ISSN 1936-5330

The Federal Reserve Bank *of* Kansas City
Economic Research Department

Phillips Curves, Monetary Policy, and a Labor Market Transmission Mechanism

Robert R. Reed and Stacey L. Schreft

December 2007
RWP 07-12

Research Working Papers

Phillips Curves, Monetary Policy, and a Labor Market Transmission Mechanism

Robert L. Reed and Stacey L. Schreft*

December 2007

RWP 07-12

Abstract: This paper develops a general equilibrium monetary model with performance incentives to study the inflation-unemployment relationship. A long-run downward-sloping Phillips curve can exist with perfectly anticipated inflation because workers' incentive to exert effort depend on financial market returns. Consequently, higher inflation rates can reduce wages and stimulate employment. An upward-sloping or vertical Phillips Curve can arise instead, depending on agents' risk aversion and the possibility of capital formation. Welfare might be higher away from the Friedman rule and with a central bank putting some weight on employment.

Key words: Phillips curve; Efficiency wages; Involuntary unemployment; Labor and financial market frictions; Central Bank mandate

JEL classification: E24, E31, E52, E58, J21, J64, M5

*Corresponding author.
Reed, Department of Economics, Finance and Legal Studies, 200 Alston Hall, Box 870224, University of Alabama, Tuscaloosa, AL 35487, 205-348-8667, rreed@cba.ua.edu; Schreft, Economic Research Department, Federal Reserve Bank of Kansas City, Kansas City, MO 64198, 816-881-2581, Stacey.L.Schreft@kc.frb.org.
The authors thank Joydeep Bhattacharya, Roberto Billi, Craig Hakkio, and Pu Shen for many helpful conversations, seminar participants at the Federal Reserve Banks of Atlanta and Kansas City, and Bonnie Brewer, Alice Chiu, and Danielle White for able research assistance.

1. Introduction

Some central banks (e.g., the U.S. Federal Reserve System) explicitly have a dual mandate that requires balancing price stability and economic growth. Others (e.g., the Bank of Canada) have only an explicit inflation target. No central bank, however, can ignore macroeconomic stability. As Federal Reserve Board Chairman Ben Bernanke (2007) has explained,

> In fact, the practice of monetary policy in an inflation-targeting regime is not necessarily inconsistent with a dual mandate such as that given to the Federal Reserve; indeed, most if not all inflation-targeting central banks today practice "flexible"inflation targeting, meaning that they take account of other economic goals besides price stability—notably economic growth, employment, and financial stability—when making policy decisions.

Little is known, however, about how monetary policy affects unemployment or is constrained by a concern about unemployment. Mishkin (2007a) surveys the many channels or transmission mechanisms that have been proposed to explain the effect of monetary policy on the real economy. None of these channels works through wage or employment determination directly. Most operate through interest rates, exchange rates, or stock prices, which directly affect some component of output, and employment is determined implicitly, as firms adjust output. Some channels instead work through credit markets, affecting the supply of credit directly or the presence of moral hazard and adverse selection in credit markets.

Omission of the microfoundations of employment and wage determination from transmission mechanisms is significant. Levin et al. (2005) finds that the optimal monetary policy is extremely sensitive to the wage-contracting mechanism. Similarly, Blanchard and Gali (2007) finds that the optimal policy design depends on the interaction between frictions and shocks. All of these authors call for more research into the microfoundations linking wage determination and the nonneutrality of money.

This article fills the gap in the existing literature by presenting a general-equilibrium monetary model in which the impact of monetary policy is transmitted to real variables through the wage-contracting/employment process in the labor market. The model gives rise to either a downward-sloping, vertical, or upward-sloping long-run Phillips curve. The model is used to address three questions: First, how does monetary policy affect labor-market contracts and thus employment? Second, what is the optimal monetary policy in the face of involuntary unemployment? Third, what weight should a central bank with a dual mandate put on each mandate?

The model used is a variant of the overlapping generations model with random-relocation (e.g., Schreft and Smith, 1997, 1998). Monetary policy has real effects because the model's risk-averse agents face two types of risk—liquidity risk (modeled as relocation shocks) and income risk (modeled as uncertainty about whether agents will find employment)—and because frictions exist in both labor and financial markets. Private information about an agent's effort on the job results in firms setting efficiency wages and hiring only a subset of available workers, leaving the rest involuntarily unemployed. Spatial separation, limited communication, and restrictions on asset portability prevent private credit markets from operating and, along with the risk of relocation, create a role for banks to provide insurance against liquidity shocks. These financial-market frictions are present in the standard random-relocation model, but here they interact with the parameters governing the frictions in the labor market and agents' opportunity cost of being employed. Agents can self-insure against income risk by exerting effort on the job, and asset returns affect agents' desire to do so. For example, if workers earn high returns from their savings, they have less incentive to insure themselves against the risk of unemployment by exerting labor effort.[1] The Phillips curve is the set of inflation rate-unemployment rate pairs that are steady states. The slope of the Phillips curve and the optimal monetary policy (modeled as a choice of an inflation target) depend on fundamentals of labor and financial markets, on agents' risk aversion, and on whether capital formation is possible.

Section 2 follows the New Keynesian models of the Phillips curve in presenting a version of the model that abstracts from capital formation. The model has three general features. First, varying any of the parameters that characterize the labor-market or financial-market frictions or affect their impact on unemployment, holding the others fixed, generates a family of long-run Phillips curves.[2] At a higher steady-state inflation rate, agents know that the return on their

---

[1] In contrast, in Reed and Waller (2006), agents have random endowments, which is an alternative form of income risk, and information frictions prevent them from contractually sharing that risk. Fiat currency, the only asset in the model, is shown to allow agents to self-insure against income risk.

[2] There are different possible interpretations of the time period to which the long-run Phillips curves in the model pertain. One interpretation relies on the fact that in a steady state, the economy's growth rate equals its potential growth rate. This translates to a period of about three years, as reflected by the Federal Open Market Committee's November 2007 decision to release its members' projections over a three-year horizon (Bernanke 2007). A second interpretation takes a period in the model to be about 20 years long, and the Phillips curves pertain to periods of similar length. A third interpretation is that the model captures the part of the human life cycle that occurs shortly before and after retirement. Under this interpretation, a period in model is five to 10 years, and that is the period to

assets will be lower; hence, they have greater incentive to put forth full effort on the job. Firms, in turn, can offer lower wages and still induce full effort, so the equilibrium unemployment rate is lower. Second, points along each Phillips curve can be ranked based on welfare, which is taken to be the expected utility of a representative agent. Third, the optimal monetary policy, defined as the inflation target that maximizes welfare, can be identified.

The model of Section 2 yields striking results. If agents are relatively risk averse (meaning more risk averse than with logarithmic utility), the long-run Phillips curves are always downward sloping.[3] The inflation target that maximizes welfare can vary dramatically across Phillips curves within a given family, being the target that achieves either full employment or the Friedman rule (the rate that drives the gross nominal interest rate to one), or some inflation rate in between. For a central bank with a dual mandate that requires balancing an inflation goal against an unemployment goal, the weight that should be given to each goal depends on the parameters for the frictions, the opportunity cost of employment, and the disutility of labor. Equal weight on each goal is not in general optimal. For example, if the disutility of labor is sufficiently high, the Friedman rule is always optimal.

If instead agents are less risk averse (meaning that they have log preferences), the Phillips curves are vertical at a natural rate of unemployment that depends only on real variables and the labor-market frictions but is independent of financial-market frictions. The Friedman rule is always optimal, in contrast to the case where agents are relatively risk averse. As a result, a central bank should put weight only on its inflation objective.

Comparing the optimal inflation target across Phillips curves in the Section 2 economy shows that, regardless of agents' risk aversion, agents are better off when the frictions are low or when agents have good alternatives to employment (that is, the opportunity cost of employment is high). A central bank, however, cannot control those factors.

Section 3 presents a variant of the model that allows for capital formation. If agents are relatively risk averse, the additional complexity of the model makes closed-form solutions

---

which the Phillips curves apply. Whatever the interpretation chosen, the Phillips curves in the economy of Section 2, which can be though of as having a fixed capital stock, should pertain to a shorter period of time than the curves in the Section 3 economy, with capital formation. In addition, monetary policy should have effects in the short run as well as the long run in this model.

[3] This finding is consistent with Fair (2000), Akerlof et al. (2000), and King and Watson (1994). The latter also shows that the Phillips curve's elasticity adjusts to changing economic circumstances, while Akerlof et al. finds that the tradeoff is strongest at lower inflation rates and disappears at sufficiently high inflation rates. DiNardo and Moore find a long-run tradeoff for nine OECD countries.

unobtainable. Numerical analysis indicates, however, that the long-run Phillips curves are upward sloping.[4] The welfare-maximizing inflation target is the lowest inflation rate achievable and for which an equilibrium exists, which means that a central bank with a dual mandate should put all its emphasis on its inflation goal. This inflation rate might be the one that achieves the Friedman rule, but it might not be. Along some Phillips curves, there is no steady-state equilibrium under the Friedman rule. Intermediate values of the parameters governing the frictions and the opportunity cost of employment are optimal as well because if those parameters are too low, then no steady-state equilibrium exists, and if they are too high, then the steady-state inflation rate is associated with a higher steady-state unemployment rate than the ones achievable if those parameters were lower. Alternatively, if agents have log preferences (i.e., are not too risk averse), then as in the model without capital formation, the Phillips curve is vertical at the natural rate of unemployment, and the Friedman rule is always optimal. However, with capital formation, the natural rate of unemployment depends on financial-market frictions as well as labor-market frictions.

In addition to these results, several lessons stand out from Sections 2 and 3. The first is that monetary policy influences the performance incentives (e.g., the wage rate, incentive pay) that firms offer through policy's effect on worker's financial-market returns and thus their incentive to exert full effort on the job. This is explicit in the model and not surprising. The decline in equity prices after the September 11, 2001, terrorist attacks in the United States drove some retired workers back into the labor market and caused others to postpone retirement. That type of impact on worker incentives reduces the wages firms must offer to induce work effort.

A second and related lesson is that monetary policy can have real effects even in the long run without any nominal rigidities or unanticipated inflation. This is consistent with some research indicating that most U.S. firms adjust prices fairly frequently, suggesting few nominal rigidities, at least for prices.[5] In addition, theoretical research suggests that with perfect foresight, there would be no inflation-unemployment trade-off; the Phillips curve would be vertical.[6]

A third lesson from the model is that parameters characterizing fundamental features of

[4] This is consistent with Friedman (1977), which finds evidence in cross-country data supporting an upward-sloping Phillips curve.
[5] Bils and Klenow (2004) find that half of prices are changed every 5.5 months or less. Dennis (2006) finds that about 60 percent of firms change prices each quarter, although most also use a rule-of-thumb in setting prices.
[6] See Lucas (1972) and Kydland and Prescott (1977).

labor and financial markets are critical to an economy's inflation-unemployment relationship, altering the position and slope of the long-run Phillips curves.[7] These same parameters, along with the disutility of labor effort, are critical to determining the welfare-maximizing inflation target. The implication is that the impact of a central bank's choice of a particular inflation target on employment is dependent on such parameters. Because these parameters are difficult to estimate, a central bank is challenged in choosing its inflation target, regardless of the formality with which such a target is specified. In particular, the Friedman rule is not necessarily optimal.[8] In some cases a low and positive inflation rate is optimal, and in other cases an even higher inflation rate is optimal.[9]

Fourth, the analysis when agents are not too risk averse (i.e., have log preferences) highlights features of vertical Phillips curves. First, the position of a Phillips curve that is vertical at the natural rate of unemployment depends on labor-market and financial-market fundamentals. Consequently, when parameters characterizing these frictions and fundamentals shift, the vertical Phillips curve also shifts. This is consistent with empirical evidence on the volatility of the natural rate of unemployment.[10] Second, the possibility of capital formation enhances the impact of financial-market frictions on the natural rate of unemployment.

A final striking lesson from the model is the pervasiveness of the effect that unemployment has on the economy. The model teaches that when the possibility of involuntary unemployment is added to a random-relocation model, aggregate income and aggregate deposits depend on the level of employment. The lower the unemployment rate, the more deposits banks have available to invest in the economy's assets. Therefore, the employment and unemployment rates, the wage rate, and the opportunity cost of employment, which jointly determine the volume of deposits, enter into banks' asset demand functions. In contrast, when capital is added to a simple random-relocation model, abstracting from any employment decisions, banks have an

---

[7] Similarly, although in a model abstracting from unemployment, Davig (2007) finds that changing costs of price adjustment change the slope of the Phillips curve, with lower costs making the curve steeper.

[8] Mishkin (2007b) summarizes the reasons why models have found that a zero or negative inflation rate is not optimal.

[9] These findings are consistent with existing research. Billi (2007) finds the optimal inflation rate to be positive but robustly below 1 percent. Akerlof et al. (1996) and Andersen (2002), for example, find that a positive and low inflation rate, something around 2 percent or 3 percent, is optimal. Cavalcanti and Villamil (2003) finds that in an economy with structural imperfections, such as the existence of an underground economy, the optimal inflation rate is between 0 percent and 22 percent.

[10] King and Morley (2007) finds that in post-war U.S. data, that the natural rate of unemployment, measured as the time-varying steady state of a structural vector autoregression, is quite volatile, and its movements can be related to variables associated with labor-market search, labor productivity, real wages, and sectoral shifts in the labor market.

additional asset (capital) in which they can invest and thus an additional first-order condition, but their demand functions for other assets are no more complex.[11]

This article is not the first to attempt to incorporate labor-market fundamentals into a model of the Phillips curve so that the inflation-unemployment relationship can be analyzed directly. Others who have attempted this include Blanchard and Gali (2007), Ravenna and Walsh (2007), Rocheteau, Rupert, and Wright (2006), and Andolfatto, Hendry, and Moran (2004). The first two rely on sticky prices, distinguishing them from the latter two and from this article.[12] The latter two, however, do not model performance incentives in the labor market and reach different, or no, conclusions about the optimal monetary policy. Rocheteau, Rupert, and Wright (2006) embeds indivisible labor and Rogerson (1988) employment lotteries into a Lagos-Wright (2005) model with the usual frictions to generate a monetary equilibrium and with preference shocks instead of random matching. The result is an equilibrium long-run Phillips curve that can be downward or upward sloping or vertical. The authors assume that buyers and sellers have the same bargaining power and find that the Friedman rule is always optimal, in contrast to this article. Andolfatto et al. (2004) develops a model with information frictions in financial markets (agents must make asset choices before monetary policy is known) and with search frictions in labor markets that generates a downward-sloping Phillips curve in the short run and an upward-sloping curve in the long run. The relationship arises from a monetary transmission mechanism that is most similar to the one in this article: an unanticipated monetary policy action affects interest rates, which affect firm liquidity and recruiting activity, and thereby employment. In addition, the change in interest rates alters workers' desired deposits, which affects liquidity further, although at the next date, leading to a second round of employment effects. Banks do not arise endogenously in Andolfatto et al., however, and optimal monetary policy is not analyzed, which distinguishes it from this article.

