

## Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?

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It is a pleasure to be asked to comment upon Lars Svensson's thoughtful and ambitious paper, though it raises far too many issues for me to attempt to address them all in the limited time available here. Svensson offers a thorough review of the recent scholarly literature on the conduct of monetary policy, and also reviews one of the most important recent developments in central bank practice as well, namely the evolving methodology of "inflation forecast targeting."

In my own remarks, I would like to develop further a single theme, which is the advantage of central bank commitment to a systematic approach to monetary policy. This theme also figures in Svensson's discussion, but I believe that its consequences extend even further than he indicates.

### **The advantages of credible policy commitments**

One of the most important issues in the conduct of monetary policy, that should attain particular significance in an era of price stability, is the need to take account of the effects of the central bank's conduct upon private-sector expectations. In general, there is every reason to believe that the aspects of economic behavior that are central to the transmission mechanism for monetary policy are critically dependent upon people's expectations, including their expectations regarding future policy. If the central bank commits itself to a systematic pattern

of behavior and can make this *credible* to the private sector, then the private sector's expectations regarding future policy should be strongly affected by the bank's commitment.

Of course, one might doubt exactly how credible a central bank should expect a contemplated change in its pattern of behavior to be. Credibility is likely to be imperfect in the transition to a new regime, and so Mervyn King's paper (King, 1996) at the conference here three years ago, on "Achieving Price Stability," rightly gave considerable attention to the consequences of lags in the adjustment of private sector expectations during a process of disinflation. However, the ability of central banks to achieve a high degree of credibility with the public for their policy commitments ought to be greater in an era of price stability. Such an era would presumably be one in which the goals of macroeconomic stabilization policy were reasonably well achieved, so that there would be no need for dramatic policy experiments. In such a stable environment the chances that the public would come to understand well what the central bank was doing ought to be greatest.

As Svensson rightly stresses, the ability to credibly commit itself provides a central bank with great advantages, in terms of the degree of macroeconomic stability that is attainable in principle. It is by now widely appreciated that, as first explained by Kydland and Prescott (1977) and Barro and Gordon (1983), commitment to a low average rate of inflation can achieve lower inflation than would result from purely discretionary optimizing behavior on the part of the central bank, with little loss of output. Indeed, this understanding of the value of commitment to a low average rate of inflation is one of the main reasons for the popularity in recent years of explicit inflation targets.<sup>1</sup>

What is less widely understood, however, is that discretionary optimizing behavior leads to suboptimal outcomes in general, *even when* there is no problem with the average rate of inflation that is achieved—as, for example, when a central bank that exercises pure discretion seeks to stabilize output around a level consistent with stable prices, rather than a higher level, as proposed by Blinder (1998). For discretion also generally leads to incorrect dynamic responses to temporary shocks.<sup>2</sup> The reason is that when the private

sector is forward-looking, and the central bank’s commitments can be credible, commitments regarding future policy can often affect the short-run constraints facing the bank in a desirable way, by affecting the expectations that determine private behavior in the present. However, under pure discretion, there will be no incentive *later* to act according to such commitments, since the prior expectations that one wished to affect are at this point historically given. And if this sort of behavior is anticipated by the private sector, then its expectations will not shift in response to the shock in the way that the central bank would prefer.

This can be illustrated with a simple example, further explained in the appendix. Let the economy’s aggregate supply relation be of the familiar “New Keynesian” form

$$(1) \quad \pi_t = \kappa\chi_t + \beta E_t \pi_{t+1} + u_t$$

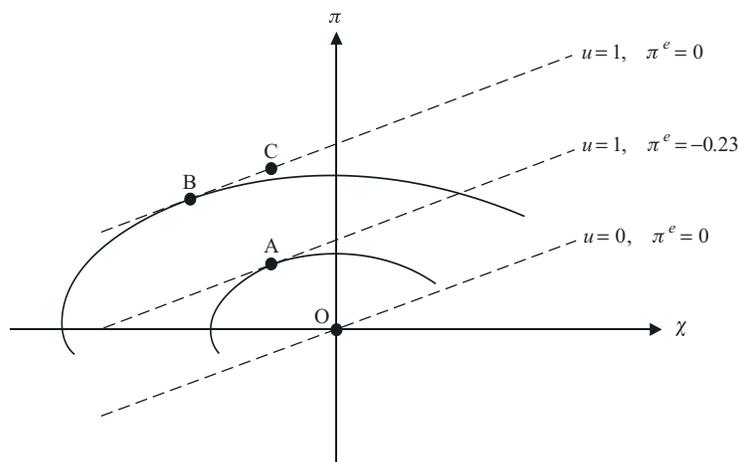
Here,  $\pi_t$  is the rate of inflation,  $\chi_t$  is the output gap,<sup>3</sup>  $\kappa$  is a positive coefficient,  $\beta$  is the discount factor (slightly less than one) by which suppliers of goods discount future real earnings, and  $u_t$  is an inefficient “supply shock” that creates a temporary discrepancy between the level of output consistent with price stability and the economically efficient level.<sup>4</sup> This relation can be derived from a model of optimal price-setting with imperfect synchronization of price changes originally proposed by Calvo (1983).<sup>5</sup> For present purposes, its most important feature is the effect of expected *future* inflation on the current short-run trade-off between output and inflation. The supply shocks are assumed to have mean zero and, to make the point most simply, I shall also assume that they are i.i.d.

Let us further suppose, as Svensson does, that the goal of monetary policy is to minimize a discounted sum of losses of the form

$$(2) \quad E \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda \chi_t^2 \right] \right\},$$

where  $\lambda$  is a positive weight. Because it is assumed that one desires to stabilize the output gap around the value zero—i.e., the average of

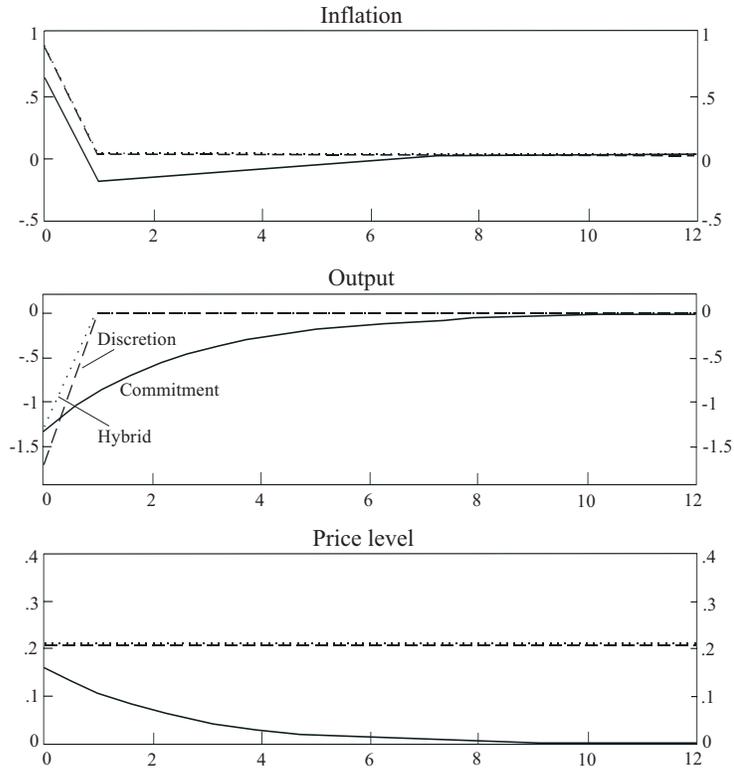
**Figure 1**  
**Short-run Phillips Curves**



the values consistent with stable prices—rather than a positive level, there is no bias in the average inflation rate resulting from discretionary optimization in this case. Yet, the equilibrium *response to supply shocks* is still inefficient under discretion.

