Monetary Conservatism and Fiscal Policy

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Abstract: Does an inflation conservative central bank à la Rogoff (1985) remain desirable in a setting with endogenous fiscal policy? To provide an answer we study monetary and fiscal policy games without commitment in a dynamic, stochastic sticky-price economy with monopolistic distortions. Monetary policy determines nominal interest rates and fiscal policy provides public goods generating private utility. We find that lack of fiscal commitment gives rise to excessive public spending. The optimal inflation rate internalizing this distortion is positive, but lack of monetary commitment generates too much inflation. A conservative monetary authority thus remains desirable. When fiscal policy is determined before monetary policy each period, the monetary authority should focus exclusively on stabilizing inflation. Monetary conservatism then eliminates the steady state biases associated with lack of monetary and fiscal commitment and leads to stabilization policy that is close to optimal.

Keywords: sequential non-cooperative policy games, discretionary policy, time consistent policy, conservative monetary policy

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1 Introduction

Which monetary institutions can overcome the problems associated with lack of monetary commitment? A prominent early answer is Rogoff’s (1985) proposal to appoint an inflation conservative central banker. This and other well-known proposals\textsuperscript{1} assume, however, that fiscal policy can be treated as exogenous when studying the design of monetary institutions. While this assumption is a useful starting point, it is an unsatisfactory aspect of previous studies. In this paper we ask whether installing a conservative central banker remains desirable when fiscal policy is endogenous and equally subject to a commitment problem.

We analyze a non-cooperative monetary and fiscal policy game within a standard stochastic general equilibrium model without capital, along the lines of Rotemberg (1982) and Woodford (2003). This economy features three sources of distortions: (1) firms operate under monopolistic competition, which causes output to be inefficiently low; (2) prices are rigid in the short-term, which gives rise to real effects of monetary policy; (3) policymakers cannot credibly commit to a path for future policy, but instead determine policy sequentially.

In line with recent monetary policy models, the monetary authority determines the short-term nominal interest rate. We add to this setting a fiscal authority which decides the level of public goods provision. Public goods generate utility for private agents and are financed by lump sum taxes, so as to balance the government’s intertemporal budget.\textsuperscript{2}

While the monetary and fiscal authority are benevolent, i.e., maximize the utility of the representative agent, lack of commitment gives rise to suboptimal policy outcomes. Since output is inefficiently low, both policymakers are tempted to increase output, either via lowering real interest rates (monetary authority) or via increasing public spending (fiscal authority).

\textsuperscript{1}See for example Svensson (1997) and Walsh (1995).

\textsuperscript{2}Our results extend to a setting with distortionary labor taxes, as shown in a companion note. See Adam and Billi (2008).
This results in an inflationary bias and in overspending on public goods, compared to a situation with policy commitment, because with sequential decision making both policymakers fail to fully internalize the welfare cost of generating inflation today.

In our setting monetary and fiscal policy interact in interesting ways. Specifically, taking the lack of fiscal commitment as given makes it optimal for monetary policy to aim at positive inflation rates. We show that positive inflation rates reduce the fiscal spending bias and thereby increase welfare. This suggests that, unlike in the standard case with exogenous fiscal policy, a conservative central bank may not always be desirable. A quantitative exercise suggests, however, that the optimal deviations from price stability tend to be small. Moreover, in the non-cooperative Markov-perfect Nash equilibrium with sequential monetary and fiscal policy, the inflation rate lies significantly above the optimal inflation rate for a wide range of model parameterizations. This result suggests that installing an inflation conservative central banker is desirable also with endogenous fiscal policy.

We then formally introduce a conservative monetary authority that maximizes a weighted sum of an inflation loss term and the representative agent’s utility. And we characterize the resulting Markov-perfect equilibria.

When policies are determined simultaneously or when monetary policy is determined before fiscal policy each period, monetary conservatism alone cannot eliminate entirely the steady state distortions from sequential policymaking. There is positive inflation or fiscal overspending, or both. Nevertheless, we find that a sufficiently high degree of monetary conservatism eliminates most of the steady state welfare loss arising from the lack of monetary and fiscal commitment.

Monetary conservatism turns out to be even more desirable if fiscal policy is determined before monetary policy each period (arguably the most relevant timing protocol). In such

3Markov-perfect Nash equilibria, as defined in Maskin and Tirole (2001), are a standard refinement used in the applied dynamic games literature, e.g., Klein et al. (2008).
a setting monetary conservatism is internalized by fiscal policy, which makes it possible to eliminate entirely the steady state biases stemming from lack of monetary and fiscal commitment, provided the monetary authority cares exclusively about inflation. Overall, the case for a conservative monetary authority thus remains stronger in a setting with endogenous fiscal policy.

We also address the issue of how the conduct of stabilization policy is affected by monetary conservatism. We show that fiscal leadership in combination with a fully conservative central banker can achieve the flexible-price Ramsey allocation following technology and mark-up shocks. This result suggests that monetary conservatism is desirable also from the viewpoint of cyclical stabilization policy.

Following the work of Kydland and Prescott (1977) and Barro and Gordon (1983), the monetary policy literature has extensively studied time-inconsistency problems in dynamic settings and its potential solutions, see Rogoff (1985), Svensson (1997) and Walsh (1995). In this literature, fiscal policy is typically absent or exogenous to the model. At the same time, a number of contributions analyze the time-consistency of optimal fiscal plans in dynamic general equilibrium models, e.g., Lucas and Stokey (1983), Chari and Kehoe (1990) or Klein, Krusell, and Ríos-Rull (2008). But this literature typically studies models without money.

The next section introduces the model and the implementability constraints characterizing private-sector behavior. Section 3 derives monetary and fiscal policy with and without commitment and some analytical results about the policy biases. In section 4 we quantify the biases and their welfare implications. Section 5 studies monetary conservatism and section 6 explains the robustness of the results to distortionary taxation. The technical material is available in the web appendix to this paper.
2 The Model

The setting is a sticky-price economy with monopolistic competition, similar to the one studied in Schmitt-Grohé and Uribe (2004).

2.1 Private Sector

There is a continuum of identical households with preferences given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, g_t) \]  

(1)

where \( c_t \) is consumption of an aggregate consumption good, \( h_t \in [0, 1] \) labor effort, \( g_t \) public goods provision by the government in the form of aggregate consumption goods, and \( \beta \in (0, 1) \) the discount factor. Utility is separable in \( c, h, g \) and \( u_c > 0, u_{cc} < 0, u_h < 0, u_{hh} \leq 0, u_g > 0, u_{gg} < 0, \) and \( \left| \frac{c_{uu}}{u_c} \right|, \left| \frac{h_{uu}}{u_h} \right| \) are bounded.

Each household produces a differentiated intermediate good. Demand for this good is \( y_t d(\tilde{P}_t/P_t) \), where \( y_t \) is (private and public) demand for the aggregate good and \( \tilde{P}_t/P_t \) the relative price of the intermediate good compared to the aggregate good. The demand function \( d(\cdot) \) satisfies \( d(1) = 1 \) and \( d'(1) = \eta_t \), where \( \eta_t \in (-\infty, -1) \) is the price elasticity of demand for the different goods. This elasticity is time-varying and induces fluctuations in the monopolistic mark-up charged by firms. The demand function is consistent with optimizing individual behavior when private and public consumption goods are a Dixit-Stiglitz aggregate of the goods produced by different households. The household chooses \( \tilde{P}_t \), then hires the necessary amount of labor effort \( \tilde{h}_t \) to satisfy the resulting product demand, i.e.,

\[ z_t \tilde{h}_t = y_t d \left( \frac{\tilde{P}_t}{P_t} \right) \]  

(2)

where \( z_t \) is an aggregate technology shock. The mark-up shock \( \eta_t \) and the technology shock \( z_t \) follow independent AR(1) stochastic processes with autocorrelation coefficients \( \rho_\eta \) and \( \rho_z \) and steady state values \( z = 1 \) and \( \eta < -1 \).
Following Rotemberg (1982), sluggish nominal price adjustment by firms is described by quadratic resource costs for adjusting prices according to
\[
\frac{\theta}{2} \left( \frac{\tilde{P}_t}{P_{t-1}} - 1 \right)^2
\]
where \( \theta > 0 \) indexes the degree of price stickiness. The households’ budget constraint is
\[
P_t c_t + B_t = R_{t-1} B_{t-1} + P_t \left[ \frac{\tilde{P}_t}{P_t} y_t d_t \left( \frac{\tilde{P}_t}{P_t} - \frac{\theta}{2} \left( \frac{\tilde{P}_t}{P_{t-1}} - 1 \right)^2 \right] + P_t w_t h_t - P t l_t \tag{3}
\]
where \( R_t \) is the gross nominal interest rate, \( B_t \) are nominal bonds that pay \( R_t B_t \) in period \( t + 1 \), \( w_t \) is the real wage paid in a competitive labor market, and \( l_t \) are lump sum taxes. Although we consider lump sum taxes, section 6 shows that the main results extend to the case with distortionary taxes. Lump sum taxes allow us to derive many results analytically.