---

[11] Schreft and Smith (1997) model the latter. Agents are endowed with one unit of time that they supply inelastically to production.

[12] Blanchard and Gali (2007), for example, combine real wage rigidities in the labor market with Calvo staggered price setting in the goods market. Ravenna and Walsh (2007) model an economy with a wholesale and retail sector to generate a downward-sloping Phillips curve. Agents can gain employment with wholesale firms via a matching technology and face some exogenous probability of separation from those firms. Retail firms only sell goods produced by wholesalers, which requires no labor input, and are Calvo price-setters. Parameters characterizing workers' bargaining power and labor-market tightness affect the coefficients in the Phillips curve. The inclusion of sticky prices in Blanchard and Gali and Ravenna and Walsh models is interesting because Nason and Slotsve (2004) finds that a model with labor-market search and flexible prices matches actual price level movements in the data better than a sticky price model does.

2. A Model of the Phillips Curve

This section presents a two-asset overlapping generations model in which the presence of moral hazard and use of performance incentives in the labor market results in monetary policy having real effects and generates either a downward-sloping or vertical long-run Phillips curve, depending on agents' risk aversion, which affects the elasticity of the demand for money. The model follows the precedent set by most New Keynesian models of the Phillips curve in abstracting from capital formation.

2.1 The Environment

The economy exists at discrete dates $t = 1, 2, \ldots$ and consists of two separate geographic locations. Each location is populated by an infinite sequence of identical two-period-lived generations. Each new generation of young agents is of unit mass. Each location also has two sectors, known as the primary and secondary sectors, each with its own labor market, following Bulow and Summers (1986).

Agents born at $t \geq 1$ are each endowed with one unit of labor time when young, which they supply to the primary labor market unless they are unemployed. If they obtain employment in the primary labor market, they either exert full effort or shirk, exerting no effort. Their effort on the job, represented by $e_t \in \{0,1\}$, is private information as in Shapiro and Stiglitz (1984) and is the source of one friction in the economy. An agent's disutility from formal employment is given by $v > 0$, where v is assumed to be identical for all young agents and known to firms. Those young agents who are unemployed, having failed to obtain a job in the primary labor market, "work" in the secondary labor market, which can be thought of as home production.[13] Effort exerted in home production is costless.

Agents when old derive utility from consumption of the economy's single consumption good. The utility of a member of generation t is represented by the function

$$U\left(c_{t+1}, e_t\right) = \begin{Bmatrix} u\left(c_{t+1}\right) - ve_t & \text{if } e_t = 1 \\ u\left(c_{t+1}\right) & \text{if } e_t = 0 \end{Bmatrix}, \tag{1}$$

[13] Alternatively, the secondary sector can be thought of as the minimum-wage sector, and unemployment can be interpreted as the lack of employment in the primary sector.

where c_{t+1} denotes the agent's consumption. In particular, utility from consumption is:

$$u(c_{t+1}) = \begin{cases} \dfrac{c_{t+1}^{1-\theta}}{1-\theta} \text{ if } \theta > 1 \\ \ln c_{t+1} \text{ if } \theta = 1 \end{cases}. \tag{2}$$

Attention is restricted to values of $\theta \geq 1$, consistent with Schreft and Smith (1997, 1998) and empirical estimates of individuals' degree of risk aversion, which are typically around two.[14]

At the end of each period, a randomly selected fraction, $\pi \in (0,1)$, of young individuals learns that they will be forced to relocate to the other island at the beginning of the next period. The probability of relocation, π, is exogenous, publicly known, the same across both islands, and independent of a worker's employment status.

2.1.1 Employment and the Labor Market

In the primary labor market, firms have monitoring technologies that sometimes allow them to detect shirking behavior.[15] The conditional probability of a worker being caught shirking is q. Workers caught shirking are fired, while those not caught receive the real wage w_t from the firm. Unemployed workers supply their labor time to home production, for which they receive a "wage" of τ. Without loss of generality, τ is time invariant. It is also assumed to be positive and finite, which is necessary for wages in the primary market to be positive and finite, as will be apparent below. All unemployment is involuntary in that agents are ex ante identical, but only a fraction of them is employed in the primary sector and earns the higher wage rate w. See Bulow and Summers (1986).

In standard efficiency-wage models, a worker's effort is determined by (i) the severity of the information friction (i.e., the inefficiency of monitoring) and (ii) the disutility of labor effort. Here, in contrast, the financial returns from working also play a role. For firms to maximize profits, the wage rate must be set to induce workers to exert full effort. This occurs when the

[14] For example, using Italian data, Chiappori and Paiella (2006) compute a median estimate equal to 1.7. From the consumer expenditures survey, Mazzocco (2006) obtains similar findings.

[15] In subsequent sections, where environments with capital are considered, primary sector firms operate constant-returns-to-scale technologies and earn zero profit. However, in this section, where labor is the only explicit factor of production, firms can earn positive profits. To keep the model tractable, firms in this section are taken to be a second type of agent, each with a linear utility function, that consumes any profits earned. These agents will be ignored in the welfare analysis to follow; however, as will be shown, profits are higher when the expected utility of the two-period lived agents is higher, so monetary policies that maximize the latter's expected utility also would make these firms better off.

utility from working and exerting full effort is at least as great as the expected utility from working and shirking, each of which depends on the rate of return on deposits, R_t^d:[16]

$$U\left(R_t^d w_t\right) - v \geq (1-q)U\left(R_t^d w_t\right) + qU\left(R_t^d \tau\right). \tag{3}$$

This expression reduces to

$$q\left[U\left(R_t^d w_t\right) - U\left(R_t^d \tau\right)\right] = v \tag{4}$$

because profit maximization requires that (3) be satisfied with equality. The connection between the rate of return on deposits and labor-market performance parameters is apparent, and can be summarized as follows:[17]

Lemma 1: *The Nonshirking Wage Rate*

Given workers' preferences, as represented by (1), satisfaction of (4) yields the nonshirking wage:

$$w_t^{ns} = \left[\frac{\tau^{(\theta-1)}}{1-(\theta-1)\frac{v}{q}\left(R_t^d\right)^{(\theta-1)}\tau^{(\theta-1)}}\right]^{\frac{1}{\theta-1}}. \blacksquare \tag{5}$$

For a given rate of return on deposits and $\theta > 1$, the response of the nonshirking wage to labor-market conditions follows standard efficiency-wage models. If labor-market frictions, measured by the ratio v/q, are more severe, workers have less incentive to work and firms must pay a higher wage to prevent shirking. These frictions are higher when the disutility of labor effort (v) is higher and/or when the technology for monitoring workers is less efficient (q lower). The nonshirking wage is also higher when the alternative to work in the primary labor market, measured by τ, the secondary market wage, is higher.

Interestingly, in contrast to standard efficiency-wage models, financial-market conditions and the degree of risk aversion also affect the nonshirking wage. If agents are sufficiently risk

[16] Although the nonshirking wage provides conditions under which workers will not shirk, it does not guarantee that workers will choose to work instead of being unemployed (employment in the secondary sector). Workers participate in the primary labor market if $U\left(R_{t+1}^d w_t\right) - v \geq U\left(R_{t+1}^d \tau\right)$ is satisfied. This participation constraint always holds because $v(1-q)/q \geq 0$.

[17] Campbell (1993) presents empirical evidence supporting firms' payment of efficiency wages.

averse $(\theta > 1)$, a higher rate of return to deposits lowers the cost of shirking. Those who obtain a job in the primary sector can self-insure against the risk of job and income loss by exerting effort on the job. However, asset returns affect the desire to self-insure. For example, when returns to saving are higher, agents have less need to insure against risky labor income, so firms must pay workers more to elicit labor effort. Thus, the model has a novel channel or transmission mechanism through which monetary policy operates to have real effects, as discussed further below.

2.1.2 Production and the Goods Market

Production of the economy's single consumption good occurs in both sectors. In the secondary sector, output is simply τ. In the primary sector, the good is produced by firms that have access to a technology that converts effective labor, $(1-\psi)L_t$, into output, Y_t, of the consumption good according to the function

$$Y_t = F\left((1-\psi)L_t\right) = A\left((1-\psi)L_t\right)^{1-\eta}, \tag{6}$$

where ψ is the fraction of employees who exert no effort (shirk) and L is the total number of employees. A is a productivity factor, which can be interpreted as incorporating the contribution of an exogenous and nondepreciating fixed stock of capital.[18] Satisfaction of (5) implies that $\psi = 0$.

The population of firms is of mass one. Therefore, the demand for labor by a representative firm is the same as the market demand. The number of workers hired comes from the implicit demand function for labor:

$$w_t = A(1-\eta)(1-\psi)^{1-\eta}(L_t)^{1-\eta}. \tag{7}$$

In turn, the unemployment rate is given by

$$\mu_t \equiv 1 - L_t. \tag{8}$$

2.1.3 Assets and the Banking Sector

There are two primary assets in this simple economy: money (fiat currency) and a linear storage technology. The real rate of return to the storage technology is constant and given by R;

[18] See, for example, Gray (1976).

in nominal terms, it is represented by I_t. The aggregate nominal stock of money in each location at date *t* is given by M_t. The monetary authority selects a gross inflation target of σ and achieves that target by allowing the money supply to grow at the fixed rate of σ; hence, $M_t = \sigma M_{t-1}$. The price level at *t* is p_t, and $m_t = M_t / p_t$ is the real money stock. The initial money supply held by each old agent of the initial generation is $M_0 > 0$.

Two additional features of the environment give rise to trading frictions. First, only currency can be transported across locations. Second, agents cannot communicate across islands. These features prevent private credit markets from operating and, along with the risk of relocation, give rise to a transactions demand for money in that agents demand a liquid asset that they can take with them when they move, even if money is dominated in rate of return. Because the fraction of agents who relocate is known even though the identity of who must move is not, a role for banks arises to provide insurance against the risk of relocation, as in Diamond and Dybvig (1983). Banks hold the economy's assets directly, with young agents depositing all of their earnings, whether from work in the primary sector or home production.

In contrast to standard random-relocation models (e.g., Schreft and Smith 1997, 1998), deposits in this model vary across agents because some are employed in the primary sector while the others, unemployed, engage in home production. Employed agents deposit $w_t^{ns} L_t$, and unemployed agents deposit $\tau\left(1 - L_t\right)$. Agents' employment status when young can be shown to be immaterial for a bank's allocation of their deposits across assets. Banks acquire money balances of m_t^d and invest i_t^d in the storage technology. As a result, the bank's balance sheet condition is:

$$m_t^d + i_t^d \leq w_t^{ns} L_t + \tau\left(1 - L_t\right), \quad t \geq 1. \tag{9}$$

There is free entry into the banking sector, and banks are Nash competitors in the market for deposits, announcing rate of return schedules for deposits and taking the announced schedules of other banks as given. Banks thus announce deposit-return schedules $\left(r_t^m, r_t^n\right)$, where r_t^m and r_t^n are the rates of return paid to relocated agents, "movers," and to "nonmovers," respectively, and

$$R_t^d \equiv \pi r_t^m + \left(1 - \pi\right) r_t^n$$

is the rate of return on deposits.

Profit-maximization requires that banks maximize the expected utility of a representative depositor. That is, $\left(r_t^m, r_t^n\right)$ is chosen to maximize

$$L_t\left[\frac{\pi}{1-\theta}\left(r_t^m w_t^{ns}\right)^{1-\theta}+\frac{1-\pi}{1-\theta}\left(r_t^n w_t^{ns}\right)^{1-\theta}\right]-vL_t+\left(1-L_t\right)\left[\frac{\pi}{1-\theta}\left(r_t^m \tau\right)^{1-\theta}+\frac{1-\pi}{1-\theta}\left(r_t^n \tau\right)^{1-\theta}\right], \quad (10)$$

given the preferences specified in (2) for $\theta>1$ and the fact that a fraction L_t of agents is employed and a fraction $1-L_t$ is unemployed. The deposit-return schedule must satisfy two conditions in addition to the bank's balance sheet constraint. First, a bank's currency holdings must be sufficient to meet the demand for money by the fraction π of agents that must move:

$$\pi r_t^m \leq \left(\frac{m_t^d}{w_t^{ns} L_t+\tau\left(1-L_t\right)}\right)\left(\frac{p_t}{p_{t+1}}\right), \quad (11)$$

where p_t / p_{t+1} is the rate of return on real balances between t and t +1. Second, because the focus in analyzing this economy is on equilibria in which money is dominated in rate of return, which requires $I_t>1$ for all t, banks hold only enough currency to satisfy the withdrawal demand of movers, which implies that (11) is satisfied with equality and that investment in storage on behalf of nonmovers satisfies

$$(1-\pi) r_t^n \leq\left(\frac{i_t^d}{w_t^{ns} L_t+\tau\left(1-L_t\right)}\right) R. \quad (12)$$

Since bank profit-maximization requires that (9) be satisfied at equality, constraints (11) and (12) can be rewritten as

$$r_t^m \leq \frac{\gamma_t p_t}{\pi p_{t+1}}, \quad (13)$$

and

$$r_t^n \leq \frac{\left(1-\gamma_t\right)}{1-\pi} R, \quad (14)$$

where $\gamma_t \equiv m_t^d / \left(w_t^{ns} L_t+\tau\left(1-L_t\right)\right)$ is the bank's reserve-deposit ratio and $1-\gamma_t$ is its storage investment-deposit ratio. Since holdings of currency and storage must be nonnegative, $\gamma_t \geq 0$.

It follows that the solution to the bank's optimization problem is

$$\gamma_t \equiv \gamma(I_t) = \frac{1}{1+\left(\frac{1-\pi}{\pi}\right)(I_t)^{\frac{1-\theta}{\theta}}}, \quad \forall I \geq 1. \tag{15}$$

When $I_t = 1$, which occurs under the Friedman rule, γ is indeterminate because there is no opportunity cost of holding currency, leaving banks indifferent between currency and storage investments. Hereafter, although $\gamma_t \geq \pi$ when $I_t = 1$, $\gamma_t = \pi$ is taken to be the demand for real balances in that case.