The impulse responses of inflation and output to a positive (adverse) supply shock in period zero, for the equilibrium associated with discretionary optimization, are shown in Chart 1.<sup>6</sup> After period zero, there is no longer expected to be a supply disturbance in any period,<sup>7</sup> and so there is no problem with stabilizing both inflation and the output gap at their “target” values, zero in each case. Under discretion, there is no constraint upon policy as a result of past events, and so this is what is expected to occur. Given expected future inflation of zero, the short-run Phillips curve in period zero shifts up as a result of the supply shock, from the lowest curve in Figure 1 (the steady-state position) to the highest. Given preferences indicated by the indifference curves drawn in the figure, an optimizing central bank chooses point B in period zero; this corresponds to the temporary burst of inflation and negative output gap shown by the dashed lines in Chart 1.

**Chart 1**  
**Impulse Responses to an Inefficient Supply Shock**



If, however, the central bank can commit itself to an alternative dynamic response to such a shock, and make this credible (so that private sector expectations respond accordingly), it is possible to achieve a lower value for the discounted sum of losses (2). The optimal commitment is shown by the solid lines in Chart 1. It involves a smaller initial output gap but a *more persistent one*. Under this commitment, policy keeps the output gap negative for several quarters, even after the supply shock no longer affects the economy, resulting in a period of inflation *below* its long-run target level. The advantage of this is that the supply shock is accompanied by an expectation of *lower* future

**Table 1**  
**Performance Under Alternative Policies**

Policy	$\text{var}(\pi)$	$\text{var}(\chi)$	E/L
Commitment	.51	2.86	65
Discretion	.71	2.75	84
Hybrid	.77	1.66	85

inflation, which partially offsets the adverse shift in the short-run Phillips curve. (Even though there is temporary cost pressure, price increases in period zero are restrained by the anticipation of future deflation.) This allows better performance in period zero with regard to *both* inflation and the output gap.

The cost, of course, is that the deflationary policy must be pursued in later periods, even though it is bad for the economy then (and so would *not* be pursued by a discretionary optimizer). Yet, a certain amount of such pain is worthwhile if it can be made credible in advance, in order to restrain earlier price increases. Table 1 shows the overall variance of inflation and the output gap under the two policies, and the resulting expected values of the discounted sum of losses (2).<sup>8</sup>

This example illustrates an important general point, which is that optimal policy will generally be *history-dependent* in ways that are unrelated to any constraints that past events impose upon what is technically achievable in the present (Woodford, 1999c). Its failure to make the conduct of monetary policy history-dependent in this way is one of the crucial respects in which purely discretionary policy-making is suboptimal, unrelated to the better-understood “inflation bias” problem. But other popular current proposals for the conduct of monetary policy—such as the “Taylor Rule” (Taylor, 1993) and the “inflation forecast targeting” procedure recommended by Svensson—are *equally* lacking in history dependence, and, thus, suffer from the same difficulty. The design of a decision-making procedure for the conduct

of monetary policy that incorporates a greater degree of history dependence (of the right sort) is an important challenge, if we are to fully reap the potential benefits of an era of price stability (interpreted here as an era of highly-credible monetary policy).

Closer attention to this issue would have consequences for a number of the issues taken up in Svensson's survey. I briefly review several of these in turn.

### **Inflation stabilization or price-level stabilization?**

Svensson rightly calls attention to the possibility that, even if the central bank's loss function is of the form (2) assumed above—and, therefore, depends only upon the variability of a relatively short-run measure of the inflation rate, not upon cumulative changes in the price level—a policy that responds to deviations in the price *level* from some target value (or deterministic trend path) may, nonetheless, have advantages over one that pays attention only to the inflation rate. The reason for this is intimately connected with the desirability of history dependence in the central bank's conduct.

The example just presented provides a simple illustration. Under discretionary optimization, Chart 1 shows that the rate of inflation allowed by the central bank should depend solely upon the current supply shock  $u_t$ . Each change in the price level resulting from a transitory disturbance of this kind will be permanent, as later policy is not conditioned upon it; and so the price level follows a random walk, though the inflation rate is brought quickly back to its long-run target level after each disturbance. But this is not optimal policy; a superior outcome can be obtained by commitment to a history-dependent policy, under which inflationary disturbances are followed by periods of *deflation*, which eventually brings the price level back to its initial level. (Panel (c) of Chart 1 shows this clearly.) In this equilibrium, the price level is a stationary random variable (or, in the case of a non-zero target inflation rate, trend-stationary). As Table 1 shows, this policy achieves a lower variance of inflation, and while the variance of output is somewhat higher (because of the greater persistence of output fluctuations under this policy), total expected losses (2) are reduced.<sup>9</sup>

The common argument—that subsequently “undoing” deviations of the price level from the path that it would follow in the absence of a shock increases the variance of inflation, because it adds the unnecessary subsequent reduction in inflation to the initial inflation warranted by the shock—is seen to be incorrect in the case of a forward-looking model. For this argument neglects the fact that expectations regarding the subsequent path of inflation can help improve stabilization at the time of the shock.<sup>10</sup> In fact, it will be recognized that the common argument against price-level stabilization is simply a special case of the more general argument (quite generally incorrect) against history-dependent policy.<sup>11</sup>

Not only is price-level stabilization (at least in the sense of maintaining stationarity) a *feature* of optimal policy, even though policy is assumed not to care about price-level stabilization as an ultimate *goal*, but policy rules involving a price-level *target* may very well be useful as a way of achieving the desired type of history-dependence in central bank conduct.<sup>12</sup> For example, a simple rule that would lead to the impulse responses shown by the solid lines in Chart 1 is given by

$$(3) \quad \chi_t = -\theta(p_t - p^*),$$

where  $p_t$  is the log price level,  $p^*$  is a constant target price level, and  $\theta$  is a positive coefficient. Here, we treat the output gap as if it was directly the central bank’s instrument, in order to avoid having to model aggregate demand determination. The rule described by (3) is an example of what Hall (1984) calls an “elastic price target.”<sup>13</sup> The value of the price level target  $p^*$  is irrelevant, as far as the goal of inducing the desired equilibrium response to supply shocks is concerned.<sup>14</sup>

Commitment to a rule like (3) is an especially simple way of achieving the optimal equilibrium responses to shocks. Implementation of such a rule does not require the central bank to observe the supply shocks (though it does require it to know the current level of “potential” output in order to measure the output gap  $x_t$ ). Furthermore, the existence of an explicit price level target would make it easy for private-sector inflation expectations to come to respond to shocks in the desired way. Whenever the price level rose above the target level, peo-

ple would easily understand that they should, therefore, expect price declines soon.

The result that it is desirable for people to expect unexpected price-level increases to subsequently be offset by (predictable) price declines is not special to the type of shock considered in the above example.

Suppose, for example, that we extend the above model by adding a simple specification of aggregate demand, through an intertemporal Euler equation of the form

$$(4) \quad \chi_t = E_t \chi_{t+1} - \sigma \left( i_t - E_t \pi_{t+1} - r_t^n \right),$$

as in Clarida et al. (1999) or Woodford (1999c). Here,  $i_t$  is the short-term nominal interest rate instrument of the central bank, and  $r_t^n$  indicates exogenous variation in Wicksell's "natural rate of interest," the real rate that would be required in order for output to be kept continuously at potential.

In the event that there are no obstacles to or costs associated with interest rate variation, the addition of equation (4) makes no difference for our analysis of optimal policy. However, it is reasonable to suppose that minimization of the volatility of the short-term nominal interest rate should also be a goal of monetary policy. Woodford (1999b) shows that when one takes account of the transactions frictions that lead people to hold money balances, the correct quadratic approximation to expected utility is of the form

$$(5) \quad E \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda_{\chi} (\chi_t - \chi^*)^2 + \lambda_i i_t^2 \right] \right\}$$

instead of (2), where  $\lambda_i$  is another positive weight. (Elimination of transactions frictions requires a zero nominal interest rate at all times, for the reason stressed by Friedman, 1969; and the costs associated with these frictions are a convex function of the tax on money balances represented by the nominal interest rate.) Policies that involve less interest rate volatility are also desirable in that they make a

lower average inflation rate consistent with the zero lower bound on nominal interest rates, discussed below.