Finally, the no-Ponzi scheme constraint on household behavior is:
\[
\lim_{j \to \infty} E_t \prod_{i=0}^{t+j-1} \frac{1}{R_i} B_{t+j} \geq 0 \tag{4}
\]

The household’s problem consists of choosing \( \{c_t, h_t, \tilde{h}_t, \tilde{P}_t, B_t\}_{t=0}^{\infty} \) to maximize (1) subject to (2), (3) and (4) taking as given \( \{y_t, P_t, w_t, R_t, g_t, l_t\}_{t=0}^{\infty} \). The first order conditions of the household’s problem are then equations (2), (3) and (4) holding with equality and also
\[
-\frac{u_{ht}}{u_{ct}} = w_t
\]
\[
\frac{u_{ct}}{R_t} = \beta E_t \frac{u_{ct+1}}{\Pi_{t+1}}
\]
\[
0 = u_{ct} \left[ y_t d'(r_t) + r_t y_t d''(r_t) - \frac{w_t}{z_t} y_t d'(r_t) - \theta \left( \Pi_t \frac{r_t}{r_{t-1}} - 1 \right) \frac{\Pi_t}{r_{t-1}} \right] + \beta \theta E_t u_{ct+1} \left( \frac{r_{t+1}}{r_t} \Pi_{t+1} - 1 \right) \frac{r_{t+1}}{r_t^2} \Pi_{t+1}
\]
where \( r_t \equiv \frac{\tilde{P}_t}{P_t} \) is the relative price and \( \Pi_t \equiv \frac{B_t}{P_{t-1}} \) the gross inflation rate. Furthermore, the transversality condition \( \lim_{j \to -\infty} E_t (\beta^{t+j} u_{ct+j} B_{t+j} / P_{t+j}) = 0 \) has to hold at all contingencies.

\[4\] Using instead the Calvo approach to nominal rigidities would considerably complicate matters because price dispersion then becomes an endogenous state variable.

\[5\] We abstract from money holdings and thus seignorage by considering a ‘cashless limit’ economy à la Woodford (1998); money only imposes a lower bound on nominal interest rates (\( R_t \geq 1 \)).
2.2 Government

The government consists of a monetary authority setting nominal interest rates $R_t$ and a fiscal authority determining the level of public good provision $g_t$. The budget constraint is

$$B_t = R_{t-1}B_{t-1} + P_t(g_t - l_t) \quad (5)$$

With lump sum taxes, tax versus debt financing decisions do not matter for equilibrium determination as long as the implied paths for the debt level satisfy the no-Ponzi scheme constraint (4) and the transversality condition. For sake of simplicity, taxes are set such that the level of real debt $\frac{B_t}{R_t}$ remains bounded from below and asymptotically grows at a rate less than $\frac{1}{\beta}$. Constraint (4) and the transversality condition are then always satisfied and can be ignored from now on. Fiscal policy is thus ‘passive’ in the sense of Leeper (1991).

2.3 Private Sector Equilibrium

In a symmetric price setting equilibrium the relative price is given by $r_t = 1$ for all $t$. From the assumptions made, it follows that the first order conditions of households behavior can be condensed into a price setting equation

$$u_{ct}(\Pi_t - 1)\Pi_t = \frac{u_{ct}z_th_t}{\theta} \left( 1 + \eta_t + \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) + \beta E_t u_{ct+1}(\Pi_{t+1} - 1)\Pi_{t+1} \quad (6)$$

often referred to as a ‘Phillips curve’, and a consumption Euler equation

$$\frac{u_{ct}}{R_t} = \beta E_t \frac{u_{ct+1}}{\Pi_{t+1}} \quad (7)$$

A private-sector rational expectations equilibrium consists of a plan $\{c_t, h_t, B_t, P_t\}$ satisfying equations (6) and (7), the government budget (5), and the market-clearing condition

$$z_t h_t = c_t + \theta \frac{1}{2}(\Pi_t - 1)^2 + g_t \quad (8)$$

given the policies $\{g_t, l_t, R_t \geq 1\}$, the exogenous processes $\{\eta_t, z_t\}$, and the initial conditions $(R_{-1}B_{-1}, P_{-1})$. 
2.4 Time Inconsistency Problems

Under commitment policymakers determine state-contingent future policies at the beginning of time. Policymakers that cannot commit decide about policies at the time of implementation, i.e., period by period. We refer to such behavior as sequential decision making.

Sequential policy is suboptimal because it fails to fully internalize the welfare cost of generating inflation today. Since past prices can be taken as given at the time current policy is determined, sequential decision making ignores the link between current policy decisions and past pricing decisions. Such a link does exist because the private sector rationally anticipates current inflation and is forward-looking in its pricing decision, see equation (6). As a result, sequentially deciding policymakers underestimate the welfare costs of generating inflation today and are ‘tempted’ to move output closer to its first-best level.

3 Monetary and Fiscal Policy Regimes

In this section we study the steady state outcomes associated with lack of commitment. The implications for stabilization policy will be discussed in section 5. We start by analyzing the first-best allocation, which abstracts from monopoly distortions and nominal rigidities. Then we determine the Ramsey allocation, which takes into account both distortions, but still allows for policy commitment. In a final step we relax the commitment assumption.

3.1 First-Best and Ramsey Allocation

The first-best allocation solves

$$\max_{\{c_t, h_t, g_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, g_t) \quad \text{s.t.} \quad z_t h_t = c_t + g_t$$

(9)

where equation (9) is the resource constraint. The steady state first-order conditions deliver

$$u_c = u_g = -u_h$$
showing that, given the technological constraints, it is optimal to equate the marginal utility of private and public consumption to the marginal disutility of labor effort. This simple result ceases to be optimal in the Ramsey allocation, which takes into account the presence of price setting and monopoly distortions, as summarized by the implementability constraints (6) and (7). Specifically, the Ramsey allocations solves\(^6\)

\[
\max_{\{c_t, h_t, \Pi_t, R_t \geq 1, g_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, g_t) \tag{10}
\]

s.t. Equations (6), (7), (8) for all \(t\)

Note that the Ramsey allocation still allows for commitment to policies at time zero and full cooperation between monetary and fiscal policymakers. Deriving the first order conditions of (10) and imposing steady state conditions delivers\(^7\)

\[\Pi = 1 \quad \text{and} \quad R = \frac{1}{\beta}\]

as well as the marginal conditions

\[u_c = -\frac{\eta}{1 + \eta} u_h \tag{11}\]
\[u_g = -\frac{1 + \eta}{1 + q} u_h \tag{12}\]

where \(q \equiv -\frac{c}{h} \frac{h u_{hc}}{u_h} \frac{u_c}{c u_{cc}} \geq 0\). Equation (11) shows that monopolistic competition creates a wedge between the marginal utility of private consumption and the marginal disutility of work. This wedge reflects the fact that labor fails to receive its marginal product when firms have monopoly power, which causes households to reduce consumption of produced goods and to increase consumption of leisure.\(^8\)

\(^6\)Since Ricardian equivalence holds we ignore the financing decisions of the fiscal authority and the initial debt level \(R_{-1}B_{-1}\), which do not matter for equilibrium determination of the other variables. Since the initial condition \(P_{-1}\) simply normalizes the implied price level path, it can equally be ignored.

\(^7\)Details of the derivation are in web appendix A.1.

\(^8\)Steady state real wages fall short of their marginal product because \(w = (1 + \eta) / \eta < 1\).
For $u_{hh} < 0$, one has $q > 0$ and equation (12) implies that the optimal level of public spending falls short of equating the marginal utility of public consumption to the marginal disutility of work, unlike in the first best allocation. One might think that the optimal provision of public goods should be unaffected by the presence of a monopolistic mark-up, since lump sum taxes allow to finance any price mark-up without generating additional distortions. However, public spending (and taxes) below the first-best level reduce the marginal disutility of work, which increases the inefficiently low level of private consumption.

3.2 Sequential Policymaking

We now consider separate monetary and fiscal authorities that cannot commit to future policy plans and decide about policies at the time of implementation. To facilitate the exposition, each policymaker takes as given the current policy choice of the other policymaker, as well as all future policies and future private-sector choices. We verify the rationality of these assumptions at the end of this section.

3.2.1 Sequential Fiscal Policy: Spending Bias

Given the assumptions made above, the fiscal authority’s problem in period $t$ is

$$\max_{\{c_{t+j}, h_{t+j}, \Pi_{t+j}, g_{t+j}\}} E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, h_{t+j}, g_{t+j})$$

s.t. Equations (6), (7), (8) for all $t$

$$\{c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j-1} \geq 1, g_{t+j}\} \text{ given for } j \geq 1$$

Eliminating Lagrange multipliers from the first order conditions delivers\(^9\)

$$u_{gt} = -\frac{u_{ht}}{z_t} \frac{2\Pi_t - 1}{2\Pi_t - 1 - (\Pi_t - 1)} (1 + \eta_t + \frac{u_{ht}}{u_{ct}} \frac{\Pi_t}{z_t} + h_t \frac{u_{ht}}{u_{ct}} \frac{\Pi_t}{z_t})$$

\(^9\)Details of the derivation are in web appendix A.2.
The fiscal authority sets the level of public goods provision \( g_t \) such that this fiscal reaction function (FRF) is satisfied, each period.

Consider a steady state in which the inflation rate is equal to the one chosen by the Ramsey planner \((\Pi = 1)\). The fiscal reaction function then simplifies to

\[
u_g = -u_h
\]

showing that fiscal policy equates the marginal utility of public consumption to the marginal disutility of labor effort. While such behavior is consistent with the first-best allocation, it is generally suboptimal in the presence of monopolistic distortions, as argued in section 3.1. Sequential fiscal policy thus gives rise to a suboptimally high level of public spending. This ‘fiscal spending bias’ causes the Ramsey allocation to be unattainable in the presence of sequential fiscal policy, because inflation or fiscal spending, or both must deviate from their Ramsey values. This result is summarized in the following proposition.

**Proposition 1** For \( u_{hh} < 0 \), sequential fiscal policy implies excessive fiscal spending in the presence of price stability.

