-share with fewer data requirements or optimization assumptions. Second, the extra effect of time-loading growth comes from a dissaving response to an anticipated steeper

[A-9] This second step is necessary to make sure we have consistent units.

life-cycle profile of income, and it is unclear to us whether households in the postwar U.S. have responded to fluctuations in growth in this way. More generally, the strong asset demand effects of productivity growth in the time-loading case, which imply very large real interest rate increases in equilibrium, seem difficult to reconcile with the observed experience in fast-growing countries, where increases in growth rates do not lead to large increases in $r - g$, nor to large negative net foreign asset positions.[A-10]

C Appendix to section 4

Asset supply shifters. We obtain all supply parameters as described in appendix A. We then write asset supply as $\mathcal{A}^s(r, \boldsymbol{\eta})$, where $\boldsymbol{\eta}$ is a vector of all eight parameters, and decompose it nonlinearly according to

$$\mathcal{A}^s(r, \boldsymbol{\eta}_t) - \mathcal{A}^s(r, \boldsymbol{\eta}_{1950}) = \sum_i \int_0^1 \frac{\partial \mathcal{A}^s}{\partial \eta_i} \left(r, x\boldsymbol{\eta}_t + (1-x)\, \boldsymbol{\eta}_{1950}\right) \cdot \left(\eta_{it} - \eta_{i,1950}\right) dx \tag{A.12}$$

The i-th term of the sum on the right hand side is our contribution of parameter η_i to asset supply.

Asset demand shifters. On the asset demand side, for demographics, we use the U.N. World Population Prospects data for π_{jt}. We use the low fertility scenario as a baseline, since this is the only central scenario that does not assume a recovery in fertility rates, which seems reasonable in light of the continued decline in fertility rates observed in the past decades. We use net worth data a_j by age from the U.S. Survey of Consumer Finances (SCF) 2016 wave. We measure pre-tax labor income we_j by age in the SCF as well.

For inequality, we think of the permanent types θ as percentiles in the distribution of labor income early in life, since labor income early in life is known to be very predictive of income later in life (e.g. see Guvenen et al. 2017). For any cohort t, we define the factor $\lambda_t(\theta)$ as the ratio of labor income early in life for type θ to average labor income early in life. Thus, by construction, we have $\sum_\theta p(\theta)\lambda_t(\theta) = 1$. We further assume that all permanent types have the same life-cycle income profile (up to scale), that is, $e_j(\theta) = e_j$, independent of θ. To discipline $\bar{a}_j(\theta)$, we regress log net worth on log labor income between ages 25 and 44 in the Panel Study of Income Dynamics (PSID) after 1999. This has a slope that is much larger than one in the data, as illustrated by the binscatter in figure C.1, with a regression point estimate of 1.61. Theoretically, this suggests that $\lambda_{t-j}(\theta)\bar{a}_j(\theta)$ scales, on average, with $\lambda_{t-j}(\theta)^{1.61}$ for periods t after 1999. This motivates us to express $\bar{a}_j(\theta) = \bar{a}_j \cdot \lambda_{t_0 - j}(\theta)^{0.61}$ for a reference period t_0 which we assume to be 2016. With this in hand, we

[A-10] For instance, with our baseline $\epsilon^s + \epsilon^d \approx 30$, an asset demand semielasticity of 100 with respect to growth, as in the long run with time effects calculated here, would imply a 3.33 pp increase in real interest rates for each 1 pp increase in productivity growth.