Once we recognize the existence of a cost of interest-rate variations, as posited in (5), it is no longer optimal to completely stabilize inflation, even in the absence of inefficient supply shocks. For then, variations in the natural rate of interest  $r_t^n$  create a conflict between the goals of inflation and output-gap stabilization on the one hand, and interest-rate stabilization on the other. Furthermore, Woodford (1999c) shows that, in this case again, the optimal pattern of responses to shocks is one in which unexpected increases in inflation (due to unexpected increases in the natural rate of interest) are followed by subsequent periods of inflation *below* its long-run target level.

In this case, the advantage of a credible commitment to such a policy is that the size of nominal interest-rate increase required to keep such a shock from greatly increasing inflation is reduced. This occurs for several reasons. First, lower expected future inflation reduces the incentives for current price increases, so that less increase in real interest rates is required to restrain inflation. Second, the expectation that real rates will remain high even after the shock has subsided (in order to bring about the subsequent deflation) restrains aggregate demand at the time of the shock (since spending depends upon long-term rather than solely short-term real rates), so that a smaller increase in short-term real rates is needed to achieve a given degree of demand restraint. And third, lower expected future inflation means that less of an increase in nominal rates is required to achieve a given increase in real rates. Thus, once again, it is desirable not simply that the central bank be expected to eventually bring inflation back down to its long-run target level; it is better if the private sector can count on its actually *undershooting* the long-run target level for a time.<sup>15</sup>

This does not mean that it is crucial that people expect the price level to return precisely to its original trend path. In fact, in the analysis of Woodford (1999c), the optimal commitment involves eventual *over-compensation* for the initial price-level surprise: an unexpected price level increase should lead to subsequent predictable price-level declines that imply that the price level will eventually end up *below* its

original level. The same is true in the case of the more complicated model that Rotemberg and Woodford (1997) fit to U.S. time series. Rotemberg and Woodford (1999) show that, in that model, the optimal commitment is one in which an unexpected increase in inflation (relative to what would have been predicted a quarter earlier) implies, on average, an eventual decline in the price level (relative to what would have been forecast at the same earlier date) that is twice as large as the unexpected price rise.

There is, thus, no intrinsic significance, according to such a model, to achieving trend-stationarity of the price level. Nonetheless, a policy that stabilized the long-run price level—so that inflation innovations would at least imply subsequent disinflation sufficient to undo the initial price-level increase—would be a significant step in the right direction, relative to actual U.S. policy that has allowed positive serial correlation in the inflation rate, so that positive inflation innovations lead to an expectation of further price-level *increases* thereafter. According to the Rotemberg-Woodford model, commitment to the “Taylor Rule” would result in a pattern of this kind as well, and it seems likely that “inflation forecast targeting” as currently practiced should have the same result. Relative to any of these approaches to policy-making, a policy aimed at price-level stabilization might well be an improvement.

Indeed, Rotemberg and Woodford find that a simple instrument rule that responds to deviations of the price level from a deterministic target path, with appropriately chosen coefficients, could achieve most, though not quite all, of the reduction in deadweight loss (relative to actual U.S. policy) that is theoretically achievable according to their model. Ease of communication with the public about a commitment of this kind might then make such an approach attractive. Thus, as Svensson concludes, rules of this kind certainly deserve further study.

### **Inflation forecast targeting as a policy framework**

As a framework for policy decision-making, Svensson advocates a procedure that he calls “inflation forecast targeting.” This framework, for which he has argued elsewhere (Svensson, 1997; 1999a, 1999b),

seeks to formalize the approach to policy-making that seems to be followed currently at several of the inflation-targeting central banks. The approach has several clear advantages. It allows all of the information available to the decision-makers at each point in time to be brought to bear upon the decision at hand. But it allows these diverse sources of information to be used in a disciplined and focused way. In practice, it has allowed a greater degree of transparency than has been associated with other central bank decision frameworks, through the publication of periodic “inflation reports” that detail the central bank’s forecasts and the conclusions drawn from them.

Nonetheless, this procedure, as described by Svensson, does not properly take account of the forward-looking character of private-sector action, or of the effects of central-bank policy commitments upon private-sector expectations. Essentially, Svensson advocates a dynamic-programming approach that is appropriate to the optimal control of a purely backward-looking system that evolves mechanically as a function of its own past state and the current actions of the central bank, independently of any commitments regarding future policy. In such a case, an optimal program for the central bank satisfies the dynamic-programming principle: it has the property that the continuation of the optimal program would be chosen again at any later date, if the central bank were to reoptimize given the state of the economy at that time.

Thus, there is no harm in adopting a decision-making framework that involves reoptimization in each decision cycle (i.e., discretionary policy-making). In any given decision cycle, there is no need to recall decisions or announcements made in the past, or any aspect of past conditions that does not matter for the current and future evolution of the goal variables (inflation and output). On the other hand, it *is*, in general, necessary to look forward to the decisions that one anticipates making in the future in order to make the best current decision. Thus, an optimal decision-making framework is highly prospective in character (requiring forecasts, in principle, of how the economy is expected to evolve over an unbounded future), and completely unconstrained by past commitments or expectations.

But matters are significantly different if the system to be controlled

is forward-looking, and the decision-making framework adopted by the central bank affects private-sector forecasts.<sup>16</sup> Then, as noted above, discretionary policy-making generally leads to an inferior outcome to what can be obtained under a suitably chosen policy commitment, if the latter can be made credible to the private sector.

There are several respects in which Svensson's description of inflation forecast targeting fails to take account of the nature of policy analysis using a forward-looking model. First, the optimizing procedure that he describes contains an internal inconsistency. The central bank is directed to make conditional forecasts of the paths of the goal variables, conditional upon alternative paths of its interest-rate instrument, using its model of the economy and all relevant information about current and future conditions. These conditional forecasts extend much farther into the future than the time of the next decision cycle (say, for two years, while policy may be re-evaluated monthly and inflation reports published quarterly). A path for interest rates is chosen over at least this entire horizon, and the effects of private-sector anticipation of the chosen path are taken into account in judging the desirability of the resulting paths for the goal variables. However, during the next decision cycle, the same procedure is repeated—with no constraint that the interest rate actually chosen in the next cycle correspond to a continuation of the *path* for interest rates that had been judged desirable in the previous cycle.

Svensson writes as if the procedure will not lead to a contrary decision in the next cycle, except as a result of new information in the meantime,<sup>17</sup> so that any such deviations should be unforecastable. But this amounts to an assumption that the optimal *commitment* path, chosen once-and-for-all in a given decision cycle, is *time* consistent. This is exactly what Kydland and Prescott (1977) showed is *not* generally true, when the private sector is forward-looking. The procedure described by Svensson will generally result in the choice of interest-rate paths in subsequent cycles that differ systematically and predictably from the continuation of the path chosen earlier. But then there is no reason for the private sector to actually expect the path considered at the earlier time, so that the conditional forecasts made under the assumption that the path is credible are incorrect.

Such a procedure could actually lead to an outcome that is even *worse* than discretionary optimization (under which the central bank correctly recognizes what the private sector expects its future policy to be like). This can be illustrated using the simple forward-looking model of aggregate supply introduced above. For simplicity, let us suppose that the level of GDP is itself directly controlled by the central bank, so that we do not need to model the transmission mechanism. Under the Svensson procedure, the central bank in period zero should consider alternative paths for the output gap from then on, given the occurrence of the adverse supply shock in that period. It should compute the implied paths of inflation in each case (using the aggregate supply relation (1)), and evaluate the intertemporal loss function (2). It will then find that the optimal path is the one indicated by the solid line in panel (a) of Chart 1, and accordingly will choose to allow an output gap of only - 1.13 percent in period zero.

In period one, however, the Svensson procedure directs the central bank to repeat this process, now understanding that the supply disturbance is no longer present and with no expectation of further shocks. It will then conclude that choosing a zero output gap from period one on should lead to zero inflation from then on, and this will obviously be optimal. It will then choose a zero output gap in period one and thereafter. But if the private sector correctly anticipates this pattern of conduct in period zero, expected future inflation will equal zero, as in the equilibrium resulting from discretionary optimization. Because there are no deflationary expectations, the small output decline results in *more inflation* under this policy than does the same size output gap under the optimal commitment. The result is an even higher initial burst of inflation than occurs under discretionary policy, shown by the dotted line (labeled “hybrid policy”) in Chart 1. (It corresponds to point C in Figure 1.) This policy is even worse than discretionary optimization, for the discretionary policy is at least optimal among those policies that assume no ability to commit to a non-zero output gap after the disturbance has subsided. (The expected value of the discounted loss criterion for this policy is shown on the third line of Table 1.)