The economic intuition underlying this result is as follows. By taking future decisions and the current monetary policy choice \( R_t \) as given, the fiscal authority considers private consumption \( c_t \) to be determined by the Euler equation (7). The fiscal authority thus perceives labor input \( h_t \) to move one-for-one with government spending \( g_t \). In a situation with price stability, the inflation costs of public spending are zero (at the margin) and can be ignored, so that the sequential spending rule (14) appears optimal. When \( \Pi \neq 1 \) the marginal costs of inflation fail to be zero, leading to the optimality condition FRF.
3.2.2 Sequential Monetary Policy: Inflation Bias

Given the previous assumptions, the monetary authority’s problem in period $t$ is

$$\max_{\{c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j}\geq 1\}} E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, h_{t+j}, g_{t+j})$$ (15)

s.t. Equations (6),(7),(8) for all $t$

$$\{c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j} \geq 1, g_{t+j-1}\} \text{ given for } j \geq 1$$

Eliminating Lagrange multipliers from the first order conditions of this problem delivers the monetary reaction function\(^{10}\)

$$-z_t \eta_t (\Pi_t - 1) - (\Pi_t - 1) \eta_t (1 + h_t \frac{u_{ct}}{u_{ht}}) + 2 \Pi_t - 1 - \frac{u_{ct}}{u_{ht}} (\Pi_t - 1) (\theta (\Pi_t - 1) \Pi_t - z_t h_t (1 + \eta_t)) = 0$$ (MRF)

where the monetary authority sets the nominal interest rate $R_t$ such that MRF is satisfied, each period. In a steady state with $\Pi = 1$ this policy reaction function simplifies to

$$-\frac{u_c}{u_h} = 1$$

where the expression on the l.h.s. corresponds to the real wage. Monopoly power implies that real wages fall short of the marginal product of labor. As a result, the monetary reaction function MRF is inconsistent with price stability. We appendix A.4 shows that the steady state inflation rate must be positive.

**Proposition 2** For $\beta$ sufficiently close to 1, sequential monetary policy implies a strictly positive rate of inflation in steady state.

Sequential monetary policy generates an ‘inflation bias’ as in the standard case with exogenous fiscal policy, see for example Svensson (1997). Intuitively, the monetary authority

\(^{10}\)Details of the derivation can be found in web appendix A.3.
is tempted to stimulate demand by lowering nominal interest rates. Since price adjustments are costly, the price level will not fully adjust, and real interest rates fall, which stimulates demand. The real wage increase required to satisfy this additional demand generates inflation.

### 3.2.3 Sequential Monetary and Fiscal Policy

We now define a Markov-perfect Nash equilibrium with sequential monetary and fiscal policy. We also verify the rationality of our initial assumptions that sequentially deciding policymakers can take as given the current policy choice of the other policymaker, as well as all future policies and future private-sector decisions.

The private sector’s optimality conditions (6) and (7), the feasibility constraint (8), as well as the policy reactions functions FRF and MRF, all depend on current and future variables only. This observation suggests the existence of an equilibrium where current play is a function of the current exogenous variables $z_t$ and $\eta_t$ only. Future play then depends on future exogenous variables only, which justifies the assumption that future equilibrium play (and off-equilibrium play) is independent of current play. If each period, in addition, monetary and fiscal policy are determined simultaneously, Nash equilibrium requires taking the other player’s current decision as given. This justifies the assumptions made in deriving FRF and MRF and motivates the following definition.

**Definition 3 (SP)** A stationary Markov-perfect Nash equilibrium with sequential monetary and fiscal policy consists of time-invariant policy functions $c(z_t, \eta_t)$, $h(z_t, \eta_t)$, $\Pi(z_t, \eta_t)$, $R(z_t, \eta_t)$, $g(z_t, \eta_t)$ solving equations (6), (7), (8), FRF and MRF.

We now show that Stackelberg leadership by one of the policy authorities (with regard to the within-period moves) is consistent with the same equilibrium outcome. While the policy problem of the Stackelberg follower remains unchanged, the Stackelberg leader takes into account the reaction function of the follower. Yet, the Lagrange multipliers associated
with additionally imposing MRF in the sequential fiscal problem (13) or with imposing FRF in the sequential monetary problem (15) are zero. These reaction functions can be derived from the first order conditions of the leader’s policy problem even when the follower’s reaction function is not being imposed.

Intuitively, the leadership structure does not matter for the equilibrium outcome because both authorities are pursuing the same policy objective. Any departure from the Ramsey solution is thus entirely due to sequential decision making. The presence of distinct policymakers and the sequence of moves will matter in section 5 when we consider a monetary authority that is more inflation averse than the fiscal authority.

4 How Much Inflation is Optimal?

We have shown that a steady state with sequential monetary and fiscal policy (SP) involves positive rates of inflation (proposition 2). We now show that price stability ($\Pi = 1$) is not optimal whenever fiscal policy faces a commitment problem, i.e., is described by FRF. Monetary policy should then aim at positive rates of inflation because this increases the perceived costs of public spending for the fiscal authority and reduces the fiscal spending bias. This leads to a first order welfare gain\(^{11}\), while the costs of locally deviating from price stability are of second order only. This is formally shown in web appendix A.5:

**Proposition 4** Assume $u_{hh} < 0$. In a steady state with sequential fiscal policy, welfare increases and fiscal spending decreases with the steady state inflation rate, locally at $\Pi = 1$.

\(^{11}\)This is so because with sequential fiscal policy $u_c > u_g = -u_h$ when $\Pi = 1$, see FRF.
What is then the welfare maximizing inflation rate that correctly internalizes the sequential fiscal policy distortion? This inflation rate can be obtained as the solution to the problem

\[
\max_{\{c_t, h_t, R_t, R_t \geq 1, g_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, g_t)
\]

(OI)

s.t. Equations (6), (7), (8), (FRF) for all \( t \)

where monetary policy can commit, while fiscal behavior is described by FRF. We will refer to this situation as the optimal inflation (OI) regime. If the optimal steady state inflation rate is below (above) the rate emerging in the steady state of the Markov-perfect Nash equilibrium, then an inflation conservative (liberal) central banker would appear desirable.\(^{12}\)

4.1 Quantifying the Policy Biases

In this section we quantify the steady state under sequential monetary and fiscal policy (SP) and compare it to the Ramsey allocation and also to the allocation achieved under the optimal inflation (OI) regime.

We consider the following preference specification, which is consistent with balanced growth,

\[
u(c_t, h_t, g_t) = \log (c_t) - \omega_h \frac{h_t^{1+\varphi}}{1+\varphi} + \omega_g \log (g_t)
\]

with \( \omega_h > 0, \, \omega_g \geq 0 \) and \( \varphi \geq 0 \). As a baseline calibration, we set \( \beta = 0.9913 \) implying an annual real interest rate of 3.5%. The steady state price elasticity of demand \( \eta \) is set at -6, so that there is a 20% mark-up over marginal costs. The degree of price stickiness \( \theta \) is chosen to be 17.5, such that the log-linearized version of the Phillips curve (6) is consistent with the one in Schmitt-Grohé and Uribe (2004). Labor supply elasticity is set to \( \varphi^{-1} = 1 \) and the utility weights \( \omega_h \) and \( \omega_g \) are chosen such that in the Ramsey steady state agents work 20% \(^{12}\)Since steady state inflation depends on steady state nominal interest rates only, see equation (7), the optimal inflation rate implicitly determines optimal monetary policy.
of their time \((h = 0.2)\) and spend 20% of output on public goods \((g = 0.04)\).\(^{13}\)

We tested the robustness of our numerical results by considering a wide range of alternative model parameterizations and by using different starting values.\(^{14}\) The baseline results are robust, except for the flexible price limit \((\theta \to 0)\) and for inelastic labor supply \((\text{large } \varphi)\). In both cases the time-inconsistency problems of policy disappear and the real allocations under SP approach the Ramsey steady state.

The first row of table 1 presents information on the steady state in the SP regime. All variables are expressed as percentage deviations from their corresponding Ramsey steady state values.\(^{15}\) The last column of the table reports the steady state welfare loss, expressed in terms of the permanent reduction in private consumption making the Ramsey steady state welfare equivalent to the considered policy regime.\(^{16}\) In line with proposition 2, the sequential policy outcome is characterized by an inflation bias, which turns out to be sizable. In addition, there is a small fiscal spending bias. Overall, the welfare losses generated by the sequential conduct of policy are fairly large, in the order of 1% of steady state consumption per period.

The second row of table 1 shows the outcome under the OI regime. The optimal inflation rate is not only lower than the inflation rate in the SP regime but also very close to price stability. As suggested by proposition 4, reducing inflation from the level of the SP regime to the optimal level increases the fiscal spending bias. While the fiscal spending increase is fairly large, the optimal inflation rate nevertheless eliminates large part of the welfare losses associated with the SP regime. This suggests that the fiscal spending bias, despite being

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\(^{13}\)This requires \(\omega_h = 26.042\) and \(\omega_g = 0.227\), see equations (44) and (45), in web appendix A.6.

\(^{14}\)To keep the Ramsey steady state invariant to the parameterization, we adjusted the weights \(\omega_h\) and \(\omega_g\).

\(^{15}\)In the Ramsey steady state \(c = 0.16, h = 0.2, g = 0.04\) and \(\Pi = 1\).

\(^{16}\)Web appendix A.8 explains the computation of the welfare losses.
sizable in absolute value, is not very detrimental in welfare terms. Clearly, this result hinges partly on the availability of lump sum taxes.

The results from table 1 show that the optimal inflation rate is well below the one emerging in the SP regime. This holds for a wide range of alternative model parameterizations, which suggests that installing a conservative monetary authority should be desirable on welfare grounds whenever sequential fiscal policy is described by FRF.

5 Conservative Monetary Authority

This section analyzes whether the distortions stemming from sequential monetary and fiscal policy decisions can be reduced by installing a monetary authority that is more inflation averse than society. Rogoﬀ (1985) and Svensson (1997) have shown this to be the case if fiscal policy is treated as exogenous. Following Rogoﬀ (1985), we consider a ‘weight conservative’ monetary authority with period utility function

\[
(1 - \alpha) u(c_{t+j}, h_{t+j}, g_{t+j}) - \alpha \frac{(\Pi_{t+j} - 1)^2}{2}
\]

where \( \alpha \in [0, 1] \) is a measure of monetary conservatism. For \( \alpha > 0 \) the monetary authority dislikes inflation (and deflation) more than society; if \( \alpha = 1 \) the policymaker cares about inflation only. The preferences of the fiscal authority remain unchanged.