Given $\gamma(I_t)$, the rate of return on deposits is

$$R_t^d = \left(1-\gamma(I_t)\right)R + \gamma(I_t)\frac{p_t}{p_{t+1}}. \tag{16}$$

With $\theta > 1$, $\partial\gamma(I)/\partial I > 0$, indicating that the income effect from a change in the nominal interest rate on the demand for real balances dominates the substitution effect. When $\theta = 1$, money demand is perfectly inelastic. Both cases will be considered below.

2.1.4 The Timing of Economic Activity

The features of the economy described above are consistent with the following timing of events each period. At the beginning of each date, a new generation of workers is born. The central bank injects currency to implement its chosen monetary policy, and the return to storage investments made the previous period is realized. Old agents who did not relocate withdraw their deposits. Next, firms choose the number of workers to hire and a wage rate to attract the desired number of workers and ensure full effort from each. Young agents either receive job offers at the going wage or are involuntarily unemployed and involved in home production. Production occurs in the primary sector and within the homes of the unemployed, and then the goods market operates. Old agents who relocated trade their currency holdings for goods, and all old agents then consume. Young workers receive their earnings in the form of goods, all of which they deposit. All young agents, regardless of their employment status, learn whether they must relocate. Those who must move withdraw their deposits in the form of currency before moving.

2.2 Equilibrium with $\theta > 1$: A Phillips Curve Tradeoff

In analyzing equilibria of this economy, attention is restricted to steady states.

With an inflation-targeting central bank conducting monetary policy according to the fixed-money-growth-rate rule specified in section 2.4, in a steady state the real rate of return on currency, p_t / p_{t+1}, equals $1/\sigma$, and $I = R\sigma$. It follows that monetary policy affects the steady-state rate of return on deposits, as shown in (16), in two ways. First, a higher inflation rate lowers money's rate of return, $1/\sigma$, which directly lowers the return to deposits given the bank's demand for money. Second, higher inflation drives up the nominal interest rate. With $\theta > 1$, agents are relatively risk averse (more so than with logarithmic preferences), so the income effect of a change in the nominal interest rate dominates the substitution effect. Consequently, higher inflation causes banks to partially insure movers against inflation's effects by holding more real balances. Because money is dominated in rate of return, this results in a lower rate of return on deposits and thus a lower rate of return to nonmovers.

2.2.1 Steady States

To obtain a closed-form solution for the steady state, $\theta = 2$ is assumed. With this assumption, the steady-state expected rate of return on deposits is

$$R^{d*} = \left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right), \tag{17}$$

and the nonshirking wage is

$$w^{ns} = \left(\frac{\tau}{1 - \frac{v}{q} R^d \tau} \right). \tag{18}$$

Substituting the expected return to deposits into (18) leads to the following lemma:

Lemma 2: ***The Steady-state Nonshirking Wage Rate***

If the disutility of effort (v) is sufficiently low or the probability of detecting shirking (q) is sufficiently high, then the nonshirking wage rate is finite and given by

$$w^{ns*} = \left[\frac{\tau}{1 - \tau \frac{v}{q} \left(\left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right) \right)} \right]. \tag{19}$$

Proof. For the wage rate to be positive and finite, the denominator of (19) must be positive. Therefore, it must be the case that

$$\frac{q}{v} > \tau \left(\left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right) \right). \blacksquare$$

In this economy, the nonshirking wage is decreasing in the economy's inflation rate. As the inflation rate rises, the expected return to deposits falls, which reduces the return to labor effort. To offset this loss, agents must be willing to put forth full labor effort. The resulting impact on employment is provided in the following proposition:

Proposition 1: ***Existence of a Long-run Phillips Curve Trade-off***

If the disutility of labor effort (v) is sufficiently low or the probability of detecting shirking (q) is sufficiently high, so that the nonshirking wage (w^{ns} *) is positive and finite, then a steady state with equilibrium unemployment exists and is unique. Steady-state employment is given by*

$$L^* = \left(\frac{(1-\eta) A}{\tau} \right)^{\frac{1}{\eta}} \left[1 - \tau \frac{v}{q} \left(\left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right) \right) \right]^{\frac{1}{\eta}}. \tag{20}$$

From Proposition 1, it follows that $\partial\mu/\partial\sigma \equiv \partial(1-L_t)/\partial\sigma < 0$: the Phillips curve is always downward sloping.$\blacksquare$

Because at higher inflation rates the rate of return to deposits (saving) is lower, agents have more incentive to work at such inflation rates and equilibrium employment is higher. This is the case even though firms pay lower wages, giving workers less incentive to perform. With less employment, output is lower. This is the mechanism that generates a steady-state inflation-unemployment tradeoff and real effects of monetary policy.

Two employment levels will be of particular interest in analyzing the welfare-maximizing monetary policy. Full employment (that is, L^* = 1) requires, from (20), that

$$\frac{v}{q}=\left[\frac{A(1-\eta)-\tau}{A(1-\eta)\tau}\right]\left[\frac{\sqrt[2]{\sigma R}+\frac{1-\pi}{\pi}}{\left(\frac{1-\pi}{\pi}\right)R+\sqrt[2]{\frac{R}{\sigma}}}\right]. \tag{21}$$

At higher values of *v/q* and τ , the inflation target that achieves full employment is also higher. For π < 0.5, as π increases, the full-employment inflation rate is higher. For π > 0.5, the opposite is the case.

When inflation is set to achieve the Friedman rule (that is, $\sigma = 1/R$),

$$L^* = \left(\frac{A(1-\eta)}{\tau}\right)^{\frac{1}{\eta}}\left(1-\tau\frac{v}{q}R\right)^{\frac{1}{\eta}}. \tag{22}$$

At the Friedman rule, agents are fully insured against liquidity risk; hence, the financial-market friction does not affect agents' incentives on the job or unemployment. However, higher *v/q* and τ are associated with higher unemployment.

2.2.2 Optimal Monetary Policy

Closed form solutions cannot be obtained for the optimal inflation target because welfare is sensitive to the parameters reflecting the labor and financial-market frictions and corner solutions abound. As in standard random-relocation models, the gross nominal interest rate must exceed one in any equilibrium because the demand for real balances is indeterminate otherwise. In addition, with the possibility of unemployment, as here, and a population of young agents of mass 1, employment must be between zero and one in equilibrium. Consequently, the optimal monetary policy is identified numerically instead.

For the numerical analysis, $\theta = 2$, $\eta = 0.3$, and R = 1.02 are used, consistent with U.S. data or empirical estimates. As a baseline, $\pi = 0.5$, $\tau = 0.5$, and $v/q = 1.0$. The Phillips curves, consisting of inflation rate-unemployment rate combinations that are steady states, depend on the ratio *v/q*, not on *v* or *q* independently. Welfare, however, measured as the expected utility of a representative agent, must be calculated to identify the optimal monetary policy along and across the Phillips curves.[19] It does depend on *v* through the disutility of labor effort, as indicated in (1) . The welfare analysis is discussed for two representative values of *v*: 0.001 and 0.5. *A*, generally calculated as a residual (Abel and Bernanke, 1998), is treated as a residual here too and set to 1.25 so there exists an equilibrium with a reasonable unemployment rate at an inflation rate (target) of 2 percent (that is, $\sigma = 1.02$). Because agents are either employed or unemployed in the model—no one is out of the labor force—the unemployment rate in the model is the equivalent of the measured share of the civilian population that is unemployed plus the share not in the labor force. Thus, a reasonable equilibrium unemployment rate is 36 percent, given a labor force participation rate of about 64 percent in the data.[20]

For these parameterizations, families of Phillips curves are graphed as functions of the labor-market friction parameters (τ and *v/q*) and the financial-market friction parameter (π), given the disutility of labor (*v*). For each Phillips curve, welfare is calculated for each point along the curve, and the welfare-maximizing combination is identified. Figures 1 through 6 show the steady states and optimal monetary policies for *v* = 0.001, while Figures 7 through 12 do so for *v* = 0.5. As is discussed below, an equilibrium with full employment, at the Friedman rule, or somewhere in between, can be optimal, implying a corresponding optimal monetary policy.[21]

Figure 1 displays part of the family of Phillips curves obtained by varying the opportunity

---

[19] This concept of the optimal monetary policy is consistent with the Federal Reserve's (Mishkin 2007b).

[20] The assumed values for σ, π, *R*, and θ with *v* = 0.001 are also an equilibrium for a 5 percent unemployment rate, with *A* = 1.405. The article's qualitative conclusions do not change if this value of *A* is used in the numerical analysis.

[21] As discussed in footnote 15, firm profits can be positive in this version of the model. In measuring welfare as the expected utility of a representative agent, meaning a two-period lived agent, the expected utility of the one-period-lived firms is ignored. However, higher inflation always results in higher profits and higher utility for the firms. As a result, any weight put on the firms' expected utility in determining the optimal monetary policy would reinforce the optimality of the full-employment equilibrium when that equilibrium maximizes the expected utility of a representative agent. In contrast, when an equilibrium with a relatively low inflation rate maximizes the expected utility of a representative agent, such as when the Friedman rule is optimal, the welfare of firms is relatively low, offsetting to some extent the optimality of that equilibrium. Therefore, considering the welfare of the firms makes the Friedman rule less likely to be an optimum than the analysis here indicates.

cost of employment (the secondary wage rate, τ), given a low disutility of labor. For the parameters studied, steady states do not exist for τ < 0.47 because the opportunity cost at that point is so low that L > 1 and μ < 0 is required for a steady state to exist, which is not feasible. For $\tau \in [0.47, 0.502]$, the optimal inflation rate achieves full employment. At τ = 0.5, for example, an inflation rate of 39 percent maximizes the expected utility of a representative agent. For $\tau \in [0.503, 0.507]$, the optimal inflation rate lies between the full-employment rate and the Friedman Rule. For $\tau \in [0.508, 0.98]$, the Friedman rule is optimal, with the highest unemployment rate achievable. When τ is in that range, unemployment is a relatively desirable state for many agents. For $\tau > 0.98$, the alternative to working in the primary sector is so lucrative that unemployment is too desirable—the unemployment rate that solves the model is negative, which is not feasible—so no steady state exists.

With the optimal inflation rate so dependent on the opportunity cost of employment, a natural question to ask is how expected utility compares across Phillips curves. Figure 2 graphs expected utility under the optimal monetary policy for each Phillips curve in Figure 1 (for each τ). Expected utility is highly nonlinear in τ, depending on the fraction of agents working versus not working and the marginal utility from employment versus unemployment, and each component moves in opposite directions. Hence, expected utility under the optimal monetary policy is convex, with $\tau = 0.98$ maximizing welfare across values of τ, as shown in Figure 2.

What values of the opportunity cost of employment are most likely to be observed? τ can be thought of as the value of home production (for example, the cost that a worker not employed in the primary market saves by doing housekeeping rather than paying a housekeeper and perhaps of caring for children rather than paying a nanny), the income earned at a minimum-wage job, or unemployment compensation. This suggests that lower values of τ are more likely to be observed than higher ones, which in turn suggests that the full-employment monetary policy is more likely to be the optimal monetary policy than the Friedman rule.

Figure 3 shows the family of Phillips curves that arises as labor-market frictions associated with the disutility of labor and the effectiveness of firm monitoring of worker effort, represented by the ratio v/q, vary. For v/q < 0.85, no steady state exists with a nonnegative unemployment rate. For $v/q \in [0.85, 1.05]$, full- or near-full employment is optimal. For

$v/q \in [1.06, 1.145]$, the optimal steady state has less than full employment and inflation higher than the Friedman rule. As *v/q* increases beyond 1.145, the Friedman Rule is always optimal.

Not surprisingly, welfare is higher when the labor-market frictions represented by *v/q* are lower, as Figure 4 shows. Agents prefer full employment and the low inflation rate available when *v/q* = 0.85, which also allows the nominal interest rate to be fairly low, limiting the welfare cost of having to relocate, over all other steady states, even the one where the Friedman rule is followed and financial frictions are neutralized.

It is tempting to conclude that better firm monitoring, which lowers *v/q* given *v*, is desirable. However, in reality, *v* rises when *q* rises: more intense and accurate firm monitoring lowers job satisfaction, which equates to greater disutility of labor. See, for example, Ottensmeyer and Heroux (1991) and Wood (1998).

Figures 5 and 6 display the last set of numerical results for this economy with *v* = 0.001. Figure 5 shows how the Phillips curves vary with the financial-market friction, π. For $\pi \geq 0.5$, the full-employment steady state is optimal. The higher π is in that range, the lower the inflation rate associated with that equilibrium. For $\pi < 0.5$, the unemployment rate rises at the welfare-maximizing steady state. Initially, the inflation rate also rises, but eventually it declines with reductions in π. As π approaches zero, the optimal monetary policy approaches the Friedman rule.[22]

Figure 6 displays expected utility under the optimal monetary policy as π varies. Welfare is maximized at $\pi = 1.0$, meaning that all agents must relocate at the end of their youth. Welfare is minimized as $\pi \to 0.0$, where no agents relocate. Interestingly, the steady state achieved when $\pi = 1.0$ is much like that as $\pi \approx 0.0$ in that there is virtually no uncertainty about relocation, or equivalently, there are no liquidity shocks. Banks' portfolios are very different across the two steady states, however. When $\pi = 1.0$, banks hold all their assets in the form of currency, whereas at $\pi \approx 0.0$, hardly anyone relocates, so banks invest almost all of their assets in storage. In the latter case, the inflation rate has almost no effect on the rate of return on deposits. The welfare gain from achieving the Friedman rule when $\pi \approx 0.0$ is insufficient to offset the welfare cost of the high unemployment rate that must obtain for the Friedman rule to be the steady-state inflation rate.

---

[22] At $\pi = 0$, the wage rate is undefined.

Figures 7 through 12 show the steady states and optimal monetary policies when the disutility of labor effort is relatively high (that is, at $v = 0.5$). The Phillips curves shown in Figures 1, 3, and 5 are identical to those in Figures 7, 9, and 11, respectively: v independent of q does not affect the inflation-unemployment relationship. Only expected utility differs at the higher value of v, and differs sufficiently that the Friedman rule is the optimal policy along each Phillips curve in Figures 7, 9, and 11, and a central bank should put full weight on its inflation objective. Intuitively, as the inflation rate rises, the unemployment rate falls, so employment is higher. With a higher v, the disutility from labor effort of the representative agent is lower, even though the utility from consumption is not altered. As a result, welfare is lower at higher inflation rates for higher v than for lower v, other things equal.

Figures 8, 10, and 12 show expected utility under the optimal monetary policy across the Phillips curves in Figures 7, 9, and 11, respectively. The values of τ and v/q that maximize expected utility are the same regardless of the value of v, as seen by comparing Figure 8 to Figure 2 and Figure 10 to Figure 4. That is not the case for π, however. Expected utility is highest when $\pi = 1.0$ when the disutility of labor effort is low ($v = 0.001$, Figure 6) but is independent of π when the disutility of labor effort is high ($v = 0.5$, Figure 12). When work is sufficiently unpleasant that the welfare-maximizing steady state is always at the Friedman rule, as is the case in Figure 11, the gross nominal interest rate is 1.0, which neutralizes the impact of the financial-market friction, π, whatever its value. Hence, welfare is the same at the Friedman rule regardless of the value of π.