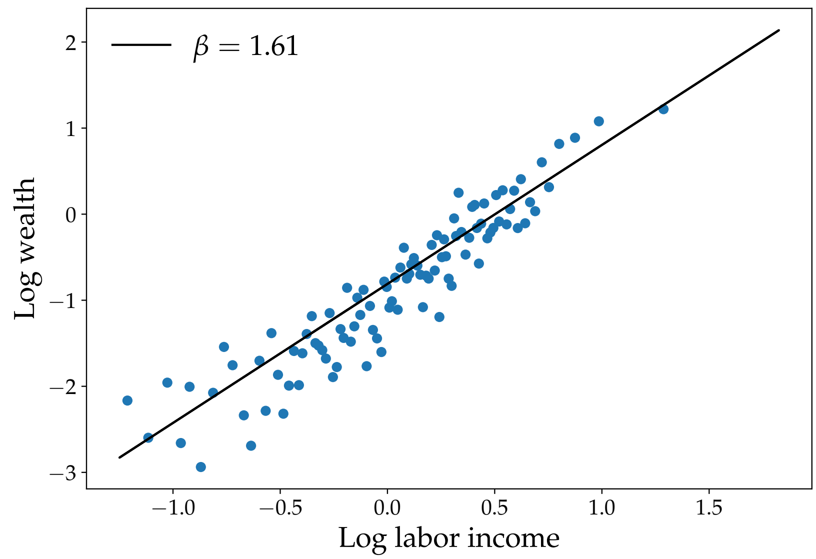

Figure C.1: Regression of log wealth on log income

construct the shift share in logs as

$$\log \frac{A^h_{t'}}{Y} - \log \frac{A^h_t}{Y} = \log \frac{\sum_j \pi_j \left(1+\gamma\right)^{-j} \bar{a}_j \sum_\theta p(\theta) \left[\lambda_{t'-j}\left(\theta\right) \lambda_{t_0-j}(\theta)^{0.61}\right]}{\sum_j \pi_j \left(1+\gamma\right)^{-j} \bar{a}_j \sum_\theta p(\theta) \left[\lambda_{t-j}\left(\theta\right) \lambda_{t_0-j}(\theta)^{0.61}\right]}$$

where we evaluate the sums over θ using the income distribution among 25 through 44 year-olds in the Distributional National Accounts (Piketty, Saez and Zucman 2018).

For productivity growth, we follow a similar strategy. We define Γ_t as the cumulative productivity growth experienced until the cohort born in year t achieves their prime working years, which for simplicity we choose to be at age 45. (The exact date here is not crucial.) We can then express the change in log asset demand between periods t' and t as

$$\log \frac{A^h_{t'}}{Y} - \log \frac{A^h_t}{Y} = \log \frac{\sum_j \pi_j a_j \frac{\Gamma_{t'-j}}{\Gamma_{2016-j}}}{\sum_j \pi_j e_j \frac{\Gamma_{t'-j}}{\Gamma_{2016-j}}} - \log \frac{\sum_j \pi_j a_j \frac{\Gamma_{t-j}}{\Gamma_{2016-j}}}{\sum_j \pi_j e_j \frac{\Gamma_{t-j}}{\Gamma_{2016-j}}}$$

Finally, for social security, we construct a profile of effective consumption $\tilde{c}_j$ by combining a consumption-age profile from the PSID for 2016 with mortality tables from the Census for 2010 and the asset-age profile from the SCF. We obtain social security transfers by age t_j from the Social Security Administration. Together, these allow us to evaluate (24).

All the other asset demand shifters are straightforward to implement.[A-11]

[A-11] In the plots below, we will continue showing level changes, not log changes. We convert from logs to levels just like in section 2, building on the idea that if we have n log shifts in a quantity $d \log x_i$, we can

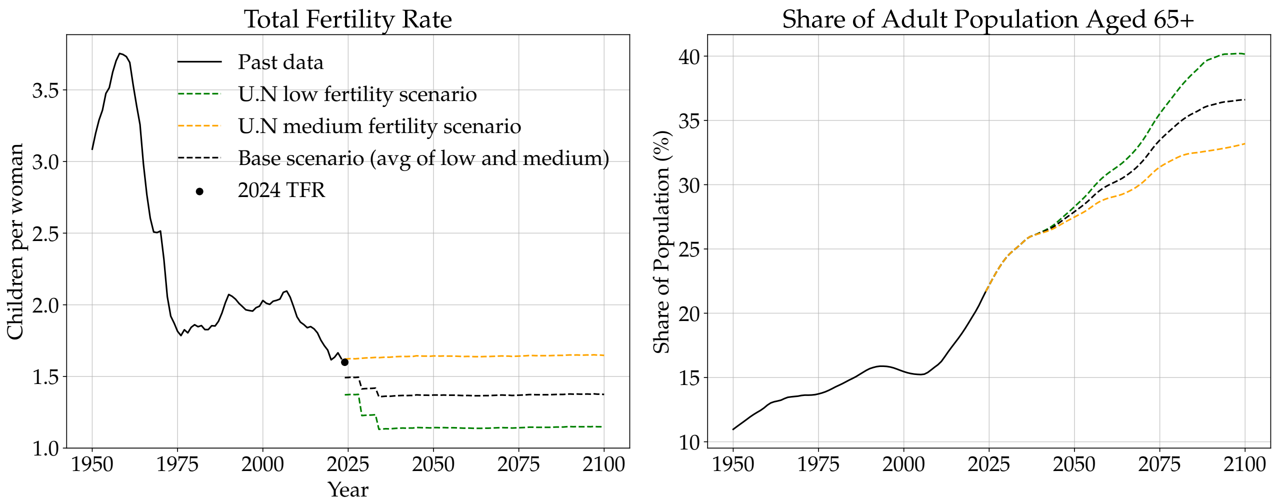

Figure D.1: Demographic assumptions

D Appendix to section 5

D.1 Fertility and demographic projections

Our baseline population projection is the geometric average of the UN's low and medium projections. Figure D.1 shows the past evolution of the total fertility rate (TFR) as well as the share of the adult population aged 65 or above, together with our central projection. The dashed lines show the U.N.'s medium and low fertility scenarios. In light of the continued decline in the TFR that has occurred since 2005, our baseline scenario seems more realistic than the U.N.'s medium scenario of immediate stabilization. For instance, the TFR projection for 2024 was 1.622, but realized TFP ended up being 1.599. Our baseline projection is less pessimistic, however, than the very low fertility level assumed in the U.N. low scenario.