With monetary and fiscal authorities now pursuing different policy objectives, the equilibrium outcome will depend on the timing of policy moves, i.e., on whether fiscal policy is determined before, after, or simultaneously with monetary policy each period. Casual observation suggests that it takes longer to enact fiscal decisions, which would imply that fiscal policy is determined before monetary policy. At the same time, the time lag between a monetary policy decision and its effects on the economy may be substantial too. It thus remains to be ascertained which of these timing structures is the most relevant for actual economies.
For these reasons, we consider Nash as well as leadership equilibria.

5.1 Nash and Leadership Equilibria

This section defines the various Markov-perfect equilibria in the presence of a conservative monetary authority. As it turns out, the equilibrium outcomes with simultaneous monetary and fiscal decisions each period (Nash case) and with monetary policy determined before fiscal policy each period (monetary leadership) are very similar. This similarity emerges because in both cases fiscal policy takes current monetary decisions as given, so that fiscal policy continues to be described by FRF, i.e., by the reaction function in the absence of a conservative authority. For space constraints we therefore only discuss the Nash case.

The situation in which fiscal policy is determined before monetary policy (fiscal leadership) differs considerably. The fiscal authority has to take into account the conservative monetary authority’s within-period reaction function. Monetary policy can then use ‘off-equilibrium’ behavior to discipline the behavior of the fiscal authority along the equilibrium path. Fiscal leadership thus opens the possibility for outcomes that are welfare superior to those achieved in the OI regime.

First, consider the case with simultaneous decisions. While the policy problem of the fiscal authority remains unchanged, the monetary authority now solves

\[
\max_{\{c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j} \geq 1\}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left( (1 - \alpha) u(c_{t+j}, h_{t+j}, g_{t+j}) - \frac{\alpha}{2} (\Pi_{t+j} - 1)^2 \right) \tag{17}
\]

s.t. Equations (6),(7),(8) for all \(t\)

\[\{c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j} \geq 1, g_{t+j-1}\} \text{ given for } j \geq 1\]

Taking first order conditions of problem (17) and eliminating Lagrange multipliers delivers the conservative monetary authority’s reaction function that we denote by CMRF.\footnote{Web appendix A.9 provides the technical details. As before, CMRF implies that current interest rates} For
\( \alpha = 0 \), CMRF reduces to the monetary reaction function without conservatism (MRF). This motivates the following definition.

**Definition 5 (CSP-Nash)** A stationary Markov-perfect Nash equilibrium with sequential and conservative monetary policy, sequential fiscal policy, and simultaneous policy decisions consists of policy functions \( c(z_t, \eta_t) \), \( h(z_t, \eta_t) \), \( \Pi(z_t, \eta_t) \), \( R(z_t, \eta_t) \), \( g(z_t, \eta_t) \) solving equations (6), (7), (8), FRF and CMRF.

Next, consider the case of fiscal leadership (FL). The fiscal authority must now take into account the conservative monetary reaction function (CMRF). The fiscal authority’s policy problem at time \( t \) is thus given by

\[
\max_{\{c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j}, g_{t+j}\}} \quad E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, h_{t+j}, g_{t+j}) \tag{18}
\]

subject to Equations (6), (7), (8), CMRF for all \( t \)

\[ \{c_{t+j}, h_{t+j}, R_{t+j}, g_{t+j} \geq 1, g_{t+j}\} \text{ given for } j \geq 1 \]

The first order conditions associated with problem (18) deliver the corresponding fiscal reaction function that we denote by CFRF-FL. We propose the following definition.

**Definition 6 (CSP-FL)** A stationary Markov-perfect equilibrium with sequential and conservative monetary policy, sequential fiscal policy, and fiscal policy deciding before monetary policy, consists of policy functions \( c(z_t, \eta_t) \), \( h(z_t, \eta_t) \), \( \Pi(z_t, \eta_t) \), \( R(z_t, \eta_t) \), \( g(z_t, \eta_t) \) solving equations (6), (7), (8), CFRF-FL and CMRF.

depend on current economic conditions only, validating the conjecture in (17) that in a Markov-perfect equilibrium future policy choices can be taken as given.
5.2 Steady State Implications

We now determine the steady state properties for the various timing arrangements with an inflation conservative central banker. Our first result is that fiscal leadership in combination with a fully conservative monetary authority achieves the Ramsey steady state.

**Proposition 7** The Ramsey steady state is consistent with sequential policymaking in a regime with fiscal leadership, if the monetary authority is fully conservative ($\alpha = 1$).

Intuitively, with fiscal leadership the fiscal authority anticipates the within-period reaction of the monetary authority. In particular, for $\alpha = 1$ the monetary authority is determined to achieve price stability at all costs. A fiscal expansion above the Ramsey spending level generates inflationary pressures and thus triggers an increase in interest rates to restrain private consumption. The fiscal authority therefore internalizes that fiscal spending crowds out private consumption one-for-one. This effect disciplines fiscal spending and allows the achievement of the Ramsey steady state despite sequential policymaking by both authorities. A formal proof is provided in web appendix A.10. The next proposition shows that this fails to be possible with other timing arrangements:

**Proposition 8** For $u_{hh} < 0$, the Ramsey steady state cannot be achieved with sequential policymaking in a regime with monetary leadership or simultaneous decisions, for any degree of monetary conservatism.

With monetary leadership or simultaneous decisions, the behavior of the fiscal authority continues to be described by the fiscal reaction function (FRF). It then follows from proposition 1 that inflation or fiscal spending, or both, must deviate from their Ramsey steady state values. Nevertheless, the quantitative findings from section 4.1 suggest that a conservative monetary authority should remain desirable.
Figure 1 illustrates this point using the baseline calibration of section 4.1. The figure displays the consumption equivalent steady state welfare losses associated with intermediate degrees of monetary conservatism $\alpha \in [0, 1]$ relative to the Ramsey steady state. The upper horizontal line indicates the welfare losses of the OI regime. Note that in the Nash case a fully conservative monetary authority ($\alpha = 1$) approximately achieves the steady state welfare level of the OI regime. Thus, even in the absence of fiscal leadership large part of the welfare losses associated with sequential monetary and fiscal policy can be recovered through a sufficient degree of monetary conservatism.

Using again the baseline calibration, figure 2 illustrates how the steady state values of private consumption, labor effort, inflation and public spending depend on the degree of monetary conservatism. While an increase in monetary conservatism reduces the inflation bias for all timing protocols, its effect on the fiscal spending bias depends on whether fiscal policy anticipates the monetary policy reaction. If fiscal policy takes monetary decisions as given, monetary conservatism results in an increased fiscal spending bias, as suggested by proposition 4. Nevertheless, an inflation conservative central banker remains desirable, as a value of $\alpha$ slightly below 1 recovers the OI outcome.

5.3 Implications for Stabilization Policy

This section extends the analysis to a stochastic economy, considering stabilization policy in response to technology and mark-up shocks. We restrict attention to the sequential policy regime that achieves the Ramsey steady state, i.e., fiscal leadership and full monetary conservatism ($\alpha = 1$).

Full monetary conservatism implies that the central bank will achieve stable prices at all times. Thus, a necessary condition for the optimality of the impulse response under this policy

---

18 The welfare level of the OI regime is achieved by a value of $\alpha$ very close but slightly below 1.
arrangement is that the Ramsey allocation can be achieved with a stable price path. The next proposition states that this is also a sufficient condition.\footnote{The proof is given in web appendix A.11 and shows that the first order conditions of the Ramsey problem with stable prices are identical to the equilibrium conditions implied by fiscal leadership and full conservatism.}

**Proposition 9** If the Ramsey response to shocks can be achieved with a stable path for prices, then it is consistent with sequential policymaking in a regime with fiscal leadership and fully conservative monetary policy ($\alpha = 1$).

The following proposition provides sufficient conditions under which the Ramsey impulse response to shocks involves a stable price path.

**Proposition 10** Assume preferences over $c_t, h_t$, and $g_t$ are of the constant relative risk class. If private and public consumption have the same relative risk aversion, then the Ramsey response to a technology shock involves no deviation from price stability.

The proof is given in web appendix A.12 and shows that price stability under Ramsey policy requires a stable private consumption to output ratio, as well as a stable public consumption to output ratio. Maintaining both ratios constant is not possible if preferences are not homogeneous in $(c_t, g_t)$. Thus, the Ramsey response to technology shocks will generally involve deviations from price stability.

The Ramsey response to mark-up shocks will equally involve deviations from price stability. This is the case even when the assumptions of proposition 10 are satisfied. We illustrate this point in figure 3 for the baseline parameterization of section 4.1 and a positive mark-up shock of three standard deviations.\footnote{Following Ireland (2004) we set the quarterly autocorrelation of mark-up shocks to $\rho_m = 0.96$. We choose Ireland’s estimate of the standard deviations of innovations, which requires multiplying the value reported in his table 1 by the price adjustment parameter $\theta$.} The Ramsey response involves an initial rise in inflation followed by a small but persistent amount of deflation, while the sequential policy implements
stable prices at all times. Overall, the deviations from price stability in the Ramsey regime seem small (in the order of less than 0.1% per quarter) and the responses differ across regimes only for the early periods following a shock.

Although the stabilization policy associated with fiscal leadership and full monetary conservatism is not fully optimal, the following proposition suggests that such a policy arrangement remains close to fully optimal:

**Proposition 11** Sequential policymaking in a regime with fiscal leadership and fully conservative monetary policy ($\alpha = 1$) is consistent with the Ramsey response to shocks under flexible prices.