Another logical problem with Svensson’s account of inflation fore-

cast targeting is that it presumes that the central bank *can* use its model to determine equilibrium paths for its goal variables corresponding to any arbitrarily specified path for its instrument. This might seem straightforward, as indeed it is in the case of a mechanically evolving (backward-looking) system of the kind assumed in conventional optimal control theory. But matters need not be so simple in the case of a forward-looking model. In the simple model consisting of equations (1) and (4), it is not the case that for given expected paths of the exogenous variables  $u_t$ ,  $r_t^n$  and a given path for the central bank's instrument  $i_t$ , one can solve for unique rational expectations equilibrium paths for inflation and the output gap. Under an exogenously specified path for the nominal interest rate, equilibrium is *indeterminate* in this model, for essentially the same reason as in the famous analysis of Sargent and Wallace (1975). This means that there is a large multiplicity of possible self-fulfilling expectations under such a policy specification.<sup>18</sup>

However, this does not mean that the model is incomplete, nor that there is any general problem with the use of an interest-rate instrument by the central bank. If one specifies central bank policy in terms of an interest-rate *feedback rule*, such as the “Taylor Rule,” determinacy is restored.<sup>19</sup> Thus, there is no problem in using this sort of model to predict the consequences of systematically following (and being expected to follow) a particular *instrument rule*, at least in the case of the kind of instrument rules that are of practical interest.

This is, in fact, the kind of policy evaluation exercise for which quantitatively realistic forward-looking models are currently used at central banks, such as the Federal Reserve Board, the Bank of Canada, and the Reserve Bank of New Zealand.<sup>20</sup>

Finally, even supposing that these problems can be resolved—so that “forecast targeting” leads to a determinate prescription, and correctly models the effect of the bank's systematic policy upon private-sector expectations—there remains the problem that a purely forward-looking analysis of this kind must yield an inferior outcome because it does not allow for the kind of *history-dependence* that generally characterizes fully optimal policy. For example, in the case of the supply shock analyzed above, *no* decision framework according

to which the previous occurrence of the supply shock becomes irrelevant after it ceases to affect current inflation and output determination can possibly result in the optimal responses of inflation and output to that shock or anything very close to them. *Any* such framework (that does not result in unnecessary randomness) will lead to choice of a zero output from period one onward, and so the best outcome that can be hoped for is for the procedure to be equivalent to discretionary optimization. Doing better than this would require taking account, in some way, of the idea that policy should be constrained to be consistent with what it was desirable in the past for the private sector to have anticipated.

### **Rule-based policy-making**

An alternative approach to the conduct of monetary policy is to use One's model of the economy to evaluate the consequences of systematic adherence to one or another *instrument rule*, such as the "Taylor Rule." And once a good rule has been found through such analysis, adjust one's interest-rate instrument accordingly. This general approach to monetary policy has been advocated, in particular, by Taylor (1993, 1998) and McCallum (1988, 1999). Examples of the kind of evaluation of alternative rules that this approach calls for, carried out in the context of forward-looking models derived to varying extents from explicit consideration of private-sector optimization, can be found in the papers cited earlier,<sup>21</sup> as well as Ireland (1997), McCallum and Nelson (1999), Woodford (1999c), Clarida et al. (1999), Svensson (1999c), and a number of the papers collected in Taylor (1999).

Such a procedure takes full account of the forward-looking character of private-sector behavior and of the advantages of credible commitment summarized above. It would also seem, in principle, to facilitate policy credibility, insofar as this involves accurate forecasting of central bank behavior by the public. A simple feedback rule would make it easy to describe the central bank's likely future conduct with considerable precision, and verification by the private sector of whether such a rule is actually being followed should be straightforward as well.

Svensson criticizes a rule-based approach to the conduct of monetary policy as both unrealistic and undesirable. Perhaps most obviously, he argues that a once-and-for-all commitment to a particular instrument rule is unattractive, no matter how careful the research that goes into the choice at that particular point in time. For such a commitment would leave no room for improvement of policy in the light of subsequent research, nor for response to events that were simply not foreseen, even as possibilities, at the earlier date. He similarly stresses the arbitrariness of being bound at all times by a commitment that appeared desirable at one single point in time. “Why,” he asks, “is period zero special?”

I believe that these objections are based upon a misunderstanding of what a rule-based approach to the conduct of policy should mean in practice. Svensson’s discussion assumes that such an approach requires a once-and-for-all commitment to a rule that is chosen at a single point in time. Such a commitment is presumably thought to be needed on the grounds that optimal commitments are not generally time consistent, as discussed above. However, the optimal commitment fails to be time consistent *only* if the central bank considers “optimality” at each point in time in a way that allows it to consider the advantages, from the vantage point of that particular moment, of a policy change at that time that was not previously anticipated. In order to resolve this problem, it is necessary, as McCallum (1999, sec. 2) argues, for the central bank to forswear “any attempt to exploit ... given inflationary expectations for brief output gains.”

The way that this can be done is for the central bank to adopt, not the pattern of behavior from now on that it now would be optimal to choose, taking previous expectations as given, but rather the pattern of behavior *to which it would have wished to commit itself to at a date far in the past*, contingent upon the random events that have occurred in the meantime. This “timeless perspective” ensures that the program of action that one would choose at date one is indeed the continuation of the program that one would choose at date zero: in each case, it is the program that one would have wished to commit to at date far in the past, conditional upon one’s reaching the state of the world that actually exists now. Thus, there is, in fact, no time-consistency problem

with a commitment of this kind. Nor is behavior constrained by what happened to appear desirable at an arbitrary past date (“period zero”).<sup>22</sup> One can allow the central bank to re-compute the optimal program during each decision cycle, and—if its model and its objective remain the same, and it remains committed to the “timeless perspective”—it will renew its commitment to *exactly the same* program of state-contingent action each time.

At the same time, if the bank’s belief about the best model of the economy changes, or if a state is realized that was not contemplated under the previous model, there is no reason for the bank to feel bound by the path for interest rates that it projected in the past, using a now-discarded model of the economy. The fact that the bank expected that path in the past, and may have communicated that forecast to the public as well, is beside the point. After all, under the “timeless perspective,” one chooses to act as one believes one *would have* wished to commit oneself to act at a date far in the past, not as one *actually did* commit oneself to act at any such distant past date (say, prior to the formulation of the natural rate hypothesis).<sup>23</sup> Thus, a rule-based approach to policy-making need not imply any obstacle to the use of all available knowledge at the time.

There is, nonetheless, no logical inconsistency involved in choosing the policy rule that one will follow by calculating its consequences, under the assumption that one will follow it *indefinitely*. For under the assumption that one’s model correctly describes the evolution of the economy over an indefinite future, one has no reason to anticipate believing otherwise later, and, hence, no reason to anticipate the choice of an alternative policy rule at any later date. If there is any reason to anticipate a change in the structure of the economy at a later date, that structural change should already be incorporated into one’s current model of the economy (which, to be complete, must describe the economy’s future evolution).

The application of such an analysis to the simple model of aggregate supply constituted by equation (1) may be sketched using results derived in the appendix. If we allow for a target output gap  $\chi^* > 0$  and assume i.i.d. supply shocks  $u_t$  each period, then the optimal state-con-

tingent commitment from some date  $t_0$  forward—chosen to minimize the expected value of the generalized version of (2), conditional upon the state of the economy at date  $t_0$ , when the commitment is chosen—involves an output gap each period given by

$$(6) \quad \chi_t = \chi^* \mu_1^{t-t_0+1} - \theta \sum_{j=0}^{t-t_0} \mu_1^j u_{t-j}.$$

This prescription embodies the responses to shocks in period  $t_0$  and later shown in panel (a) of Chart 1, but no responses to shocks in any periods prior to  $t_0$ . It also involves a deterministic component that converges asymptotically to zero, but that is initially positive, representing exploitation of the opportunity to run a stimulative policy without the resulting temporary inflation having any effect upon inflation expectations prior to date  $t_0$ .<sup>24</sup> The time inconsistency of such a commitment can be seen from the fact that it depends upon the value of  $t_0$ ; this also displays the arbitrariness to which Svensson objects.