It is not clear what values are reasonable for the disutility from labor effort. The true value could vary by job (e.g., v might be lower for empowered workers in pleasant work environments), based on national culture (e.g., differences between American workers and their European counterparts), or over time for a variety of reasons (e.g., a worker's age, trends in employer-provided benefits). There seems little way for a central bank to know the true value of v, yet the value matters if a central bank is to adopt the optimal inflation target. A central bank that knows τ, v/q, and π still might not know v and therefore would not know how to conduct monetary policy to maximize welfare.

2.2.3 Discussion

The numerical analysis of the optimal policy is striking because of the extent to which labor-market and financial-market frictions affect the optimal inflation target. The optimal monetary policy can be the inflation target that achieves full employment or the Friedman rule or some target in between. The greater the disutility of labor, the more likely it is that the Friedman rule is optimal.

One implication of these results is that an inflation target chosen without good estimates of the magnitude of the frictions, the opportunity cost of employment, or the disutility from labor could achieve a steady state far from the one that maximizes welfare, contrary to a central bank's intentions. In addition, the weight that a central bank with a dual mandate puts on inflation versus unemployment depends on the labor-market and financial-market frictions present. When the optimal policy is to achieve full employment, the central bank should emphasize its employment goal, but when the Friedman rule is the optimal monetary policy, all weight should be put on price stability. When optimal monetary policy involves an inflation target somewhere between the full-employment rate and the Friedman rule, the central bank must balance its dual goals.

2.3 Equilibrium with $\theta = 1$: A Vertical Phillips Curve

When $\theta = 1$, $\gamma = \pi$ from (15): money demand is perfectly inelastic, so the income and substitution effects of a change in the nominal interest rate offset each other. In addition, the nonshirking wage rate only depends on the severity of labor-market frictions:

$$w^{ns*} = \tau e^{\frac{v}{q}}. \tag{23}$$

This implies that agents' incentives in the labor market do not depend on financial returns. The same is true for the Phillips curve, which is vertical at a "natural" rate of unemployment that depends only on real factors:

$$\mu^* \equiv 1 - L^* = 1 - \left(\frac{\left(A(1-\eta)\right)}{\tau e^{\frac{v}{q}}} \right)^{\frac{1}{\eta}}. \tag{24}$$

With a vertical Phillips curve, the Friedman rule is always optimal, and a central bank should focus only on its price-stability goal, as the following proposition shows.

Proposition 2: ***Optimality of the Friedman Rule When $\theta = 1$.***

When $\theta = 1$, welfare is maximized when $\sigma = 1/R$.

Proof. With $\theta = 1$, $\gamma^* = \pi$, $r^{m*} = 1/\sigma$, and $r^{n*} = R$. Expected utility of the representative agent is

$$\left[L^* \ln\left(w^*\right) + \left(1 - L^*\right)\ln\left(\tau\right)\right]\left[\pi \ln\left(r^{m*}\right) + \left(1-\pi\right)\ln\left(r^{n*}\right)\right] - vL^*. \tag{25}$$

The first bracketed term is independent of the money growth rate. The second bracketed term is $\pi \ln\left(1/\sigma\right) + \left(1-\pi\right)\ln\left(R\right)$, which is maximized when the central bank sets $\sigma = 1/R$, which drives I to one.■

3. A Model of the Phillips Curve with Capital Accumulation

This section extends the analysis of Section 2 to a neoclassical growth framework in which physical capital replaces storage as the economy's real asset. As in standard monetary models such as Stockman (1981), the economy exhibits a reverse Tobin effect. However, anticipated inflation also has an impact on investment through the labor-market effects of unemployment.

3.1 The Environment

The environment is the same as in Section 2, except that in the primary sector firms have access to a constant-returns technology that combines capital, K_t, with effective labor, $\left(1-\psi\right)L_t$, to produce output:

$$Y_t = F\left(K_t, \left(1-\psi\right)L_t\right).$$

For simplicity, capital completely depreciates at the end of the period. If $k_t \equiv K_t / \left(\left(1-\psi\right)L_t\right)$ denotes the capital-*primary-sector* labor ratio (hereafter, the "capital-labor ratio"), then

$$f\left(k_t\right) \equiv A\left(k_t\right)^{\eta}, \quad A > 0, \ \eta \in \left(0,1\right). \tag{26}$$

In addition to choosing wages and employment to maximize profits, as described previously, firms also choose the amount of capital to rent. The rental decision implies that the gross real return to capital satisfies

$$R_t = f'\left(k_t\right) = \eta A k_t^{\eta-1}, \tag{27}$$

and the demand for labor is given by

$$w_t = w(k_t) \equiv f(k_t) - k_t f'(k_t) = (1-\eta) A k_t^{\eta} = (1-\eta) A \left(\frac{K_t}{L_t} \right)^{\eta}. \tag{28}$$

In all other respects, the economy is as in Section 2.

3.2 Equilibrium with $\theta > 1$: An Upward-sloping Phillips Curve

In a steady-state equilibrium,

$$R = \eta A k^{\eta - 1}, \tag{29}$$

which provides the demand for capital:

$$k = \left(\frac{\eta A}{R} \right)^{\frac{1}{1-\eta}}. \tag{30}$$

As a result, the wage rate as a function of the rental rate on capital is

$$w(R) = (1-\eta) A \left(\frac{\eta A}{R} \right)^{\frac{\eta}{1-\eta}}. \tag{31}$$

Higher rental rates result in less capital being used per worker and thus a lower marginal product of labor in the primary sector and a lower wage rate.

From the analysis in Section 2, the nonshirking wage is

$$w^{ns}(R,\sigma) = \left(\frac{\tau}{1 - \tau \frac{v}{q} \left(\left(\frac{1}{\sqrt[2]{\sigma R} + \frac{1-\pi}{\pi}} \right) \left(\left(\frac{1-\pi}{\pi} \right) R + \sqrt[2]{\frac{R}{\sigma}} \right) \right)} \right). \tag{32}$$

In a steady-state equilibrium for this economy, R^* is the unique value of R that equates (31) and (32), and from (30), pins down the steady-state capital-labor ratio, k^*.

As in Section 2, the bank's balance-sheet constraint must hold with equality. In turn, this implies that steady-state investment satisfies

$$i^* = (1-\gamma^*)\left(w^* L^* + \tau (1 - L^*) \right) \tag{33}$$

and that the demand for real balances satisfies

$$m^* = \gamma^* \left(w^* L^* + \tau \left(1 - L^* \right) \right).$$

Total investment, i^*, is equal to the capital stock, K^*, in the steady state:

$$K^* = L^* k^* \left(R^* \right) = \left(1 - \gamma^* \right) \left(w^* L^* + \tau \left(1 - L^* \right) \right). \tag{34}$$

Given the economy's steady-state capital-labor ratio, $k^* \left(R^* \right)$, steady-state employment is:

$$L^* \left(R^*, \sigma \right) = \frac{\tau \left(1 - \gamma \left(R^*, \sigma \right) \right)}{k^* \left(R^* \right) - \left(1 - \gamma \left(R^*, \sigma \right) \right) \left(w^* \left(R^*, \sigma \right) - \tau \right)}. \tag{35}$$

From the foregoing, the effect of capital accumulation on the economy can be analyzed. As in Section 2, monetary policy affects performance incentives. At higher inflation rates, workers bear a higher cost of shirking, so wages are lower and employment is higher. However, at higher inflation rates, the nominal interest rate is higher, and with $\theta > 1$, banks hold more currency and invest less in capital formation. This is a reverse-Tobin effect. Since capital and labor are complements, less capital accumulation is accompanied by less employment and a higher unemployment rate. The net impact on employment depends on which effect dominates. Although closed-form solutions cannot be obtained, it is expected that the second effect dominates when $\theta > 1$, resulting in Phillips curves that are upward sloping.

Figures 13 through 18 present the results from numerical analysis of this economy, assuming $\theta = 2$ and η = 0.335, the latter approximately the same as its value in the analysis of Section 2. As a baseline, $\pi = 0.5$, $\tau = 0.5$, and $v/q = 1.0$ are again used. Although steady states only depend on the ratio of *v* to *q*, and expected utility also depends on *v* independent of *q*, the analysis is only presented for *v* = 0.001, for reasons presented below. *R* is now endogenous. The analysis is extremely sensitive to changes in *A*, which is set at 0.725.

Figures 13, 15, and 17 show the Phillips curve families as τ, *v/q*, and π, respectively, are varied. All Phillips curves are upward-sloping, with the slope increasing at an increasing rate as the unemployment rate rises. The curves shift to the right as the opportunity cost of work and the friction parameters increase.

The analysis of the optimal monetary policy is the same qualitatively whether the impact of τ, *v/q*, or π is studied because of the model's corner solutions and the slope of the Phillips curves. In each case, for sufficiently small values of each parameter, given the baseline values of the others, the Friedman rule cannot be implemented in a steady state because it would require the unemployment rate to be negative. Welfare maximization instead requires a central bank to set its inflation target to the lowest inflation rate for which a steady state exists (Figures 13, 15, and 17). This achieves the lowest equilibrium unemployment rate. Consequently, the qualitative results regarding the optimal monetary policy do not depend qualitatively on whether the disutility of labor (v) is low or high, so only the results for v = 0.001 are presented.

The critical values for the parameters are 0.2 for τ, 1.0 for v/q, and 0.5 for π (Figures 14, 16, and 18, respectively). The Friedman rule is optimal for values at and above the critical values (Figures 13, 15, and 17, respectively), at which point I = 1.0 and the impact of the financial-market friction is neutralized.[23] However, unemployment continues to increase in τ, v/q, and π for values above the critical levels. Intuitively, any steady state in which the Friedman rule is implemented equates the intertemporal marginal rates of substitution across movers and nonmovers, who are ex ante identical. However, the Friedman rule does not eliminate the existence of the financial-market friction; a fraction π of the population still must relocate after they finish working, and agents' awareness of this, but not of which individuals must relocate, affects their incentives to work. Agents know that banks will hold more currency and invest less in capital formation the higher is π. Because labor and capital are complements, less capital formation implies that fewer agents will be employed in equilibrium and that less output will be available to consume when old. Thus, even though agents' intertemporal marginal rates of substitution are equated in all equilibria in which the Friedman rule is achieved, unemployment is higher and welfare lower when either τ, v/q, or π is higher, given baseline values of the others. Only at the critical levels of τ, v/q, and π are financial-market frictions neutralized and income risk minimized.

For all values of τ, v/q, and π studied, a central bank with a dual mandate should focus only on achieving its inflation goal. "Price stability" should be defined as the lowest inflation rate consistent with the existence of a steady-state equilibrium. That might not be consistent with the Friedman rule, but it achieves the lowest equilibrium unemployment rate.

3.3 Equilibrium with $\theta = 1$: A Vertical Phillips Curve

As in the economy without capital, when $\theta = 1$, the demand for real balances is perfectly

---

[23] In the analysis of Section 2, with a fixed capital stock, the Friedman rule requires setting σ = 1/R to achieve I = 1.0. With capital formation, a central bank must set σ > 1/R to achieve I = 1.0 if I = 1.0 is a steady state equilibrium.

inelastic and equal to π, independent of the rates of return on any assets. Consequently, the nonshirking wage rate only depends on the severity of labor-market frictions:

$$w^{ns} = \tau e^{\frac{v}{q}}, \tag{36}$$

and the wage condition is

$$w(R) = (1-\eta) A \left(\frac{\eta A}{R} \right)^{\frac{\eta}{1-\eta}}. \tag{37}$$

Because $w^{ns} = w(R)$ in a steady state, the steady-state real return to capital is

$$R^* = \frac{\eta}{\left(\tau e^{\frac{v}{q}} \right)^{\frac{1-\eta}{\eta}} (1-\eta)^{\frac{1-\eta}{\eta}} A^{\frac{1-2\eta}{\eta}}}, \tag{38}$$

and the equilibrium capital-labor ratio is

$$k^* = \left(\frac{\tau e^{\frac{v}{q}}}{(1-\eta) A} \right)^{\frac{1}{\eta}}. \tag{39}$$

As in the economy without capital formation, the Phillips curve is vertical at the natural rate of unemployment:

$$\mu^* \equiv 1 - L^* = \frac{\frac{1}{\tau}\left(\frac{\tau e^{\frac{v}{q}}}{(1-\eta) A} \right)^{\frac{1}{\eta}} - (1-\pi) e^{\frac{v}{q}} + (1-2\pi)}{\frac{1}{\tau}\left(\frac{\tau e^{\frac{v}{q}}}{(1-\eta) A} \right)^{\frac{1}{\eta}} - (1-\pi)\left(e^{\frac{v}{q}} - 1 \right)}. \tag{40}$$

This natural rate of unemployment is higher if labor-market frictions are more severe (q lower, v and τ higher). However, in contrast to the natural rate for the Section 2 economy, depicted in (24), this natural rate also depends on the intensity of the financial-market friction. The higher the probability of relocation, the higher the natural rate of unemployment. With a greater fraction of depositors having to move, banks must hold more real balances and invest less in capital formation, which implies less employment in equilibrium.

The following proposition may now be stated:

Proposition 3. ***Optimality of the Friedman Rule When $\theta = 1$.***

When $\theta = 1$ and capital formation is possible, welfare is maximized when $\sigma = 1/R$.

Proof. The proof is analogous to the proof of Proposition 2, but $r^{n^*} = R^*$ from (38).■

It follows that a central bank with a dual mandate should put all its weight on its inflation objective.

4. Concluding Remarks

This article has shown that an upward-sloping, downward-sloping, or vertical long-run Phillips curve can arise in an overlapping-generations monetary model with random relocation and involuntary unemployment. The slope of the Phillips curve and optimal monetary policy depend on fundamentals of labor and financial markets, on agents' degree of risk aversion, and on whether capital formation is possible. They are independent of any wage or price stickiness and unanticipated inflation.

Although the model studied in this article is rich in many respects, it is abstract in others. For example, because only steady states are derived, the article is silent on the short-run dynamics of the inflation-unemployment relationship. Mankiw (2001) has noted that the dynamic relationship between inflation and unemployment also has not been explained by the New Keynesian models of the Phillips curve, which are not consistent with the delayed and gradual effect that monetary policy has on inflation. This article does not resolve that mystery. Doing so must be the subject of future research.