The proof is given in web appendix A.13. Thus, fiscal leadership and full conservatism eliminate all gaps to the Ramsey equilibrium with flexible prices. The presence of sticky prices allows, however, to improve somewhat upon the flexible price allocation, see Adao et al. (2003). Since these gains are likely to be small, full monetary conservatism achieves desirable stabilization outcomes in a setting with fiscal leadership.

6 Distortionary Taxation

The discussion so far relied on the availability of lump sum taxes. While this assumption allows to derive the main results analytically, it also implies that the government could potentially eliminate the monopolistic distortion, and thereby the commitment problem, via an output subsidy to firms or a wage subsidy to workers.

In a companion note, Adam and Billi (2008), we assume that fiscal spending must be financed by distortionary labor income taxes. We find again that sequential monetary policy gives rise to a steady state inflation bias and that sequential fiscal policy results in overspending on public goods whenever prices are stable. Numerical exercises suggest that these policy
biases are considerably larger than in a setting with lump sum taxes: with the sequential fiscal authority not fully internalizing the cost of public spending and taxation, more overspending implies higher labor taxes, thereby lower output, and thus even stronger incentives to increase public spending and inflation.

When numerically evaluating the effects of monetary conservatism, we find again that fiscal leadership with fully conservative monetary policy achieves the Ramsey steady state, and close to full monetary conservatism is optimal for the other timing protocols. Monetary conservatism thus remains desirable in a setting with distortionary taxes.

7 Conclusions

This paper analyzes the policy biases stemming from sequential monetary and fiscal policymaking, asking whether an inflation conservative central banker remains desirable in a setting with endogenous fiscal policy.

While the lack of fiscal commitment can make it optimal to aim for positive inflation rates, the optimal deviations from price stability turn out to be quantitatively small. Since lack of monetary commitment generates too much inflation, installing a conservative central banker remains welfare improving when fiscal policy is endogenous.

In a setting with fiscal leadership, arguably the most relevant case, installing a fully conservative central bank which focuses exclusively on stabilizing inflation eliminates not only the inflation bias but also the fiscal spending bias in steady state. The case for monetary conservatism may thus be even stronger in a setting with endogenous fiscal policy.
References


<table>
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<tr>
<th>Policy Regime</th>
<th>$c$</th>
<th>$h$</th>
<th>$\Pi$</th>
<th>$g$</th>
<th>Consumption Losses Relative to Ramsey SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP</td>
<td>$-0.44%$</td>
<td>$0.67%$</td>
<td>$1.46%$</td>
<td>$0.48%$</td>
<td>$-1.03%$</td>
</tr>
<tr>
<td>OI</td>
<td>$-0.83%$</td>
<td>$0.85%$</td>
<td>$0.09%$</td>
<td>$7.5%$</td>
<td>$-0.07%$</td>
</tr>
</tbody>
</table>

Table 1: Steady State Effects
Figure 1: Welfare Gains From Monetary Conservatism
Figure 2: Steady State Effects of Monetary Conservatism
Figure 3: Responses to Mark-Up Shocks and Monetary Conservatism
A Web Appendix - NOT FOR PUBLICATION

A.1 Ramsey Steady State

The Lagrangian of the Ramsey problem (10) is

\[
\begin{align*}
\max_{\{c_t, h_t, \Pi_t, R_t, g_t\}} & \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, h_t, g_t) \\
+ & \gamma_t^1 \left[ u_{ct}(\Pi_t - 1)\Pi_t - \frac{u_{ct} z_t h_t}{\theta} \left( 1 + \eta_t + \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) - \beta u_{ct+1}(\Pi_{t+1} - 1)\Pi_{t+1} \right] \\
+ & \gamma_t^2 \left[ \frac{u_{ct}}{R_t} - \beta \frac{u_{ct+1}}{\Pi_{t+1}} \right] \\
+ & \gamma_t^3 \left[ z_t h_t - c_t - \frac{\theta}{2} (\Pi_t - 1)^2 - g_t \right] \right\}
\end{align*}
\]

The first-order conditions w.r.t. \((c_t, h_t, \Pi_t, R_t, g_t)\), respectively, are given by

\[
\begin{align*}
& u_{ct} + \gamma_t^1 \left( u_{ct}(\Pi_t - 1)\Pi_t - \frac{u_{ct} z_t h_t}{\theta} (1 + \eta_t) \right) \\
& - \gamma_{t-1}^1 u_{ct}(\Pi_t - 1)\Pi_t + \gamma_t^2 \frac{u_{ct}}{R_t} - \gamma_{t-1}^2 \frac{u_{ct}}{\Pi_t} - \gamma_t^3 z_t = 0 \\
& \left( \gamma_t^1 - \gamma_{t-1}^1 \right) u_{ct}(2\Pi_t - 1) + \gamma_t^2 \frac{u_{ct}}{\Pi_t^2} - \gamma_t^3 \theta (\Pi_t - 1) = 0 \\
& -\gamma_t^2 \frac{u_{ct}}{R_t^2} = 0 \\
& u_{gt} - \gamma_t^3 = 0
\end{align*}
\]

where \(\gamma_{-1}^j = 0\) for \(j = 1, 2\). We denote the Ramsey steady state by dropping time subscripts.

Equation (22), \(u_{ct} > 0\) and \(R_t \geq 1\) imply

\[
\gamma^2 = 0
\]

Equations (23) delivers

\[
\gamma^3 = u_g > 0
\]
This and (21) gives

$$\Pi = 1$$

From (7) it then follows

$$R = \frac{1}{\beta}$$

Then (6) delivers

$$1 + \eta + \frac{u_h}{u_c} \eta = 0$$

This delivers (11) shown in the main text. Using the previous results, (20) simplifies to

$$u_h - \gamma \frac{h}{\theta} u_{hh} \eta + u_g = 0$$

(24)

From (19) one obtains

$$\gamma \frac{h}{\theta} = \frac{u_c - u_g}{u_{cc} (1 + \eta)}$$

(26)

Substituting (26) into (25) delivers

$$u_h - \frac{u_c - u_g}{u_{cc} (1 + \eta)} u_{hh} \eta + u_g = 0$$

Using (24) to substitute for $u_c$ one gets

$$u_g = -u_h \frac{1 + \left( \frac{\eta}{1 + \eta} \right)^2 \frac{u_{hh}}{u_{cc}}}{1 + \frac{\eta}{1 + \eta} \frac{u_{hh}}{u_{cc}}}$$

Using (24) again to substitute $\frac{\eta}{1 + \eta}$ delivers (12) shown in the main text.
A.2 Sequential Fiscal Reaction Function

The fiscal problem (13) is

\[
\max_{\{c_{t+j}, h_{t+j}, \Pi_{t+j}, g_{t+j}\}} \sum_{j=0}^{\infty} \beta^j \left\{ u(c_{t+j}, h_{t+j}, g_{t+j}) + \gamma^1_{t+j} \left[ u_{ct+j}(\Pi_{t+j} - 1)\Pi_{t+j} - \frac{u_{ct+j}z_{t+j}h_{t+j}}{\theta} (1 + \eta_{t+j} + \frac{u_{ht+j} \eta_{t+j}}{u_{ct+j} z_{t+j}}) \right] + \beta u_{ct+j+1}(\Pi_{t+j+1} - 1)\Pi_{t+j+1} \right. \\
+ \left. \gamma^2_{t+j} \left[ \frac{u_{ct+j}}{R_{t+j}} - \beta \frac{u_{ct+j+1}}{\Pi_{t+j+1}} \right] \right. \\
+ \left. \gamma^3_{t+j} \left[ z_{t+j} h_{t+j} - c_{t+j} - \frac{\theta}{2}(\Pi_{t+j} - 1)^2 - g_{t+j} \right] \right\}
\]

taking as given \( R_{t+j-1} \) and other variables dated \( t+j \) for \( j \geq 1 \). The first order conditions w.r.t. \((c_t, h_t, \Pi_t, g_t)\), respectively, are given by

\[
u_{ct} + \gamma^1_t \left( u_{ct} (\Pi_t - 1)\Pi_t - \frac{u_{ct} z_t h_t}{\theta} (1 + \eta_t) \right) + \gamma^2_t \frac{u_{ct}}{R_t} - \gamma^3_t = 0 \tag{27}
\]

\[
u_{ht} - \gamma^1_t \frac{u_{ct} z_t}{\theta} \left( 1 + \eta_t + \frac{u_{ht} \eta_t}{u_{ct} z_t} + h_t \frac{u_{ht h t} \eta_t}{u_{ct} z_t} \right) + \gamma^3_t z_t = 0 \tag{28}
\]

\[
\gamma^1_t u_{ct} (2\Pi_t - 1) - \gamma^3_t \theta (\Pi_t - 1) = 0 \tag{29}
\]

\[
u_{gt} - \gamma^3_t = 0 \tag{30}
\]

From (29) and (30) one gets

\[
\frac{\gamma^1_t}{u_{ct} (2\Pi_t - 1)} = \frac{u_{gt} \theta (\Pi_t - 1)}{\gamma^3_t} \tag{32}
\]

Using the previous result and (30) to substitute the Lagrange multipliers in (28) delivers FRF shown in the main text.
A.3 Sequential Monetary Reaction Function