The commitment that would instead be chosen from the “timeless perspective” is obtained by letting  $t_0$  approach minus infinity in (6), so that

$$(7) \quad \chi_t = -\theta \sum_{j=0}^{\infty} \mu_1^j u_{t-j}.$$

In this case, the output gap chosen for date  $t$  is the same (as a function of the history of shocks up until that date) regardless of the date at which the policy is evaluated. There is, thus, no longer any need for a once-and-for-all commitment that cannot be reconsidered at a later date.

An expression such as (7) is unwieldy as a policy rule.<sup>25</sup> For example, implementation of this formula would require that the numerical magnitude of the supply shock  $u_t$  be determined in each period, extending arbitrarily far into the past (though shocks far in the past would matter very little). It is, instead, convenient to choose an alternative representation of the policy rule that leads equally to (7) as the equilibrium outcome, given equilibrium relation (1). It is shown in the appendix that

$$(8) \quad \chi_t = \chi_{t-1} - \theta\pi_t.$$

is a rule of this kind: in an economy where (8) has always been followed and always expected to be followed, the output gap will be the function specified in (7) of the history of shocks. (Inflation will similarly evolve as under the optimal commitment solution.)

Alternatively, the optimal once-and-for-all commitment that would be chosen at date  $t_0$  can be expressed as a commitment to set the output gap according to (8) in every period  $t > t_0$ , with the rule in period  $t_0$  only being replaced instead by

$$\chi_{t_0} = \chi^* - \theta\pi_{t_0}.$$

(One observes that integration of (8) using this initial condition yields (6).) The special stipulation for period  $t_0$ , of course, keeps such a commitment from being time-consistent; from the “timeless perspective,” one should, instead, be willing to commit to following (8) in all periods.

Another rule that might equally well be chosen from the “timeless perspective” is the rule (3) mentioned earlier, involving the price-level target  $p^*$ . Rule (8) is just a first-differenced version of (3), so the two are equivalent in terms of what they imply after the rule has been followed for a long time. Commitment to (3), for *any* choice of the price-level target  $p^*$ , implies a commitment to satisfy (8) as well, in all periods after the first. Alternatively, commitment to (8) is equivalent to commitment to a rule of the form (3), for a particular implicit price-level target given by initial conditions,

$$p^* = p_{t_0-1} + \theta^{-1}\chi_{t_0-1}$$

where  $t_0$  is the first period in which (8) is adopted.

The other objections that Svensson cites to basing policy decisions upon an optimal instrument rule are of equally doubtful import. He stresses the difficulty of deciding which instrument rule is best, given the existence of competing models and uncertainty about parameter

values. But this objection has little force once one realizes that no once-and-for-all commitment to a specific rule is required. A central bank will properly use *its own* model of the economy in determining which rule it should follow; this should represent the consensus view of its staff, at a given point in time, as to the best way of modeling the effects of alternative policies. The staff should certainly be aware of their uncertainty about the accuracy of their model, and an analysis of robustness to alternative model specifications should be an important criterion in selecting a desirable policy rule. Making a model central to the policy decision process obviously raises questions about the accuracy of the model, and this may make it appropriate for banks to expend more resources on model construction and testing; but the situation is not appreciably different than in the case of “inflation forecast targeting.”

Svensson further argues that “commitment to an instrument rule does not leave any room for judgmental adjustments and extra-model information.” But this is not so under the rule-based procedure just sketched. For example, Drew and Hunt (1998) describe how “judgment” is used in constructing the “central scenario” under the Reserve Bank of New Zealand’s Forecasting and Policy System, even though this is a projection of what should happen under (partially credible) commitment to a specific interest-rate feedback rule, to be followed indefinitely. There is no reason, when asking which conduct an optimal rule would prescribe for a situation *like the present one*, that one should not be able to supply *ad hoc* information about the special nature of current shocks and recent past shocks, as to which equations of the model have recently been disturbed (or are expected soon to be), by how much, and how long the disturbances are expected to last. It is simply important that the model be used to ask how one *would have* wished to commit oneself, at a time far in the past, to behave in the case of a shock of this kind. The answer may depend upon details of the type of shocks currently affecting the economy, which may need to be supplied on an *ad hoc* basis.

Finally, Svensson argues that in the absence of a “commitment mechanism,” commitment to an instrument rule would not be “incentive-compatible.” The suggestion seems to be that central bank behav-

ior will inevitably reduce to pure discretion, in the absence of penalties for such behavior that are unlikely to exist. Such an assumption is, indeed, common in recent theoretical analyses of central bank behavior. But it is tantamount to an argument that rational persons are, as such, incapable of self-control or ethical behavior. As Blinder (1998, p. 49) points out, problems of time consistency arise in many areas of personal and public life, and are dealt with “by creating—and then usually following—norms of behavior, by building reputations, and by remembering that there are many tomorrows. Rarely does society solve a time-inconsistency problem by rigid precommitment or by creating incentive-compatible compensation schemes for decision-makers.” As Blinder also notes (pp. 40-41), the fact that central banks in most of the industrial world succeeded in disinflating in the 1980s—in the absence of any obvious reason for the inflationary bias resulting from discretionary optimization to have disappeared—strongly suggests that these institutions are quite capable of disciplined behavior, once they come to understand the reason for it. Given the compelling arguments for the inefficiency of discretionary optimization, it is hard to see why central banks should not be capable of commitment to a systematic decision-making procedure that promises a better outcome on average, according to their own economic models.<sup>26</sup>

One advantage of straightforward commitment to a simple instrument rule, not allowing for continual re-evaluation of the rule in each decision cycle, would be that—if the commitment could be made public and credible—it would make it easy for the public to predict future policy, eliminating resource misallocations due to expectational errors. How would this be addressed under rule-based policy-making of the kind proposed here? The answer is that it would be important to explain the decision process to the public as well as possible. This might well involve an effort to describe the “baseline” policy rule that the bank intends to follow (given its current model), in the absence of special factors that would justify deviation from it—even though it would be clearly stated that the simple rule did not represent a complete description of the bank’s policy. It would also be appropriate to periodically explain the current conditions that have been taken to justify deviations from the baseline rule—for example, through an infla-

tion report." Such reports might well present simulations of the bank's model under alternative assumed rules of conduct, in order to help clarify the nature of the rule-based decision process. Such projections should themselves directly help to anchor public expectations.

### Credibility and the "liquidity trap"

This is not the occasion for a complete treatment of the difficult issues raised by the possibility of a "liquidity trap"—by which I mean the possibility that a central bank's objectives may be thwarted by an inability to lower its overnight interest-rate instrument below the floor of zero, as has recently occurred in Japan.<sup>27</sup> However, I wish to point out that in this context, once again, a credible commitment to the right kind of history-dependent policy is the key to minimizing the losses resulting from such a state.

It cannot be excluded that the zero interest-rate bound would occasionally constrain monetary stabilization policy, even under an ideally well-managed regime. Svensson speaks of inflation targeting as a regime that should prevent an economy from falling into such a trap, because both deflation and deflationary expectations will be counteracted as soon as there is any threat of either. It does seem likely that such a policy would reduce the likelihood of a central bank's facing such difficulties. But there remains the possibility that a central bank may find itself *unable* to achieve its inflation target as a result of the zero bound.

Suppose, for example, in the context of the simple model described by equations (1) and (4), that there are never any inefficient supply shocks  $u_t$ , and that the objective of stabilization policy is given by (2). Then, in the absence of any constraint imposed by the zero bound, optimal policy will clearly seek to maintain zero inflation and a zero output gap at all times. However, such an equilibrium would require that the central bank's short-term interest-rate instrument  $i_t$  equal the natural rate of interest  $r_t^n$  at all times.