The model also abstracts from asset accumulation and from adverse selection. Agents who have worked previously and already accumulated some assets presumably would be more likely to shirk if they went back to work, affecting the wage rate firms would have to offer to induce such agents to take employment in the primary sector. Agents also could differ in their ability to perform on the job, and their abilities might be private information. Firms might offer higher wages to attract at least some better workers into the labor market. Monetary policy, through its effects on asset returns, would reinforce or mitigate the impacts of asset accumulation, but analyzing this also must be left for future research.

References

Abel, A. B. and B. S. Bernanke, 1998. *Macroeconomics*, third ed. New York:Addison-Wesley, pp.62-63.

Akerlof, G. A., W. T. Dickens, G. L. Perry, R. J. Gordon, and N. G. Mankiw, 1996. The Macroeconomics of Low Inflation. *Brookings Papers on Economic Activity* 1, 1-76.

Akerlof, G. A., W. T. Dickens, G. L. Perry, T. F., Bewley, and A. S. Blinder, 2000. Near-Rational Wage and Price Setting and the Long-Run Phillips Curve. *Brookings Papers on Economic Activity* 1, 1-60.

Andersen, T. M., 2002. Nominal Rigidities and the Optimal Rate of Inflation. *European Journal of Political Economy* 18, 375-389.

Andolfatto, D., S. Hendry, and K. Moran, 2004. Labour Markets, Liquidity, and Monetary Policy Regimes. *Canadian Journal of Economics* 37(2), 392-420.

Bernanke, B. S., 2007. Federal Reserve Communications. Speech delivered at the Cato Institute 25th Annual Monetary Conference, Washington, D.C., November 14.

Billi, R. M., 2007. Optimal Inflation for the U.S. Federal Reserve Bank of Kansas City Research Working Paper 07-03, April.

Bils, M. and P. J. Klenow, 2004. Some Evidence on the Importance of Sticky Prices. *Journal of Political Economy* 112(5), 947-985.

Blanchard, O. and J. Gali, 2007. Real Wage Rigidities and the New Keynesian Model. *Journal of Money, Credit and Banking*, Supplement to 39(1): 35-65.

Bulow, J. I. and L. H. Summers, 1986. A Theory of Dual Labor Markets with Application to Industrial Policy, Discrimination, and Keynesian Unemployment. *Journal of Labor Economics* 4(3), 376-414.

Campbell, C. M., 1993. Do Firms Pay Efficiency Wages? Evidence with Data at the Firm Level. *Journal of Labor Economics* 11, 442-470.

Cavalcanti, T. V. V. and A. P.Villamil, 2003. Optimal Inflation Tax and Structural Reform. *Macroeconomic Dynamics* 7, 333-362.

Chiappori, P. A. and M. Paiella, 2006. Relative Risk Aversion is Constant: Evidence from Panel Data. Mimeo, Columbia University.

Davig, T., 2007. Phillips Curve Instability and Optimal Monetary Policy. Federal Reserve Bank of Kansas City Research Working Paper 07-04, July.

Dennis, R., 2006. The Frequency of Price Adjustment and New Keynesian Business Cycle Dynamics. Federal Reserve Bank of San Francisco Working Paper 2006-22, November.

Diamond, D., and P. Dybvig, 1983. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91(3), 401-19.

DiNardo, J. and M. P. Moore, 1999. The Phillips Curve is Back? Using Panel Data to Analyze the Relationship Between Unemployment and Inflation in an Open Economy. National Bureau of Economic Research Working Paper #7328.

Fair, R. C., 2000. Testing the NAIRU Model for the United States. *Review of Economics and Statistics* 82(1), 64-71.

Friedman, M., 1977. Nobel Lecture: Inflation and Unemployment. *Journal of Political Economy* 85, 451-472.

Gray, J., 1976. Wage Indexation: A Macroeconomic Approach, *Journal of Monetary Economics* 2, 221-235.

King, T. B. and J. Morley, 2007. In Search of the Natural Rate of Unemployment. *Journal of Monetary Economics* 54, 550-564.

King, R. G. and M. W. Watson, 1994. The Post-War U.S. Phillips Curve: A Revisionist Econometric History. *Carnegie-Rochester Series on Public Policy* 41, 157-219.

Kydland, F. and E. C. Prescott, 1977. Rules Rather Than Discretion: The Inconsistency of Optimal Plans. *Journal of Political Economy* 85, 473-492.

Lagos, R. and R. Wright, 2005. A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy* 113, 463-484.

Levin, A. T.. A. Onatski, J. C. Williams, and N. Williams, 2005. Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models. Federal Reserve Bank of San Francisco Working Paper 2005-15, July.

Lucas, R. E., 1972. Expectations and the Neutrality of Money. *Journal of Economic Theory* 4, 103-124.

Mankiw, N. G., 2001. The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment, *Economic Journal* 111, C45-C61.

Mazzocco, M., 2006. Individual Rather than Household Euler Equations: Identification and

Estimation of Individual Preferences Using Household Data. Mimeo, Department of Economics, University of California, Los Angeles.

Mishkin, F. S., 2007a. *The Economics of Money, Banking, and Financial Markets*, eighth edition. Boston: Pearson Education, Inc.

Mishkin, F. S., 2007b. The Federal Reserves' Enhanced Communication Strategy and the Science of Monetary Policy. Speech delivered to the Undergraduate Economics Association, Massachusetts Institute of Technology, Cambridge, MA, November 29, *http://www.federalreserve.gov/newevents/speech/mishkin20071129a.htm*.

Nason, J. M. and G. A. Slotsve, 2004. Along the New Keynesian Phillips Curve with Nominal and Real Rigidities. Federal Reserve Bank of Atlanta Working Paper 2004-9, April.

Ottensmeyer, E. J. and M. A. Heroux, 1991. Ethics, Public Policy, and Managing Advanced Technologites: The Case of Electronic Surveillance. *Journal of Business Ethics* 10, 519-526.

Ravenna, F. and C. E. Walsh, 2007. Vacancies, Unemployment, and the Phillips Curve. Mimeo, July 10.

Reed, R. R. and C. J. Waller, 2006. Money and Risk Sharing. *Journal of Money, Credit, and Banking* 38, 1599-1618.

Rocheteau, G., P. Rupert, and R. Wright, 2006. Inflation and Unemployment in General Equilibrium. Mimeo, October 9.

Rogerson, R., 1988. Indivisible Labor, Lotteries and Equilibrium. *Journal of Monetary Economics* 21, 3-16.

Schreft, S. L. and B. D. Smith, 1997. Money, Banking, and Capital Formation. *Journal of Economic Theory* 73, 157-182.

Schreft, S. L. and B. D. Smith, 1998. The Effects of Open Market Operations in a Model of Intermediation and Growth. *Review of Economic Studies* 65, 519-550.

Shapiro, C. and J. Stiglitz, 1984. Equilibrium Unemployment as a Worker Discipline Device. *American Economic Review* 74, 433-444.

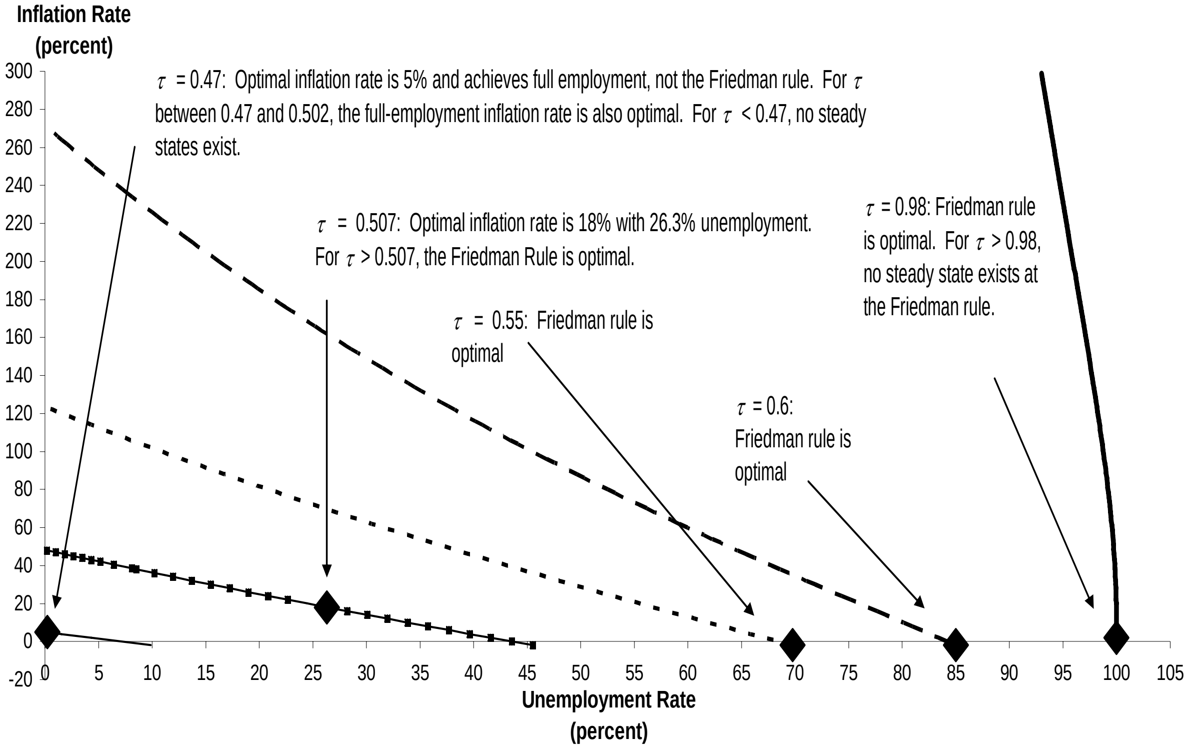
Stockman, A. C., 1981. Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy. *Journal of Monetary Economics* 8, 387-393.

Wood, A. M., 1998. Omniscent Organizations and Bodily Observations: Electronic Surveillance in the Workplace. *International Journal of Sociology and Social Policy*

18(5/6), 136-174.

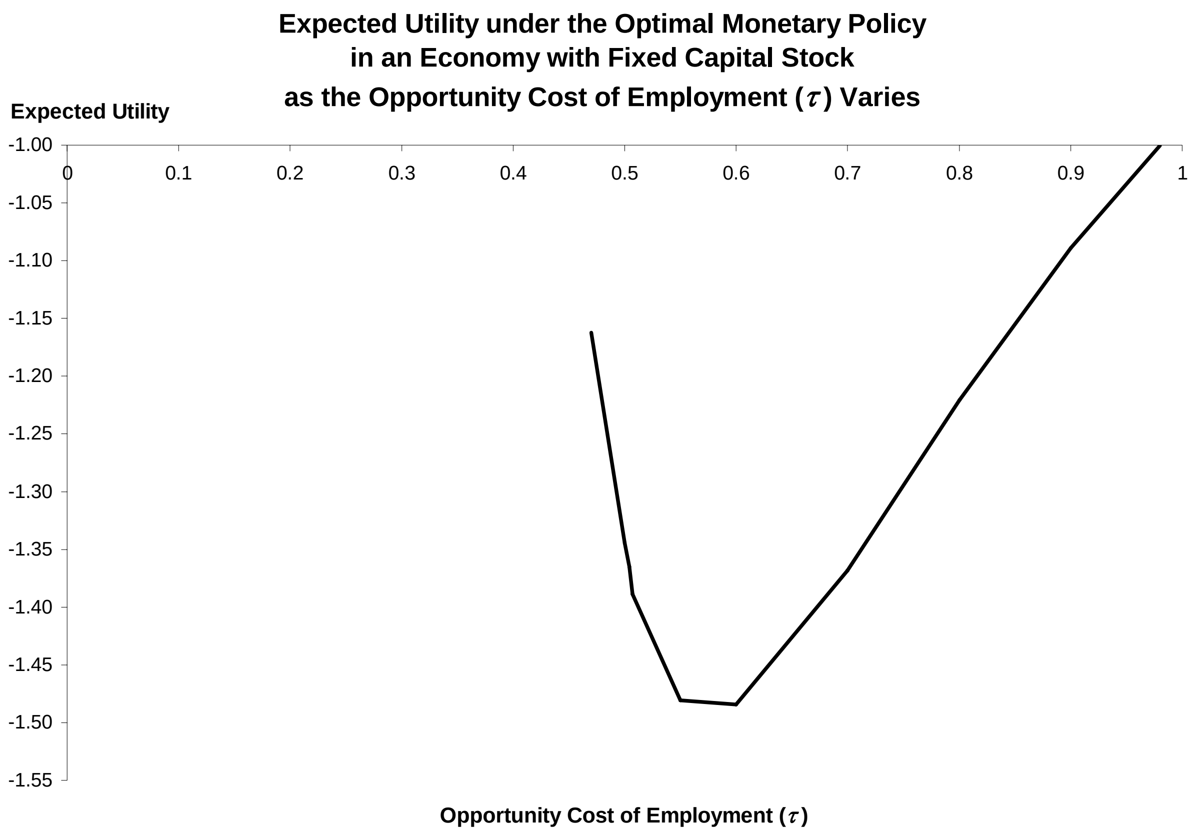
Figure 1

Phillips Curves for Economy with Fixed Capital Stock as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001. Optimal inflation rate denoted by ♦.