The monetary problem (15) is

\[
\max \left\{ E_t \sum_{j=0}^{\infty} \beta^j \left[ u_{ct+j, ht+j, g_{t+j}} + \gamma^1_{t+j} \left[ c_{t+j} (\Pi_{t+j} - 1) \Pi_{t+j} - \frac{u_{ct+j} z_{t+j} h_{t+j}}{\theta} \left( 1 + \eta_{t+j} \right) + \frac{u_{ht+j} \eta_{t+j}}{u_{ct+j} z_{t+j}} \right] \right] \right\} 
\]

\[
= \beta u_{ct+j+1} (\Pi_{t+j+1} - 1) \Pi_{t+j+1} 
+ \gamma^2_{t+j} \left[ \frac{u_{ct+j}}{R_{t+j}} - \beta \frac{u_{ct+j+1}}{\Pi_{t+j+1}} \right] 
+ \gamma^3_{t+j} \left[ z_{t+j} h_{t+j} - c_{t+j} - \frac{\theta}{2} (\Pi_{t+j} - 1)^2 - g_{t+j} \right] 
\]

taking as given \( g_{t+j-1} \) and other variables dated \( t + j \) for \( j \geq 1 \). The first order conditions w.r.t. \( (c_t, h_t, \Pi_t, R_t) \), respectively, are given by

\[
u_{ct} + \gamma^1_t \left( u_{ct} (\Pi_t - 1) \Pi_t - \frac{u_{ct} z_t h_t}{\theta} (1 + \eta_t) \right) + \gamma^2_t \frac{u_{ct}}{R_t} - \gamma^3_t = 0 
\]

\[
u_{ht} - \gamma^1_t \frac{u_{ct} z_t}{\theta} \left( 1 + \eta_t + \frac{u_{ht} \eta_t}{u_{ct} z_t} + h_t \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) + \gamma^3_t z_t = 0 
\]

\[
\gamma^1_t u_{ct} (2 \Pi_t - 1) - \gamma^3_t \theta (\Pi_t - 1) = 0 
\]

\[
-\gamma^2_t \frac{u_{ct}}{R^2_t} = 0 
\]

Equation (34), \( u_{ct} > 0 \) and \( R_t \geq 1 \) imply

\[
\gamma^2_t = 0 
\]

Then solving (31), (32) and (33) for \( \gamma^3_t \) delivers, respectively,

\[
\gamma^3_t = u_{ct} + \gamma^1_t \left( u_{ct} (\Pi_t - 1) \Pi_t - \frac{u_{ct} z_t h_t}{\theta} (1 + \eta_t) \right) 
\]

\[
\gamma^3_t = \frac{u_{ht}}{z_t} + \gamma^1_t \frac{u_{ct}}{\theta} \left( 1 + \eta_t + \frac{u_{ht} \eta_t}{u_{ct} z_t} + h_t \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) 
\]

\[
\gamma^3_t = \frac{u_{ct} (2 \Pi_t - 1)}{\theta (\Pi_t - 1)} 
\]

33
Equations (35) and (37) imply

\[ \gamma^1_t = \frac{\theta}{2n_t - 1 - \frac{u_{ct}}{u_{ct}} (\theta (\Pi - 1) \Pi_t - z_t h_t (1 + \eta_t))} \] (38)

While (36) and (37) give

\[ \gamma^1_t = \frac{\theta}{z_t u_{ct}} \left( 1 + \eta_t - \frac{2n_t - 1}{n_t - 1} + \frac{u_{ht} \eta_t}{u_{ct} z_t} + h_t \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) \] (39)

From (38) and (39) one obtains MRF shown in the main text.

A.4 Proof of Proposition 2

We first show that MRF cannot hold in the neighborhood of \( \Pi = 1 \). In steady state one can rewrite MRF as

\[ \Pi \left( 1 + \frac{u_c}{u_h} \right) + O(\Pi - 1) = 0 \] (40)

where \( O(\Pi - 1) \) summarizes terms that converge to zero as \( \Pi - 1 \rightarrow 0 \). In a steady state with \( \Pi = 1 \) equation (6) delivers \( \frac{u_c}{u_h} < -\frac{\eta}{1+\eta} < -1 \). Since the implicit function \( \frac{u_c}{u_h}(\Pi) \) defined by (6) exists, this implies that \( \frac{u_c}{u_h} \) is bounded away from \(-1\) also in a sufficiently small neighborhood around \( \Pi = 1 \). Thus, (40) cannot hold in the neighborhood of \( \Pi = 1 \). Moreover, from \( R \geq 1 \) and (7) we have \( \Pi \geq \beta \) in steady state. For \( \beta \) sufficiently close to 1, it then follows that MRF can only hold if \( \Pi > 1 \).

A.5 Proof of Proposition 4

The effect of inflation on steady state utility is given by

\[ \frac{du}{d\Pi} = u_c \frac{\partial c}{\partial \Pi} + u_h \frac{\partial h}{\partial \Pi} + u_g \frac{\partial g}{\partial \Pi} \] (41)

where \( c(\Pi), h(\Pi), g(\Pi) \) denote the steady state levels emerging under sequential fiscal policy when monetary policy implements inflation rate \( \Pi \), and the derivatives \( u_j \) \( (j = c, h, g) \) are
evaluated at this steady state. We first evaluate (41) at $\Pi = 1$. Equation (6) delivers

$$u_c = -\frac{\eta}{1 + \eta} u_h$$

(42)

Totally differentiating (8) and evaluating at $\Pi = 1$ gives

$$\frac{\partial c}{\partial \Pi} = \frac{\partial h}{\partial \Pi} - \frac{\partial g}{\partial \Pi}$$

Using this result and (14), then (41) can be rewritten as

$$\frac{du}{d\Pi} = (u_c - u_g) \frac{\partial c}{\partial \Pi}$$

Equations (14) and (42) imply $u_c > u_g$, thus

$$\text{sign} \left( \frac{du}{d\Pi} \right) = \text{sign} \left( \frac{\partial c}{\partial \Pi} \right)$$

(43)

To determine the sign of $\partial c / \partial \Pi$ we totally differentiate FRF, (6), and (8) and evaluate at $\Pi = 1$, this delivers

$$\begin{pmatrix}
0 & -u_{hh} & -u_{gg} \\
\frac{h}{\eta} u_{hh} & \frac{h}{\eta} u_{hh} & 0 \\
-1 & 1 & -1
\end{pmatrix}
\begin{pmatrix}
\frac{\partial c}{\partial \Pi} \\
\frac{\partial h}{\partial \Pi} \\
\frac{\partial g}{\partial \Pi}
\end{pmatrix} =
\begin{pmatrix}
h \eta \frac{u_{hh} u_{hh}}{u_c} \\
0 \\
0
\end{pmatrix}$$

Solving for $\frac{\partial c}{\partial \Pi}$ and $\frac{\partial g}{\partial \Pi}$ gives

$$\frac{\partial c}{\partial \Pi} = \frac{u_c u_{hh} u_{gg} - u_{cc} u_{hh} u_{gg} - u_{cc} u_{hh} u_{hh}}{h \eta \frac{u_{hh} u_{hh}}{u_c}}$$

$$\frac{\partial g}{\partial \Pi} = - \frac{u_c u_{hh} + u_{cc} u_{h}}{u_c u_{hh} u_{gg} - u_{cc} u_{hh} u_{gg} - u_{cc} u_{hh} u_{hh}} h \eta \frac{u_{hh} u_{hh}}{u_c}$$

When $u_{hh} < 0$, signing these expressions delivers $\frac{\partial c}{\partial \Pi} > 0$ and $\frac{\partial g}{\partial \Pi} < 0$, as claimed. The former inequality and (43) imply $\frac{du}{d\Pi} > 0$, locally at $\Pi = 1$. 35
A.6 Utility Weights

For the period utility specification (16), the Ramsey policy marginal conditions (11) and (12), respectively, deliver

\begin{align}
\omega_h &= \frac{1}{ch} \frac{1 + \eta}{\eta} \\
\omega_g &= \omega_h gh \frac{1 + \frac{\eta}{1+\eta} \frac{c}{h} \phi}{1 + \frac{c}{h} \phi}
\end{align}

(44) (45)

A.7 Solving for the Equilibrium with Sequential Monetary and Fiscal Policy

We show how to solve for the stochastic Markov-perfect Nash equilibrium with sequential monetary and fiscal policy. We illustrate the method for the SP case, i.e., without a conservative central banker, but the method readily extends to the case with a conservative monetary authority. The Markov perfect Nash equilibrium solves the following problem

\[
\max_{\{c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j}, g_{t+j}\}} E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, h_{t+j}, g_{t+j})
\]

\text{s.t.} \quad \text{Equations (6),(7),(8) for all } t

\[
E_t (c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j}, g_{t+j}) \text{ for } j \geq 1
\]

One should note that FRF and MRF need not be imposed, since they can already be derived from the first order conditions of this problem, see sections 3.2.1 and 3.2.2, respectively. The solution of problem (46) will always satisfy FRF and MRF.
Then, the recursive formulation of the Lagrangian of problem (46) is

\[
W (z_t, \eta_t) = \min_{(\gamma_1, \gamma_2, \gamma_3)^{(c_t, h_t, \Pi_t, R_t, g_t)}} \max \{ f (\cdot) + \beta E_t W (z_{t+1}, \eta_{t+1}) \}
\] (47)

s.t.

\[
\begin{align*}
    z_{t+1} &= (1 - \rho_z) + \rho_z z_t + \varepsilon_{zt+1} \\
    \eta_{t+1} &= \eta (1 - \rho_{\eta}) + \rho_{\eta} \eta_t + \varepsilon_{\eta t+1}
\end{align*}
\]

where the one-period return is

\[
\begin{align*}
f (\cdot) &= u(c_t, h_t, g_t) \\
&+ \gamma_1^1 \left[ u_{ct} (\Pi_t - 1) \Pi_t - \frac{u_{zt} h_t}{\theta} \left( 1 + \eta_t + \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) - E_t^{AS} \right] \\
&+ \gamma_2^2 \left[ \frac{u_{ct}}{R_t} - E_t^{IS} \right] \\
&+ \gamma_3^3 \left[ z_t h_t - c_t - \frac{\theta}{2} (\Pi_t - 1)^2 - g_t \right]
\end{align*}
\]

with the expectations functions

\[
\begin{align*}
    E_t^{AS} &\equiv \beta E_t u_{ct+1} (\Pi_{t+1} - 1) \Pi_{t+1} \\
    E_t^{IS} &\equiv \beta E_t \frac{u_{ct+1}}{\Pi_{t+1}}
\end{align*}
\] (48, 49)

taken as given. The additional control variables \( \gamma_1^1, \gamma_2^2, \gamma_3^3 \) are the Lagrange multipliers of the implementability constraints (6) and (7), and the feasibility constraint (8), respectively.