The latter quantity varies exogenously in response to real factors that affect the supply of savings and the desire to invest, and there is no economic principle as a result of which it may not sometimes be

negative. But if this ever occurs, then there does not exist an equilibrium with zero inflation and a zero output gap at all times, because nominal interest rates must remain above the natural rate during periods when the latter is temporarily negative.<sup>28</sup>

As a result, the zero bound will sometimes bind, in the case of a central bank that pursues an optimal policy. Of course, the problem could easily be avoided by maintaining a sufficiently high constant rate of inflation. For this reason, the problem is one that will deserve greater attention in an era of price stability.

What can a central bank do to minimize the degree of undesired deflation and output contraction resulting from periods in which the zero bound keeps it from reducing its interest rate instrument by as much as the natural rate has fallen? Svensson refers to “contingency plans and emergency measures” that should apply in the event of “an imminent liquidity trap,” though policy would, at all other times, be conducted according to the inflation forecast targeting procedure discussed earlier. This discussion, I believe, is inadequate in two important respects.

The first is that it implies that the special measures intended to mitigate the effects of the liquidity trap should be invoked *only while the zero bound prevents the inflation target from being achieved*. It is taken for granted that once conditions improve, so that conventional interest-rate control can once again achieve the target rate of inflation, one should return to inflation targeting. But, in fact, it is unlikely that monetary policy can do much to loosen the constraint imposed by the zero bound, except by *changing what people expect policy to be like after the constraint ceases to bind*.

It is sometimes argued that further expansion of the money supply after overnight rates have fallen to zero should be able to expand aggregate demand through channels other than the interest-rate channel. For example, the Pigou-Patinkin “real balance effect” is sometimes cited as such a channel. As shown in Woodford (1999a), it makes sense that higher real balances should, in general, increase aggregate demand by a small amount, even in the absence of any

change in real interest rates or income expectations, owing to the way in which higher money balances reduce transactions frictions. However, this effect—which should, in any event, be quantitatively small (given evidence on money demand in economies like the U.S.)—*vanishes entirely* in the case that overnight interest rates fall to zero. This is because interest rates can fall to zero only because sufficient money balances are already being held for there to be no possibility of further reduction in transactions frictions from holding more wealth in that form. Money is then equivalent in portfolios to riskless short-term government securities, so that open market operations between the two assets have no effect upon equilibrium determination.

It is often suggested that even if this is true, open market purchases of other kinds of assets should still be able to affect the prices of *those assets*, and so affect incentives to spend. For example, it is suggested that as long as longer bonds have yields above zero, purchases of such bonds by the central bank should lower longer-term interest rates. However, the expectations theory of the term structure implies that this should not be possible, *unless* such actions are taken to signal a change in the bank's commitments regarding future monetary policy (i.e., the future path of overnight rates). Indeed, given the equivalence in portfolios of money and short government securities under the circumstances of a "liquidity trap," such a policy would be effective only insofar as it would *also* be possible to lower long rates by simply changing the maturity structure of outstanding government debt, with no change in the monetary base. Such efforts to "twist" the yield curve have not been notably successful in the past, and most central banks do not currently advocate such measures as a policy tool. Alternatively, it is suggested that purchases of foreign exchange should be able to depreciate the exchange rate, and stimulate spending in that way. But here too, as Svensson notes, interest-rate parity implies that such policies should have no significant effect upon the exchange rate, except insofar as they change expectations about future monetary policy.

What remains true, as stressed by Krugman (1998), is that a central bank in such circumstances *would* be able to stimulate aggregate demand if it could credibly commit itself to a more expansionary *future* policy. Given forward-looking private sector behavior, such a

commitment would affect current spending, both because higher expectations of future inflation would lower *real* interest rates even if nominal rates cannot be lowered, and because *long* rates would fall if the private sector came to expect that short rates would remain low for a longer time. But the only commitment that matters in this regard has to do with policy *after* the central bank's hand is no longer forced by the zero interest-rate bound—i.e., in the simple model proposed above, after the natural rate of interest has again become positive. By implying that “emergency measures” should be necessary only when the central bank is not able otherwise to achieve its long-run inflation target, Svensson suggests an approach to policy-making under which expectations about later policy *will not* respond to the occurrence of the contractionary shock, and, thus, cannot play any role in mitigating its effects.

What would be needed instead, if it could be made credible, would be a commitment to a history-dependent policy, under which policy would temporarily be more expansionary than would be consistent with the long-run inflation target in the period immediately following the return of the natural rate of interest to a normal (positive) level.<sup>29</sup> This is, thus, a case in which discretionary policy-making (with loss function (2) has a *deflationary* bias, and the same is true of any purely prospective decision-making procedure, such as inflation forecast targeting.

Could a commitment to such a history-dependent policy be made credible? A mere announcement of intentions may not suffice; for there would be even greater reason for skepticism about a new commitment of this kind, announced after an economy has already fallen into a “liquidity trap,” than is true of policy reforms in general. First, an announced policy change that requires no alteration of current policy (because the zero bound binds in the short run) is particularly conducive to doubts about “cheap talk.” And in the case of an announced commitment that applies only to the aftermath of an unusual situation, past behavior following similar crises cannot be appealed to as evidence of one's seriousness, while the central bank's interest in establishing a reputation with regard to how it would act in future instances of the same kind might be doubted as well (given that recurrences are not expected to be frequent).

The best approach would surely be to commit oneself to a history-dependent policy of a kind that would mitigate such crises *before* any disturbance occurs that causes the zero bound to bind, and to follow a rule that makes policy history-dependent even when the zero bound has not yet been a constraint. In this way, the credibility of the central bank's commitment could become established before it is subjected to the severe test of a "liquidity trap." A simple example of the kind of commitment that would help, as mentioned by Svensson, would be commitment to a target for the price level, rather than the rate of inflation. Then any unavoidable deflation during a period in which the zero bound binds would automatically give rise to expectations of a subsequent period of higher-than-average inflation.<sup>30</sup>

Other rules that would lead to similar equilibrium inflation expectations, if sufficiently credible, are the "inertial" interest-rate feedback rules

$$(9) \quad i_t = (1 - \theta)i^* + \theta i_{t-1} + \phi_\pi \pi_t + \phi_\chi \chi_t$$

with  $\phi_\pi > 0$ ,  $\phi_\chi \geq 0$ , and  $\theta \geq 1$ , discussed in Rotemberg and Woodford (1999), Woodford (1999c), and Williams (1999).<sup>31</sup> Under these rules, interest rates would be kept low for a time, even if inflation and the output gap increased, simply because recent past interest rates had been low. In equilibrium, as shown in those papers, this means that deflationary shocks are followed by periods of higher-than-average inflation. Even better would be the variant rule

$$(10) \quad i_t = \sum_{j=0}^{\infty} \theta^j [\phi_\pi \pi_{t-j} + \phi_\chi \chi_{t-j}]$$

considered by Reifschneider and Williams (1999). Such a rule is equivalent to (9) as long as the zero bound never binds; but when it does, it has the advantage that interest rates remain low for a period afterward, not simply because interest rates have recently been equal to zero, but because one *wished to push them even lower than zero* in the recent past.

Rules of this kind are exactly what would result from the rule-based

approach to policy-making described above, as long as the central bank's loss function assigns a sufficient penalty to variations in its interest-rate instrument, as in (5) above. (The papers just cited all show that rules of this kind are nearly optimal under such an objective, in the context of forward-looking models of varying degrees of complexity.) If such an approach to monetary policy were followed *all* of the time, there might be little need for special "contingency plans" that apply only in the case of a "liquidity trap."