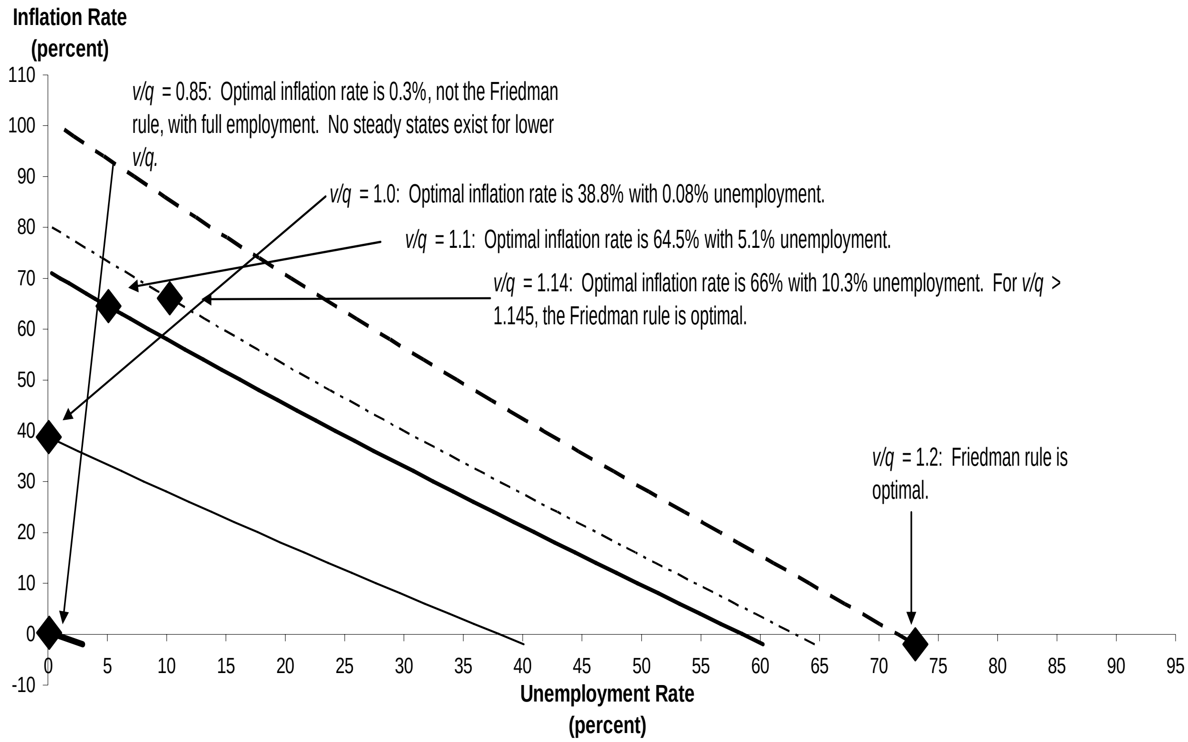
Figure 2



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; ν = 0.001

Figure 3

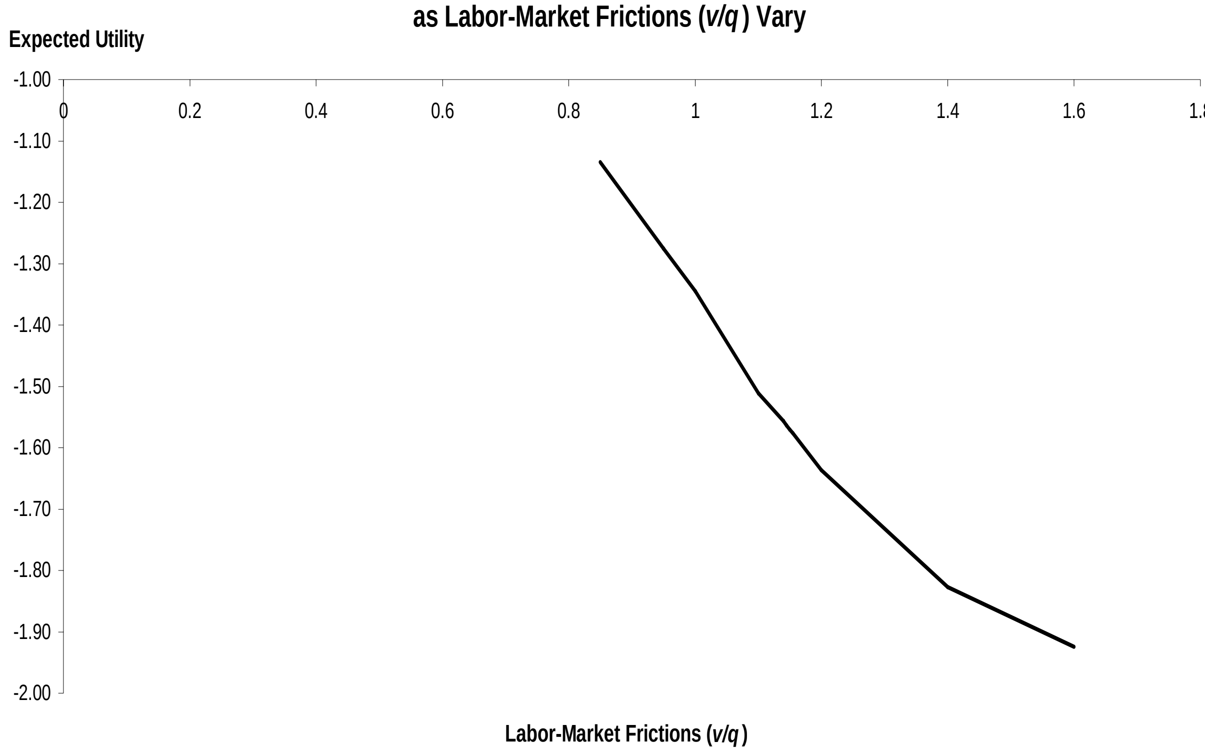
Phillips Curves for Economy with Fixed Capital Stock as Labor-Market Frictions (*v/q*) Vary



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v = 0.001; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 4

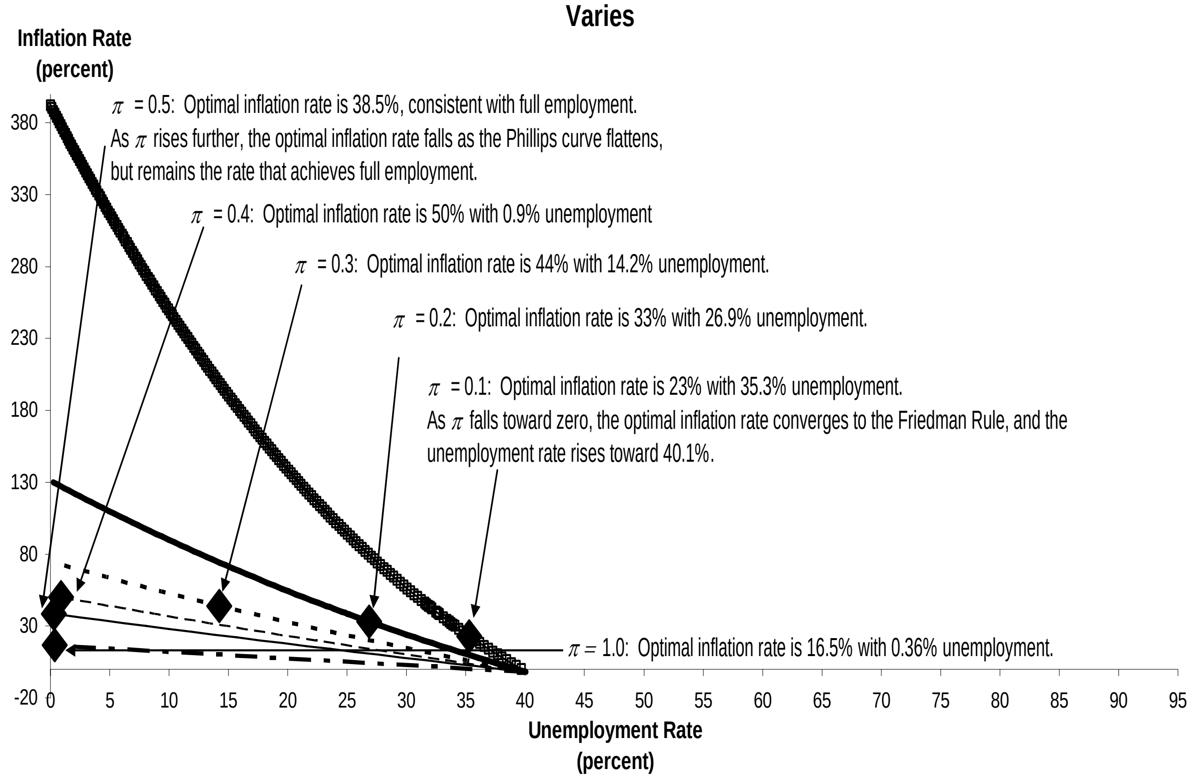
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as Labor-Market Frictions (*v/q*) Vary



Assumes: $\eta = 0.3$; $\pi = 0.5$; $R = 1.02$; $A = 1.25$; $\tau = 0.5$; $v = 0.001$

Figure 5

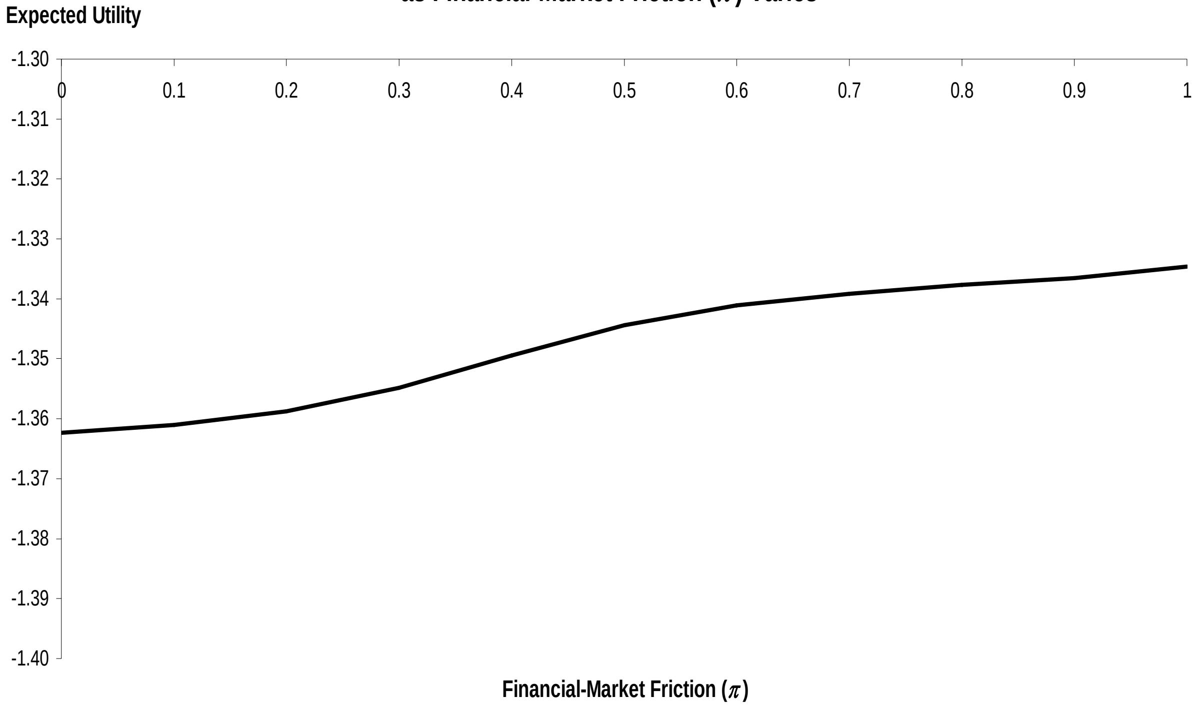
Phillips Curves for Economy with Fixed Capital Stock as Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 6

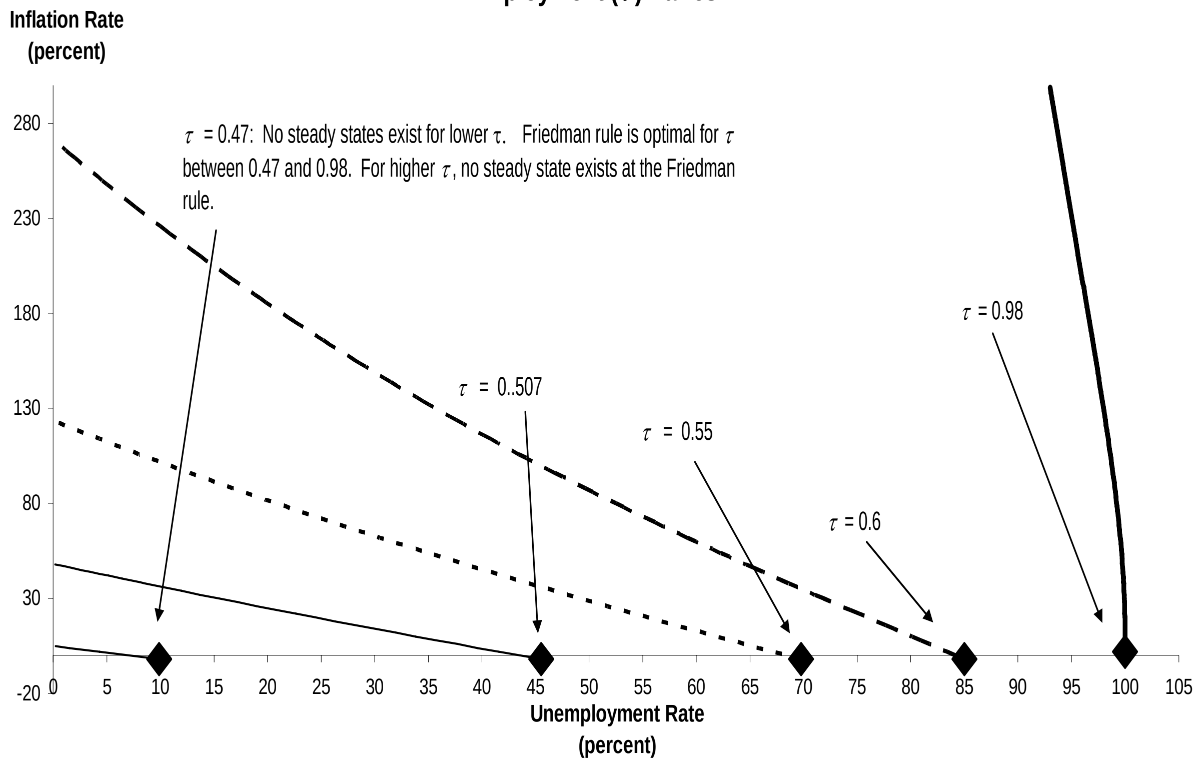
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001; τ = 0.5

Figure 7

Phillips Curves for Economy with Fixed Capital Stock as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.5. Optimal inflation rate denoted by ♦.