D.2 Projections for health expenditure and social security

To construct the age profile of medicare and medicaid spending, we use data from De Nardi et al. (2016b) for per capita costs of Medicare and Medicaid by single year age bin for individuals aged 65 and above. We merge these with the data from the age and gender tables of the CMS National Health Expenditure Accounts (Centers for Medicare and Medicaid Services, 2024). We first distribute the coarse CMS Medicaid spending totals for ages 0 - 64 across singleyear age bins proportionally to the population in each bin, effectively interpolating percapita values within each

express their contributions to the level as $dx_i = \frac{d\log x_i}{\sum_j d\log x_j} \cdot \left(e^{\sum_j d\log x_j} - 1\right) x_0$.

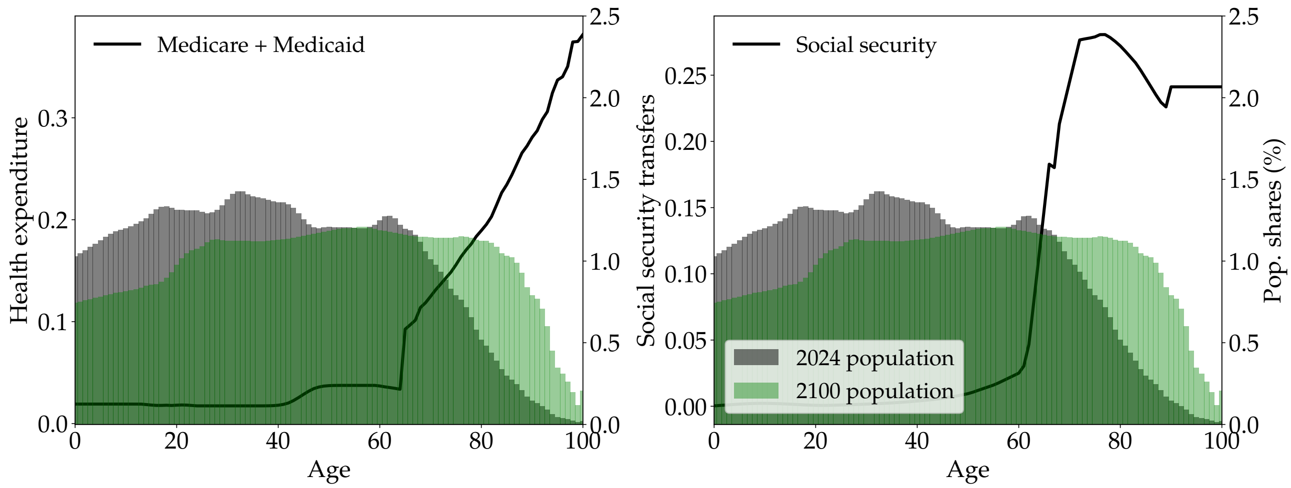

Figure D.2: Profiles used to implement equations (29) and (30)

bracket. We then smooth the resulting series with a Gaussian filter ($\sigma = 2$) to remove sharp edges and finally apply a centered 5 - year moving average to produce a smooth age profile.

To construct the age profile of social security payments, we take the total Old-Age, Survivors, and Disability Insurance payments from the 2024 SSA Annual Statistical Supplement (Social Security Administration, 2024), Table 5.A1. We divide these totals by the UN singleage population counts for 2023 to obtain percapita values. For aggregated age bins (e.g., 9094, 9599), we redistribute the reported totals evenly across single ages using population weights. Finally, each resulting series is smoothed using a centered 5 - year moving average.

Figure D.2 displays the resulting profiles, which we use to generate our projections for government spending and transfers in equations (29) and (30). The solid line on the left panel shows Medicare and Medicaid spending by age, and the bars display the population distribution in 2024 and in 2100 in our baseline scenario. Similarly, the solid line on the right panel shows spending by age on social security. Equations (29) and (30) reweigh these profiles by the age distribution of the population at each time to deliver our projections for healthcare spending and social security in figure 12.