### Appendix

Here, we consider the choice of a policy to minimize a loss criterion of the form

$$(11) \quad E \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda (\chi_t - \chi^*)^2 \right] \right\}$$

subject to the constraint that the paths of inflation and the output gap must satisfy the aggregate supply curve (1). This loss function generalizes (2) in the text to allow for the possibility that the optimal output gap takes a value  $\chi^* > 0$ . Svensson assumes that  $\chi^* = 0$  in order to eliminate the inflation bias resulting from discretionary optimization. Here, we allow for the possibility that  $\chi^* > 0$ , in order to show that when a policy rule is chosen from the “timeless perspective” advocated above, no inflation bias results from the central bank’s attempt to minimize a criterion with this feature.<sup>32</sup>

As shown in Woodford (1999c), the optimal plan under commitment can be determined in a case of this kind by writing a Lagrangian

$$(12) \quad E \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} \left( \pi_t^2 + \lambda (\chi_t - \chi^*)^2 \right) + \phi_t (\pi_t - \kappa \chi_t - \beta \pi_{t+1} - \mu_t) \right] \right\}$$

where  $\phi_t$  is a Lagrange multiplier associated with constraint (1) in period  $t$ . Note that in writing the constraint terms in (12), we are able to replace  $E\{E_t \pi_{t+1}\}$  by  $E\{\pi_{t+1}\}$ , using the law of iterated expectations. The optimal once-and-for-all commitment that would be selected as of date  $t_0$  is then given by the stochastic processes  $\{\pi_t, \chi_t\}$  for dates  $t \geq t_0$  that minimize (12), for some multiplier process  $\{\phi_t\}$  chosen so that the solution satisfies (1) at all times.

Differentiation of (12) yields the first-order conditions

$$(13) \quad \pi_t + \phi_t - \phi_{t-1} = 0,$$

$$(14) \quad \lambda(\chi_t - \chi^*) - \kappa\phi_t = 0$$

for all  $t \geq t_0$ , with the initial condition

$$(15) \quad \phi_{t_0-1} = 0.$$

Condition (15) is another way of saying that the first-order condition (13) takes a different form in period  $t_0$  only, because there is no constraint corresponding to (1) for period  $t_0 - 1$  in the Lagrangian (12). (This, in turn, reflects the fact that inflation expectations at date  $t_0 - 1$  are taken as a historical given in choosing a once-and-for-all commitment at date  $t_0$ .) This difference in the conditions for optimality in the case of period  $t_0$  indicates the way in such a commitment treats period  $t_0$  as “special,” and it is also the reason why such a commitment is almost inevitably not time-consistent.

We can use (14) to eliminate the Lagrange multipliers from (13), obtaining the optimality condition

$$(16) \quad \pi_t + \frac{\lambda}{\kappa}(\chi_t - \chi_{t-1}) = 0,$$

for periods  $t > t_0$ . In the special case of period  $t_0$ , we must also use (15), and thus obtain instead

$$(17) \quad \pi_{t_0} + \frac{\lambda}{\kappa}(\chi_{t_0} - \chi^*) = 0.$$

We then simply need to solve for the processes  $\{\pi_t, \chi_t\}$  that satisfy (1), (16), and (17), together with a transversality condition. Equivalently, we can solve equations (1) and (16) for all periods  $t \geq t_0$ ; imposing the fictitious initial condition

$$(18) \quad \chi_{t_0-1} = \chi^*.$$

Using (16) to substitute for inflation in (1), we then obtain a second-

order stochastic difference equation for the output gap,

$$(19) \quad \beta E_t \chi_{t+1} - \left( 1 + \beta + \frac{\kappa^2}{\lambda} \right) \chi_t + \chi_{t-1} = \frac{\kappa}{\lambda} u_t.$$

We wish to solve (19) imposing the initial condition (18) and the transversality condition. Because the characteristic polynomial

$$(20) \quad \beta \mu^2 - \left( 1 + \beta + \frac{\kappa^2}{\lambda} \right) \mu + 1 = 0$$

has two real roots

$$0 < \mu_1 < 1 < \beta^{-1} < \mu_2 = (\beta \mu_1)^{-1},$$

(19) has a unique bounded solution consistent with the initial condition, and this bounded solution is the one that satisfies the transversality condition.

Standard methods then imply that the solution is of the form (6), where  $\theta \equiv \kappa / \lambda$ .<sup>33</sup> Given a solution for the output gap, one can then solve (1) forward to obtain the path for inflation under the optimal commitment, given by

$$\pi_t = \frac{\kappa}{1 - \beta \mu_1} \chi_t + u_t.$$

The impulse responses to a supply shock at date zero, under an optimal commitment of this kind chosen at any date  $t_0 \leq 0$ , are then given by

$$\begin{aligned} E_0 \chi_\tau - E_{-1} \chi_t &= -\theta \mu_1^\tau u_0 \text{ for all } t \geq 0, \\ E_0 \pi_0 - E_{-1} \pi_0 &= \left( 1 - \frac{\kappa \theta}{1 - \beta \mu_1} \right) u_0, \\ E_0 \pi_\tau - E_{-1} \pi_t &= -\frac{\kappa \theta}{1 - \beta \mu_1} \mu_1^t u_0 \text{ for all } t > 0. \end{aligned}$$

These are the responses plotted in Chart 1, for the case of a shock  $u_0 = 1$ . The parameter values used are  $\beta = .99$ ,  $\kappa = .024$ , and  $\theta = 7.88$ , taken from the estimates in Rotemberg and Woodford (1997) of a model with a slightly more complicated version of this aggregate supply relation. In the model, each period represents a quarter (so that, for example, the implied rate of time preference is 4 percent per year). The value of  $\kappa$  just cited is the coefficient in (1) when  $\pi_t$  represents the quarterly first difference of the log price level. If the annual inflation rate were instead used in (1), the slope of the short-run Phillips curve would be .10. The relative weight on output-gap variability assumed in the loss function is given by  $\lambda = \kappa / \theta = .003$ , when inflation is measured as the quarterly first difference. This corresponds, if one instead uses an annualized inflation rate as one's measure of inflation, to a relative weight on output gap variations that is sixteen times this, or about .05.

An interesting feature of this solution is that

$$\lim_{t \rightarrow \infty} E_t \pi_T = 0$$

even when  $\chi^* > 0$ . Thus, an optimal commitment involves an average inflation rate of zero (the value that minimizes the first term in the loss function), despite the fact that the associated average output gap (zero) is inefficiently low, *and* despite the fact that there is a long-run Phillips curve trade-off between output and inflation in this model. Thus, even though in this model it *would* be possible to increase output on average permanently at the price of a finite positive average inflation rate, it is optimal for the central bank to commit itself not to do so. It is also interesting to note that the impulse responses imply that the effect of a supply shock on the long-run price level is equal to

$$\lim_{t \rightarrow \infty} (E_0 p_t - E_{-1} p_t) = \left[ 1 - \sum_{t=0}^{\infty} \frac{\kappa \theta}{1 - \beta \mu_1} \mu_1^t \right] u_0 = 0,$$

where the last step uses the fact that  $\mu_1$  satisfies (20).

The consequences of discretionary optimization are computed under the assumption of a Markovian equilibrium, again as in Wood-

ford (1999c). Given that  $u_t$  is the only state variable in this simple model, this means that we assume a solution in which inflation and the output gap depend only upon the current shock  $u_t$ . If  $u_t$  is i.i.d., this means that expected future inflation  $E_t \pi_{t+1}$  is equal to a constant, which we may denote  $\pi^{ss}$ . A central bank that seeks to minimize (11) under discretion will thus choose  $x_t$  each period to minimize

$$\pi_t^2 + \lambda(\chi_t - \chi^*)^2$$

subject to the constraint

$$\pi_t = \kappa\chi_t + \beta\pi^{ss} + u_t.$$

This static problem has the solution

$$(21) \quad \pi_t = (1 + \kappa\theta)^{-1} [\kappa\chi^* + \beta\pi^{ss} + u_t].$$

Imposing the consistency condition that the expected value of this equal  $\pi^{ss}$  implies that

$$\pi^{ss} = (1 - \beta + \kappa\theta)^{-1} \kappa\chi^* > 0,$$

so that there is, as usual, an inflation bias associated with discretionary optimization when  $\chi^* > 0$ .

Substituting this into (21), we obtain the equilibrium path of inflation under discretion. Note that a disturbance  $u_t$  increases inflation for one period only, resulting in a permanent increase in the price level. Substituting the solution for inflation into (1), we obtain a solution for the output gap, also as a linear function of the current supply shock. These are the responses plotted in Chart 1 with the dashed line.