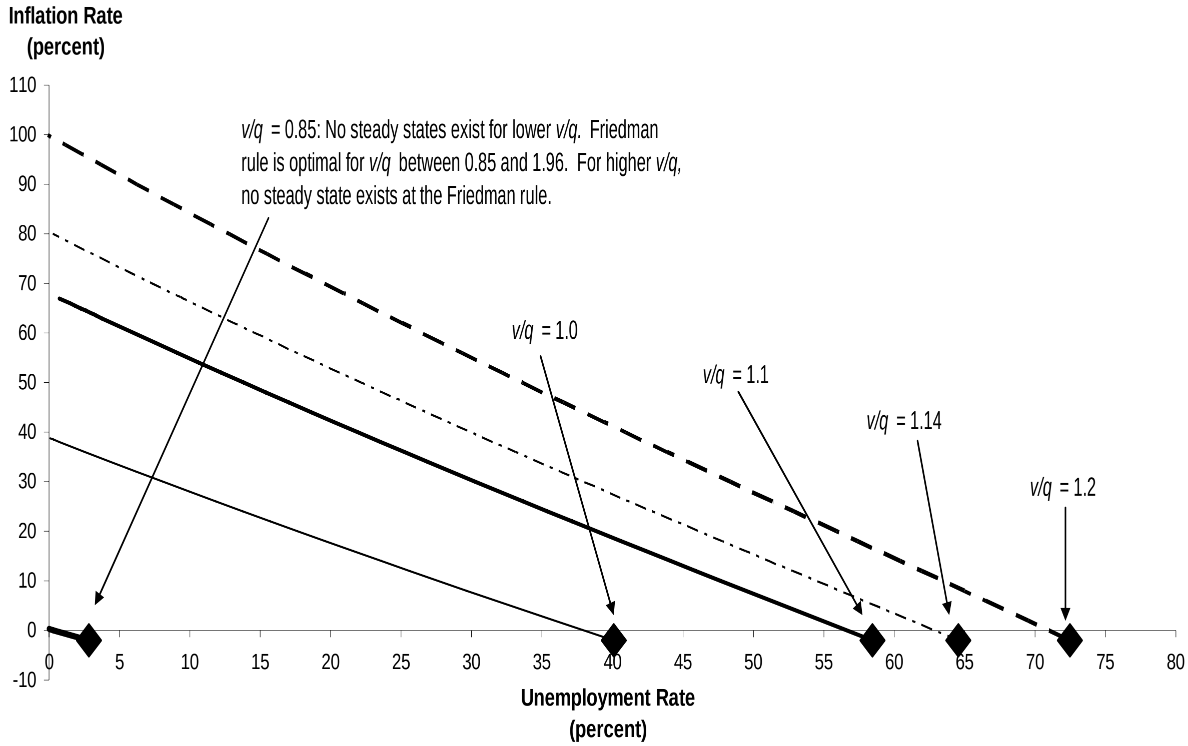
Figure 8

Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as the Opportunity Cost of Employment (τ) Varies

Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.5

Figure 9

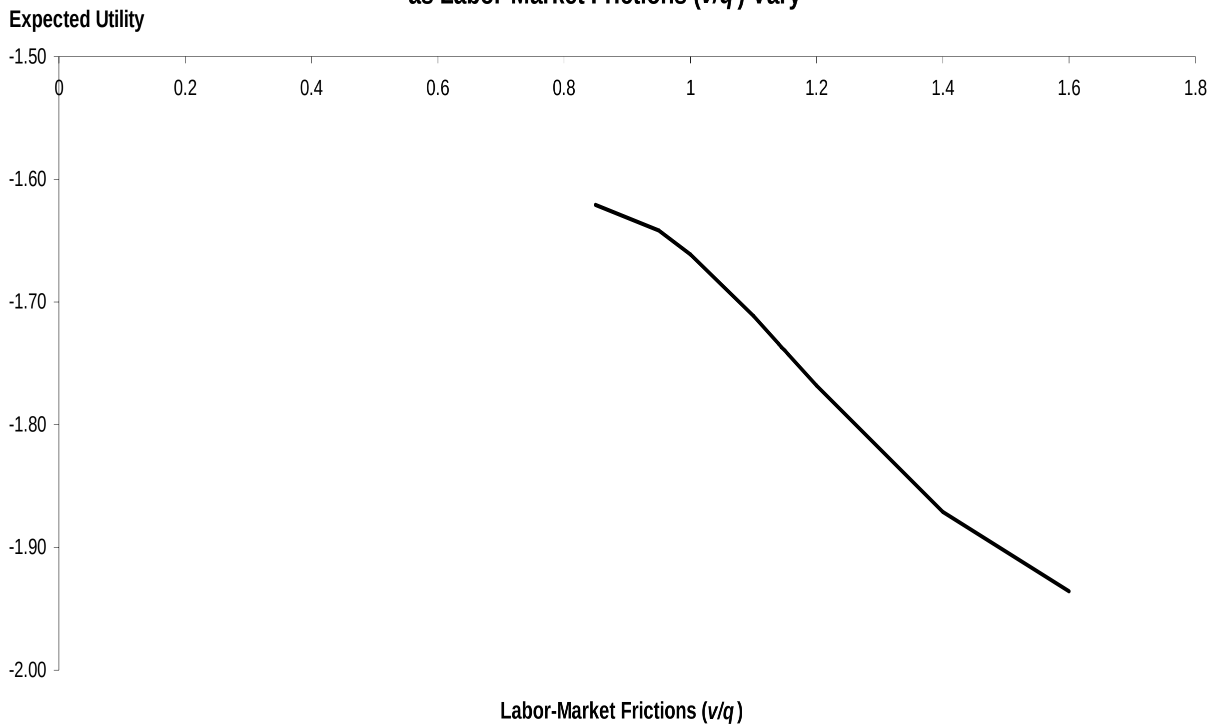
Phillips Curves for Economy with Fixed Capital Stock as Labor-Market Frictions (*v/q*) Vary



Assumes: $\eta = 0.3$; $\pi = 0.5$; $R = 1.02$; $A = 1.25$; $\tau = 0.5$; $v = 0.5$. Optimal inflation rate denoted by ♦.

Figure 10

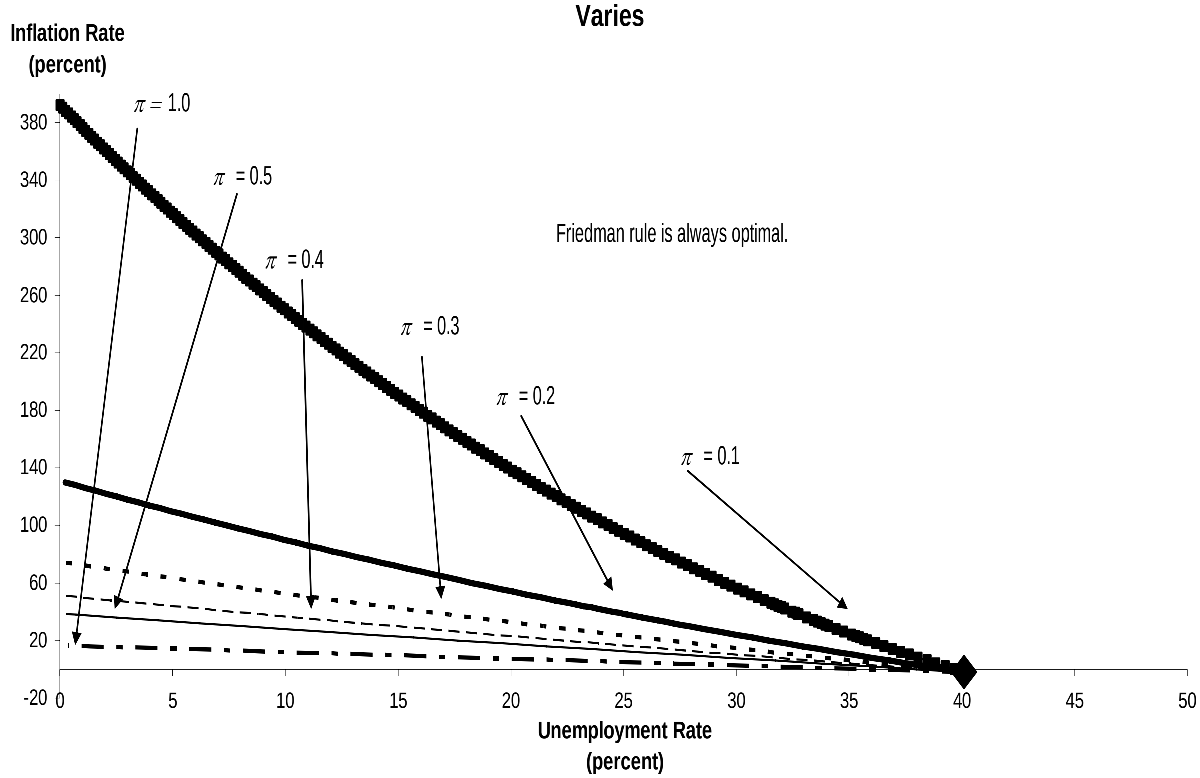
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as Labor-Market Frictions (*v/q*) Vary



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; τ = 0.5; v = 0.5

Figure 11

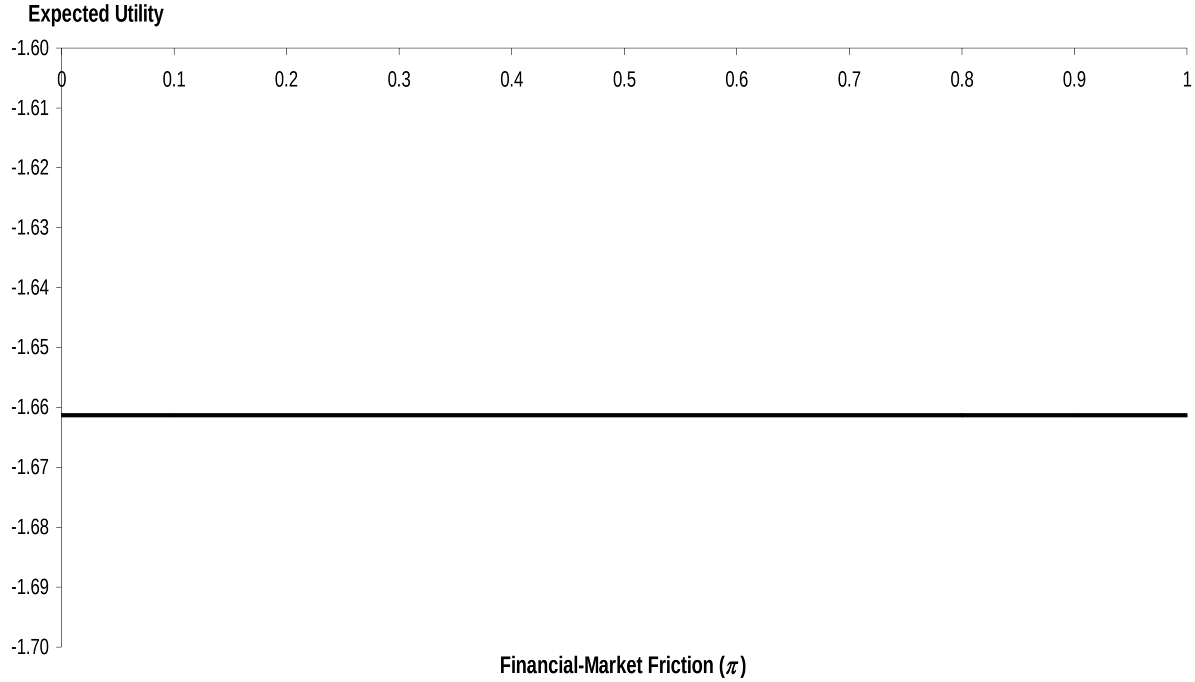
Phillips Curves for Economy with Fixed Capital Stock as Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.5; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 12

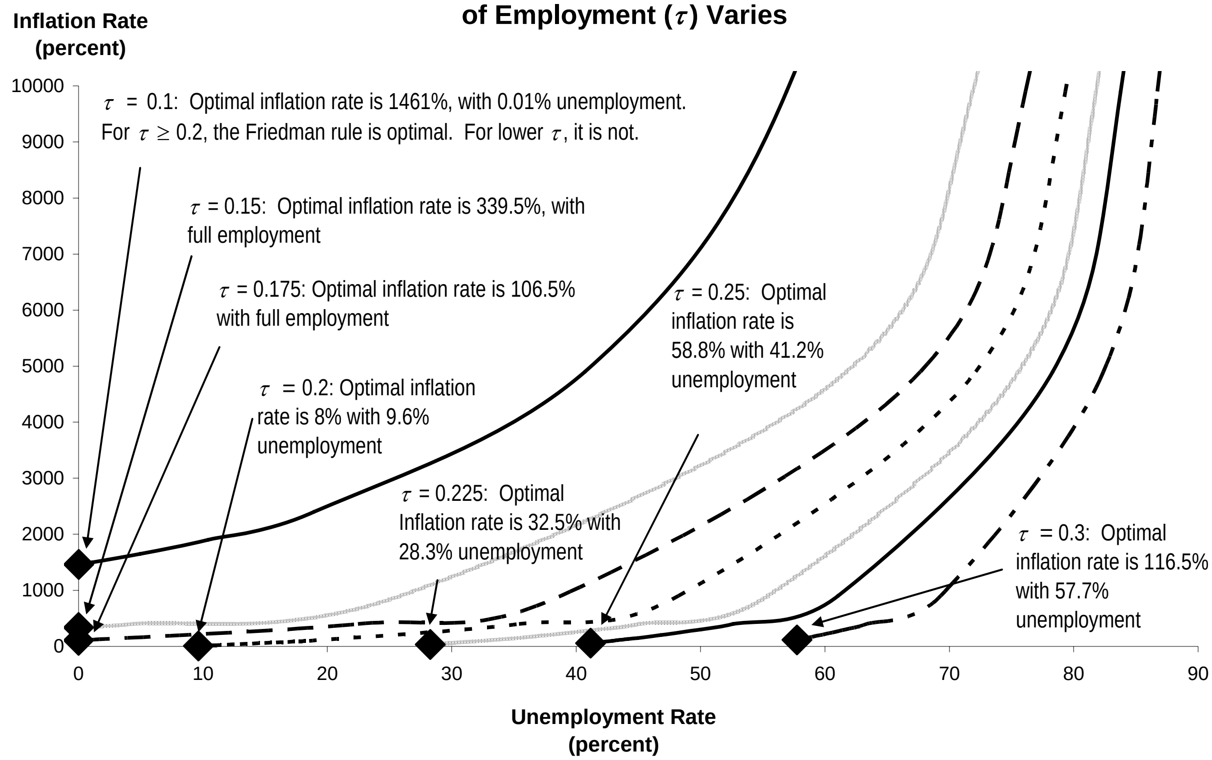
Expected Utility under the Optimal Monetary Policy in an Economy with Fixed Capital Stock as Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.5; τ = 0.5.