We then solve for the steady state using the first order conditions of the recursive formulation (47). Thereafter, we compute a quadratic approximation of the one-period return \( f(\cdot) \) around this steady state. This step involves quadratically approximating the implementability and feasibility constraints. Instead, the expectation functions \( E_t^{AS} \) and \( E_t^{IS} \) are linearly approximated as

\[
\begin{align*}
    E_t^{AS} &\approx a_0^1 + a_1^1 (z_t - 1) + a_2^1 (\eta_t - \eta) \\
    E_t^{IS} &\approx a_0^2 + a_1^2 (z_t - 1) + a_2^2 (\eta_t - \eta)
\end{align*}
\] (50, 51)
Importantly, postulating linear expectation functions is sufficient to obtain a first order approximation to the equilibrium dynamics and policy functions. The policymaker takes expectations functions as given, therefore, they do not show up in differentiated form in the first order conditions. Moreover, linear expectations functions are sufficient to evaluate the Lagrangian, i.e., utility, up to second order. This is the case because either the implementability constraints or the Lagrange multipliers are zero in a sufficiently small neighborhood around the steady state. As a result, no first order terms appear when evaluating the quadratic approximation of $f(\cdot)$ at the solution. Obviously, this is just a restatement of the fact that (47) is an unconstrained optimization problem.

We now explain how we compute the expectation functions (50) and (51). We start with an initial guess for $\alpha_i^j$ ($j = 1, 2; i = 0, 1, 2$), then we solve (47) with $f(\cdot)$ replaced by its quadratic approximation. We update $\alpha_i^j$, as explained below, and continue iterating until the maximum absolute change of the policy functions drops below the square root of machine precision, i.e., $1.49 \cdot 10^{-8}$.

Let the solution for the policy functions $c(\cdot)$ and $\Pi(\cdot)$ be given by

$$c_{t+1} - c = \delta_{cz} (z_{t+1} - 1) + \delta_{cy} (\eta_{t+1} - \eta)$$

$$\Pi_{t+1} - \Pi = \delta_{\Pi z} (z_{t+1} - 1) + \delta_{\Pi y} (\eta_{t+1} - \eta)$$

where variables without time subscript denote steady state values. A first-order approximation of the expectation functions (48) and (49) then delivers

$$E_t^{\text{AS}} \approx E_t^{\text{AS}} \bigg|_{ss} + \left. \frac{\partial E_t^{\text{AS}}}{\partial c_{t+1}} \right|_{ss} E_t (c_{t+1} - c) + \left. \frac{\partial E_t^{\text{AS}}}{\partial \Pi_{t+1}} \right|_{ss} E_t (\Pi_{t+1} - \Pi)$$

$$E_t^{\text{IS}} \approx E_t^{\text{IS}} \bigg|_{ss} + \left. \frac{\partial E_t^{\text{IS}}}{\partial c_{t+1}} \right|_{ss} E_t (c_{t+1} - c) + \left. \frac{\partial E_t^{\text{IS}}}{\partial \Pi_{t+1}} \right|_{ss} E_t (\Pi_{t+1} - \Pi)$$

where $|_{ss}$ indicates expressions evaluated at steady state. These conditions together with (52),

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(53) and

\[ E_t (z_{t+1} - 1) = \rho_z (z_t - 1) \]

\[ E_t (\eta_{t+1} - \eta) = \rho_\eta (\eta_t - \eta) \]

deliver the expectations functions consistent with the approximated policy functions

\[ a^1_0 = \beta u_c (\Pi - 1) \Pi \]
\[ a^1_1 = \beta \rho_z [(\Pi - 1) \Pi u_{cz} \delta_{cz} + u_c (2\Pi - 1) \delta_{\Pi z}] \]
\[ a^1_2 = \beta \rho_\eta [(\Pi - 1) \Pi u_{c\eta} \delta_{c\eta} + u_c (2\Pi - 1) \delta_{\Pi \eta}] \]
\[ a^2_0 = \beta \frac{u_c}{\Pi} \]
\[ a^2_1 = \beta \frac{\rho_z}{\Pi} [u_{cz} \delta_{cz} - \frac{u_c}{\Pi} \delta_{\Pi z}] \]
\[ a^2_2 = \beta \frac{\rho_\eta}{\Pi} [u_{c\eta} \delta_{c\eta} - \frac{u_c}{\Pi} \delta_{\Pi \eta}] \]

A.8 Consumption Losses Relative to Ramsey

Let \( u(c, h, g) \) denote the period utility for the Ramsey steady state and let \( u(c^A, h^A, g^A) \) represent the period utility for the steady state of an alternative policy regime. The permanent reduction in private consumption that would imply the Ramsey steady state to be welfare equivalent to the alternative policy regime \( \mu^A \leq 0 \) is implicitly defined by

\[
\frac{1}{1 - \beta} u(c^A, h^A, g^A) = \frac{1}{1 - \beta} u(c(1 + \mu^A), h, g)
\]

\[ = \frac{1}{1 - \beta} [u(c, h, g) + \log (1 + \mu^A)] \]

where the second equality uses equation (16). Therefore, one obtains

\[ \mu^A = \exp [u(c^A, h^A, g^A) - u(c, h, g)] - 1 \]
A.9 Conservative Monetary Reaction Function

The conservative monetary problem (17) is

\[
\max_{\{c_{t+j}, h_{t+j}, \Pi_{t+j}, R_{t+j}\}} E_t \sum_{j=0}^{\infty} \beta^j \left\{ (1 - \alpha) u(c_{t+j}, h_{t+j}, g_{t+j}) - \frac{\alpha}{2} (\Pi_{t+j} - 1)^2 \right. \\
\left. + \gamma^1_{t+j} [u_{ct+j}(\Pi_{t+j} - 1)\Pi_{t+j} - \frac{u_{ct+j}h_{t+j}^2}{\theta} \left( 1 + \eta_{t+j} + \frac{u_{ht+j} \eta_{t+j}}{u_{ct+j} z_{t+j}} \right) \\
- \beta u_{ct+j+1} (\Pi_{t+j+1} - 1)\Pi_{t+j+1}] \\
+ \gamma^2_{t+j} \left[ \frac{u_{ct+j}}{R_{t+j}} - \beta \frac{u_{ct+j+1}}{\Pi_{t+j+1}} \right] \right. \\
+ \gamma^3_{t+j} \left[ z_{t+j}h_{t+j} - c_{t+j} - \frac{\theta}{2} (\Pi_{t+j} - 1)^2 - g_{t+j} \right] \right\}
\]

taking as given \( g_{t+j-1} \) and variables dated \( t+j \) for \( j \geq 1 \). The first order conditions w.r.t. \((c_t, h_t, \Pi_t, R_t)\), respectively, are given by

\[
(1 - \alpha) u_{ct} + \gamma^1_t \left( u_{cc}(\Pi_t - 1)\Pi_t - \frac{u_{ct}z_t h_t}{\theta} (1 + \eta_t) \right) + \gamma^2_t \frac{u_{ct}}{R_t} - \gamma^3_t = 0 \tag{54}
\]

\[
(1 - \alpha) u_{ht} - \gamma^2_t \frac{u_{ct}}{\theta} \left( 1 + \eta_t + \frac{u_{ht} \eta_t}{u_{ct} z_t} + h_t \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) + \gamma^3_t z_t = 0 \tag{55}
\]

\[
\gamma^1_t u_{ct} (2\Pi_t - 1) - \gamma^2_t \theta (\Pi_t - 1) - \alpha (\Pi_t - 1) = 0 \tag{56}
\]

\[
-\gamma^2_t \frac{u_{ct}}{R_t^2} = 0 \tag{57}
\]

Equation (57), \( u_{ct} > 0 \) and \( R_t \geq 1 \) imply

\[
\gamma^2_t = 0
\]

Then solving (54), (55) and (56) for \( \gamma^3_t \) delivers, respectively,

\[
\gamma^3_t = (1 - \alpha) u_{ct} + \gamma^1_t \left( u_{cc}(\Pi_t - 1)\Pi_t - \frac{u_{ct}z_t h_t}{\theta} (1 + \eta_t) \right) \tag{58}
\]

\[
\gamma^3_t = -(1 - \alpha) \frac{u_{ht} \eta_t}{z_t} + \gamma^1_t \frac{u_{ct}}{\theta} \left( 1 + \eta_t + \frac{u_{ht} \eta_t}{u_{ct} z_t} + h_t \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) \tag{59}
\]

\[
\gamma^3_t = \gamma^1_t \frac{u_{ct} (2\Pi_t - 1)}{\theta (\Pi_t - 1)} - \frac{\alpha}{\theta} \tag{60}
\]
Equations (58) and (60) imply
\[
\gamma_t^1 = \frac{\theta \left( 1 - \alpha + \frac{1}{u_{ct}} \frac{\alpha}{\theta} \right)}{\frac{2^\Pi_t - 1}{\Pi_t - 1} - \frac{u_{ct}}{u_{ct}} \left( \theta (\Pi_t - 1) \Pi_t - z_t h_t (1 + \eta_t) \right)}
\]  
(61)