Figure 13

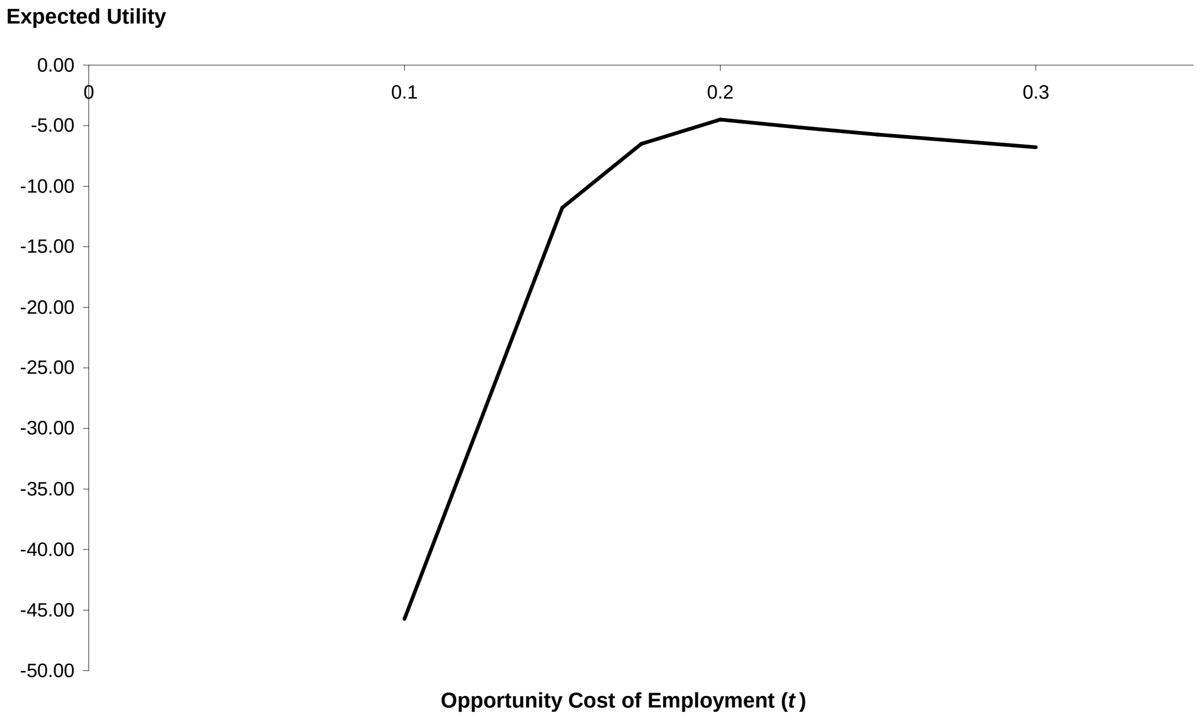
Phillips Curve for Economy with Capital Formation as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001. Optimal inflation rate denoted by ♦.

Figure 14

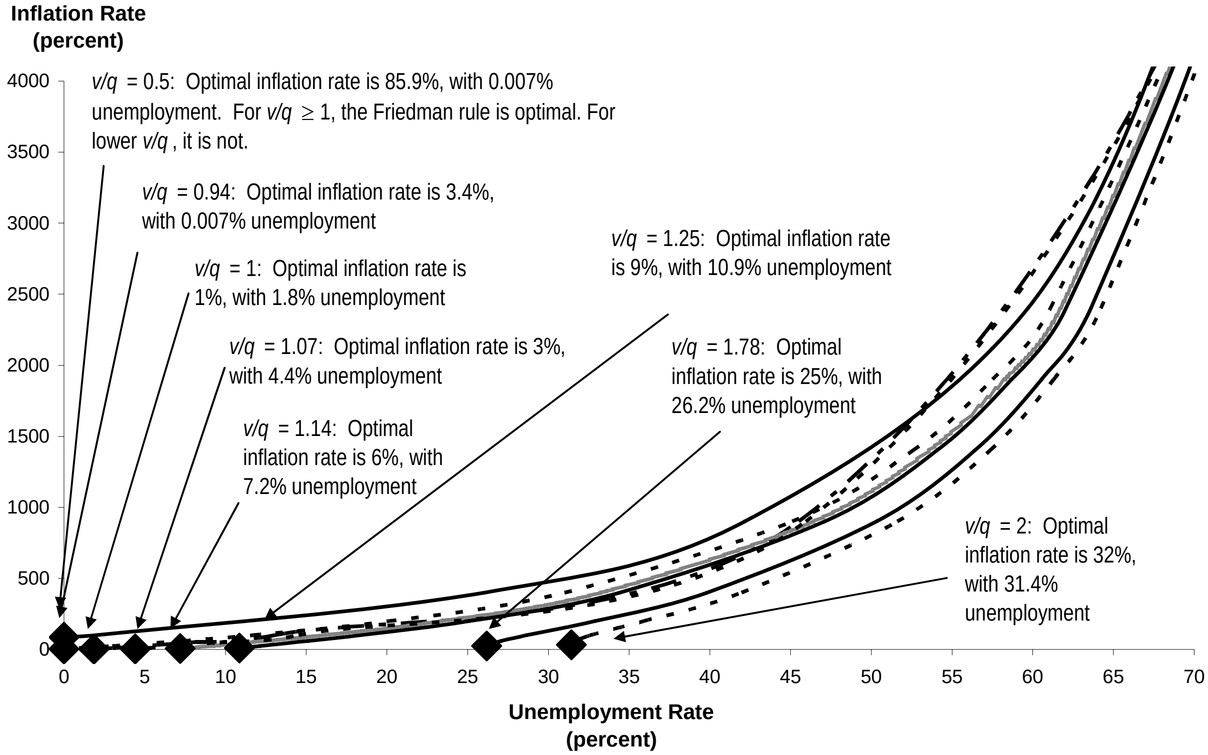
Expected Utility under the Optimal Monetary Policy in an Economy with Capital Formation as the Opportunity Cost of Employment (τ) Varies



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001

Figure 15

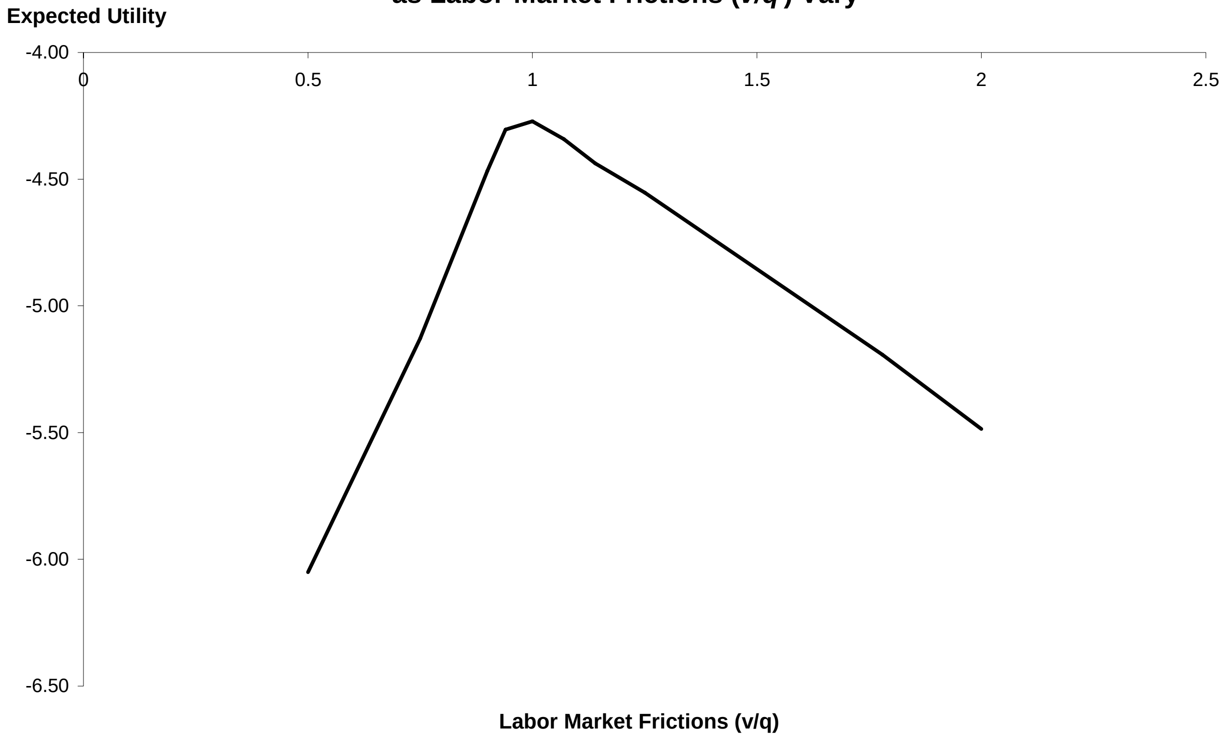
Phillips Curve for Economy with Capital Formation as Labor-Market Frictions (*v/q*) Vary



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v = 0.001; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 16

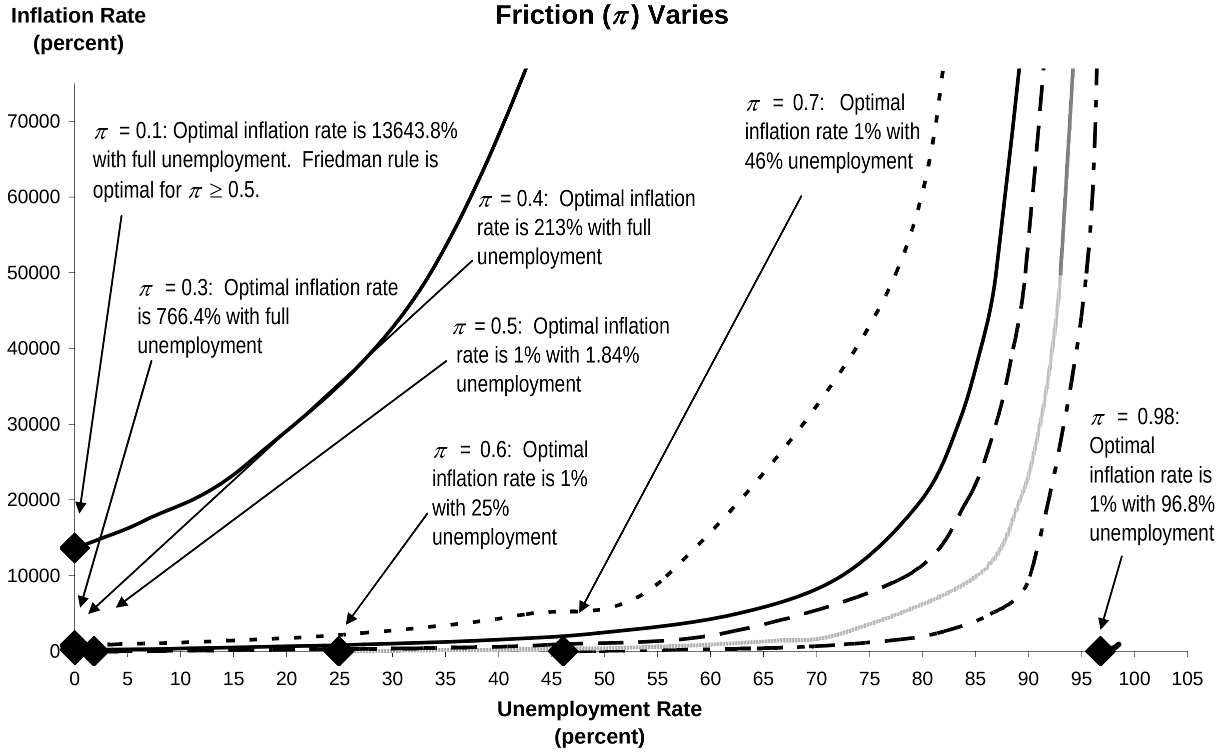
Expected Utility under the Optimal Monetary Policy in an Economy with Capital Formation as Labor-Market Frictions (*v/q*) Vary



Assumes: η = 0.3; π = 0.5; R = 1.02; A = 1.25; v = 0.001; τ = 0.5.

Figure 17

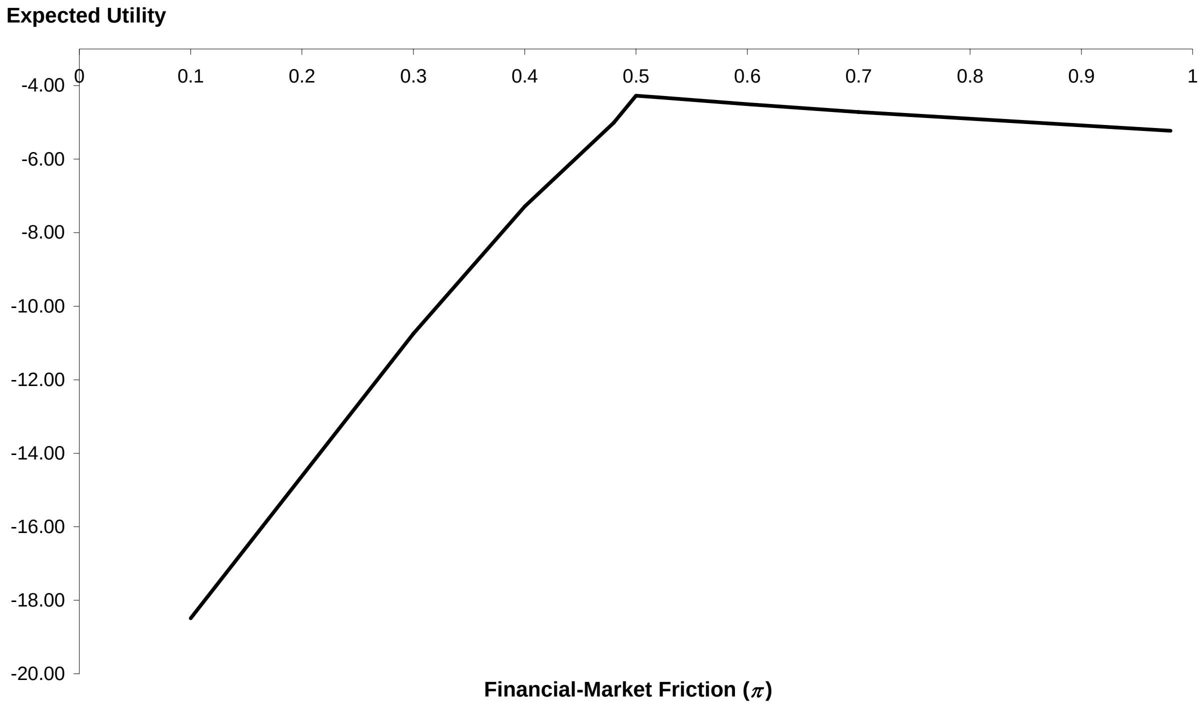
Phillips Curve for Economy with Capital Formation as the Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001; τ = 0.5. Optimal inflation rate denoted by ♦.

Figure 18

Expected Utility under the Optimal Monetary Policy in an Economy with Capital Formation as the Financial-Market Friction (π) Varies



Assumes: η = 0.3; R = 1.02; A = 1.25; v/q = 1.0; v = 0.001; τ = 0.5.