While (59) and (60) give
\[
\gamma_t^1 = \frac{\theta \left( 1 - \alpha - \frac{z_t}{u_{ht}} \frac{\alpha}{\theta} \right)}{\frac{z_t u_{ct}}{u_{ht}} \left( 1 + \eta_t - \frac{2^\Pi_t - 1}{\Pi_t - 1} + \frac{u_{ht} \eta_t}{u_{ct} z_t} + \frac{h_t u_{hht}}{u_{ct} z_t} \right)}
\]  
(62)

From (61) and (62) one obtains the conservative monetary reaction function
\[
\begin{align*}
&- \frac{z_t u_{ct}}{u_{ht}} \left( \eta_t (\Pi_t - 1) - (\Pi_t - 1) \eta_t \left( 1 + h_t \frac{u_{hht}}{u_{ht}} \right) \right) + \\
&\frac{2^{\Pi_t - 1} - \frac{u_{ct}}{u_{ct}} \left( \theta (\Pi_t - 1) \Pi_t - z_t h_t (1 + \eta_t) \right)}{(1 - \alpha) \theta - \frac{z_t}{u_{ht}}} = 0
\end{align*}
\]  
(CMRF)

A.10 Proof of Proposition 7

Full monetary conservatism implies that \( \Pi_t \equiv 1 \), see CMRF. Substituting \( \Pi_t \equiv 1 \) for CMRF, noting that (7) can be dropped as it only defines \( R_t \), and using (8) to substitute \( h_t \), one can rewrite problem (18) as
\[
\begin{align*}
\max_{\{c_{t+j}, g_{t+j}\}} & \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, \frac{c_{t+j} + g_{t+j}}{z_{t+j}}, g_{t+j}) \quad & \quad & \\
\text{s.t.} & - \frac{u_h \left( \frac{c_{t+j} + g_{t+j}}{z_t} \right)}{u_c (c_t)} = \frac{1 + \eta_t}{\eta_t} \text{ for all } t \quad & \quad & \\
& \{c_{t+j}, h_{t+j}, g_{t+j}\} \text{ given for } j \geq 1 \quad & \quad & 
\end{align*}
\]  
(63)

Letting \( \lambda_t \) denote the Lagrange multiplier on (63), the FOCs w.r.t. \( (c_t, g_t, \lambda_t) \), respectively, are given by
\[
\begin{align*}
&u_{ct} + \frac{u_{ht}}{z_t} + \lambda_t \frac{u_{hht} u_{ct} - u_{ht} u_{cct}}{(u_{ct})^2} = 0 \quad & \quad & \\
&\frac{u_{ht}}{z_t} + u_{gt} + \lambda_t \frac{u_{hht}}{u_{ct}} = 0 \quad & \quad & \\
&\frac{1 + \eta_t}{\eta_t} + \frac{u_{ht}}{u_{ct}} = 0 \quad & \quad & 
\end{align*}
\]
Eliminating $\lambda_t$ from the first FOCs delivers

$$u_{ct} + \frac{u_{ht}}{z_t} - \left( \frac{u_{ht}}{z_t} + u_{gt} \right) \left( 1 - z_t \frac{u_{ht}}{u_{ht} + u_{ct}} \right) = 0$$

Using the last FOC above to substitute the term $u_{ct}$ on the left-hand side of the previous equation gives

$$u_{gt} = -\frac{u_{ht}}{z_t} \frac{1 - \frac{\eta_t - \frac{u_{ht} u_{ct}}{u_{ht} + u_{ct}}}{1 + \eta_t}}{1 - \frac{u_{ht} u_{ct}}{z_t u_{ht} u_{ct}}}$$

(64)

The sequential equilibrium under fiscal leadership and full monetary conservatism is thus described by the solution to (63), (64), (8), $\Pi_t = 1$, and (7). Steady state versions of these equations characterize the Ramsey steady state, see section 3.1.

A.11 Proof of Proposition 9

Appendix A.10 shows that the equilibrium with fiscal leadership and full monetary conservatism is described by (63), (64), (7), (8), and $\Pi_t \equiv 1$. We now show that the same equations characterize the Ramsey equilibrium, provided $\Pi_t \equiv 1$ is optimal. Equations (7), (8) and (63) are also constraints imposed on the Ramsey problem for $\Pi_t \equiv 1$. It thus remains to be shown that the Ramsey problem also implies (64). For $\Pi_t \equiv 1$ the FOCs of the Ramsey problem (19)-(23) simplify to

$$u_{ct} + \gamma_t^1 \left( -\frac{u_{ct} z_t h_t}{\theta} (1 + \eta_t) \right) - u_{gt} = 0$$

(65)

$$u_{ht} - \gamma_t^1 \frac{u_{ct} z_t}{\theta} \left( h_t \frac{u_{ht} \eta_t}{u_{ct} z_t} \right) + u_{gt} z_t = 0$$

(66)

where we use also (63). Then from (65) we get

$$\gamma_t^1 \frac{h_t}{\theta} = \frac{u_{ct} - u_{gt}}{u_{ct} z_t (1 + \eta_t)}$$

Using this expression to eliminate $\gamma_t^1$ in (66) and employing again (63) delivers (64).
A.12 Proof of Proposition 10

The FOCs of the Ramsey problem consist of equations (19)-(23), (6), (7), and (8). Using $\gamma_t^2 = 0$ and $\gamma_t^3 = u_{gt}$, noting that the Euler equation can be dropped as it only defines $R_t$, and setting $\eta_t = \eta$ (we are interested in the impulse response to a technology shock) reduces the FOCs to

\begin{align*}
  u_{ct} + \left(\gamma_t^1 - \gamma_{t-1}^1\right) u_{ct} (\Pi_t - 1) \Pi_t - \gamma_t^1 \frac{u_{ct} z_t h_t}{\theta} (1 + \eta) - u_{gt} = 0 \\
  u_{ht} - \gamma_t^1 \frac{u_{ct} z_t}{\theta} \left(1 + \eta + \frac{u_{ht} \eta}{u_{ct} z_t} + h_t \frac{u_{ht} \eta}{u_{ct} z_t}\right) + u_{gt} z_t = 0 \\
  (\gamma_t^1 - \gamma_{t-1}^1) u_{ct} (2\Pi_t - 1) - u_{gt} \theta (\Pi_t - 1) = 0 \\
  u_{ct} (\Pi_t - 1) \Pi_t - \frac{u_{ct} z_t h_t}{\theta} \left(1 + \eta + \frac{u_{ht} \eta}{u_{ct} z_t}\right) - \beta u_{ct+1} (\Pi_{t+1} - 1) \Pi_{t+1} = 0 \\
  z_t h_t - c_t - \frac{\theta}{2} (\Pi_t - 1)^2 - g_t = 0
\end{align*}

We now show that these FOCs are satisfied for a stable price path under the assumptions stated in the proposition. Equation (70) holds for $\Pi_t = 1$ if

\begin{equation}
  \frac{u_{ht}}{u_{ct} z_t} = \frac{u_h}{u_c} = -\frac{1 + \eta}{\eta} \tag{72}
\end{equation}

where expressions without time subscript denote steady state values. From (69) follows that $\Pi_t = 1$ also requires

\begin{equation}
  \gamma_t^1 = \gamma^1 \tag{73}
\end{equation}

Equation (68) and the previous two results then deliver

\begin{equation}
  1 - \gamma^1 \frac{1}{\theta} h_t \frac{u_{ht} \eta}{u_{ht} \eta} + \frac{u_{gt} z_t}{u_{ht}} = 0 \tag{74}
\end{equation}

Under CRRA preferences for labor, $h_t \frac{u_{ht}}{u_{ht}}$ is constant and the previous equation is satisfied if

\begin{equation}
  \frac{u_{gt} z_t}{u_{ht}} = \frac{u_g}{u_h} \tag{74}
\end{equation}

From (67) and (73)

\begin{equation}
  1 - \gamma^1 \frac{u_{ct} c_t}{\theta} \frac{1}{c_t} z_t h_t (1 + \eta) - \frac{u_{gt}}{u_{ct}} = 0
\end{equation}

Note that equations (72) and (74) imply $\frac{u_{gt}}{u_{ct}} = \frac{u_{gt}}{u_{ct}}$ and CRRA utility implies that $\frac{u_{ct}}{u_{ct}}$ is constant. The previous equations thus holds provided

$$\frac{z_t h_t}{c_t} = \frac{h}{c}$$

(75)

The FOCs (67)-(70) thus hold for $\Pi_t = 1$ provided (72)-(75) hold. Therefore, if in addition to (72)-(75) also (71) holds, all FOCs of the Ramsey problem are satisfied with a stable path for prices. We now show that this is the case for utility functions of the form

$$u(c_t, h_t, g_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \omega_h \frac{h_t^{1+\varphi}}{1+\varphi} + \omega_g \frac{g_t^{1-\sigma}}{1-\sigma}$$

Equations (72) and (74) give

$$c_t = \left( -\frac{z_t}{\omega_h h_t^{\varphi} u_c} \right)^{\frac{1}{\sigma}}$$

$$g_t = \left( \frac{\omega_g z_t h_t^{\varphi} u_g}{\omega_h h_t^{\varphi} u_g} \right)^{\frac{1}{\sigma}}$$

implying that $\frac{c_t}{g_t}$ is constant. The latter and equation (71) for $\Pi_t = 1$ imply that $\frac{c_t}{z_t h_t}$ is constant, as required by (75).

**A.13 Proof of Proposition 11**

The proof of proposition 7 in section A.10 shows that the sequential equilibrium under fiscal leadership and full monetary conservatism is described by the solution to (63), (64), (8), $\Pi_t = 1$, and (7). Under flexible prices, constraint (6) has to be substituted by (63) in the Ramsey problem. The proof of proposition 9 in section A.11 shows that the same set of equations characterize the Ramsey equilibrium under flexible prices.