The equilibrium statistics reported in Table 1 are computed using the parameter values quoted above, under the assumption that  $u_t$  is an i.i.d. random variable with a standard deviation of 1 percentage point,

when (1) is written in terms of the annualized inflation rate. These statistics are computed under the assumption that  $\chi^* = 0$ , so that the statistics reported for discretionary optimization do not involve the effects of any bias in the average rate of inflation.

A policy rule that can achieve the optimal pattern of responses as a rational expectations equilibrium is given by (8). In fact, (8) is equivalent to the condition (16) that we have used above to solve for the optimal commitment. Thus, if the central bank is committed to adhere to (8) in all periods  $t > t_0$ , and to the rule (17) in period  $t_0$ , we have already shown that there is a unique bounded rational expectations equilibrium, and that it achieves the optimal feasible paths for inflation and the output gap, characterized above.

The rule (8) can also be equivalently expressed as a commitment to maintain the value of  $\chi_t + \theta p_t$  constant over time. Thus, a commitment to (8) in periods  $t > t_0$  and to (17) as an initial condition is equivalent to a commitment to a rule of the form (3), where the “target” price level  $p^*$  is given by

$$p^* = p_{t_0-1} + \chi^* / \theta.$$

Hence, this too is an example of a rule with the property that commitment to it implies a determinate equilibrium that achieves the optimal feasible outcome.

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## Endnotes

<sup>1</sup> See, e.g., King (1997).

<sup>2</sup> This is not true in certain simple cases, such as the model considered by King (1997). In that model, the responses to shocks associated with purely discretionary optimizing behavior are identical to those associated with the optimal state-contingent commitment; the only difference is in the average inflation rate associated with the two policies. On the basis of such an example, one might conclude that the optimal state-contingent commitment could be implemented by a discretionary regime, as long as the discretionary behavior is constrained to be consistent with the optimal average rate of inflation. But in general, matters are more complex, as the example considered here shows. As a result, the sense in which discretion must be “constrained” in order for it to approximate the optimal state-contingent commitment is more complex.

<sup>3</sup> This is here understood to refer to the percentage difference between actual output and the (time-varying) level of “potential” output that would represent equilibrium output if prices were completely flexible and the inefficient supply shocks  $u_t$  were not present.

<sup>4</sup> While I shall here discuss only supply shocks of this kind, it is important to remember that many kinds of disturbances to aggregate supply should also change the efficient level of output, often in roughly the same proportion as they shift the level of output consistent with stable prices (Woodford, 1999b). I emphasize the inefficient case here because it presents an instructive contrast between alternative decision procedures for monetary policy-making.

<sup>5</sup> Some authors, such as Roberts (1995) and Clarida et al. (1999), assume a specification of this form with  $\beta = 1$  for simplicity. But as shown in Woodford (1996), the correct condition for optimal price-setting in discrete time involves the discount factor. In fact, the inexact alternative specification would prevent us from obtaining some important results, such as the fact that even in the case of a non-zero output gap target, optimal policy involves commitment to an average inflation rate of zero.

<sup>6</sup> The shock is described by  $u_0 = 1$ , i.e., the shock would increase the annualized inflation rate by 1 percentage point, in the absence of any change in the output gap or in inflation expectations. The three panels of the figure show the responses of output (in percent deviation from the steady-state level), the inflation rate (in annualized percentage points), and the price level (in percent deviation from the initial level). The time unit on the horizontal axis is quarters.

<sup>7</sup> The impulse responses, as usual, plot conditional expectations as of date zero.

<sup>8</sup> The numerical parameter values used in this exercise are explained in the appendix. The figure and table also present results for a “hybrid” policy that combines aspects of discretionary optimization and the optimal commitment; this policy is discussed below.

<sup>9</sup> There also exist policies of the same kind—in which the price level eventually returns to its initial level—that *dominate* the discretionary equilibrium, in that both the variance of inflation and of the output gap can be reduced. It simply happens that for the parameter values assumed in this numerical exercise, the optimal commitment, in the

sense of minimizing (2), is not one of those.

<sup>10</sup> Black et al. (1998) find that the inflation-output variance trade-off can be improved by stabilizing the price level, through numerical analysis of alternative simple rules using a simplified version of the Bank of Canada's Quarterly Projection Model. They also show that this effect does not obtain if the private sector's expectations are not assumed to adapt to the new policy regime.

<sup>11</sup> Arguments of this kind are also commonly heard in discussions of the desirability of gradualism in the adjustment of a central bank's interest-rate instrument. See Goodhart (1998) for a review of the debate, Woodford (1999c) for a demonstration that an optimal commitment involves interest-rate inertia in a model like that considered here, and Amato and Laubach (1999) for a non-technical discussion of the argument. Levin et al. (1999) and Williams (1999) find similar advantages of rules characterized by interest-rate inertia through numerical analyses of simple rules in the context of larger, more empirically realistic forward-looking models.

<sup>12</sup> Here, I consider price-level targeting in the sense of a commitment to respond systematically to deviations of the price level from the target path. One may, in some cases, also achieve the optimal equilibrium responses to shocks by assigning the central bank an objective that penalizes deviations of the price level from its target path (rather than deviations of inflation from its target rate, as in (2)), and then allowing the central bank complete *discretion* in the pursuit of this objective, as shown by Vestin (1999). In such a case, a price-level stabilization objective may be desirable, even though the true *social* objective corresponds to minimization of (2), exactly because discretionary optimization does not optimally achieve the objective that the central bank with unconstrained discretion pursues, as discussed in the context of an interest-rate smoothing objective in Woodford (1999c). Possible advantages of a price-level stabilization objective, in the case of a central bank that optimizes under discretion, are also discussed in Svensson (1999d), Kiley (1998), and Dittmar et al. (1999a, 1999b).

<sup>13</sup> If one assumes the particular weight  $\lambda$  in (2) that can be justified as an approximation to expected utility (as shown in Woodford, 1999b) in the case of the model of monopolistically competitive pricing that underlies the aggregate supply relation (1), then the coefficient  $\theta$  in (3) corresponds specifically to the elasticity of demand facing the typical supplier. In this case, (3) has the following interpretation: the central bank should maintain macroeconomic conditions under which the demand curve facing each supplier is such that the price that would allow the supplier to sell output exactly equal to capacity is constant over time. Note that this involves a much larger coefficient  $\theta$  than Hall proposes.

<sup>14</sup> The price-level target chosen does, of course, have a transitory, deterministic effect upon inflation and output in the period immediately following adoption of the rule. There is, thus, a particular, unique choice of  $p^*$  as a function of initial conditions that corresponds to the optimal once-and-for-all commitment that might be adopted at a given point in time. However, as discussed below, rule-based policy-making should not have the aim of enforcing a commitment of that kind in any event.

<sup>15</sup> Similar conclusions are reached regarding the advantages of price-level stabilization on the basis of numerical analysis of alternative policy rules in the context of forward-looking models in Levin et al. (1999) and Williams (1999). These authors also stress the connection between the assignment of a penalty to interest-rate variability and this conclusion.

<sup>16</sup> Of course, if the central bank's approach to its task has *no effect* upon private-sector forecasts—which are formed, say, as the same moving average of past observations no matter how policy is conducted—then the system is effectively a backward-looking one to which standard dynamic-programming methods apply, even if private actions do follow from forecasts. Our discussion is, therefore, entirely premised upon the *possibility* of central-bank credibility. However, as noted earlier, even *partial* credibility suffices to make discretionary policy-making sub-optimal.

<sup>17</sup> “If no new information has arrived, the forecasts and the interest rate path are the same, and interest rate setting follows the same interest rate path” (p. 17).

<sup>18</sup> Svensson (1999b) proposes that the problem of indeterminacy can be resolved by specifying the arbitrary path for the interest rate only for some finite horizon (say, for two years), after which the model is “closed” by assuming that a particular equilibrium obtains after that date. This latter equilibrium is intended to represent the outcome of conducting policy according to the “forecast targeting” procedure, after the terminal date. But there remains an important degree of circularity to this argument: one obtains a determinate result from the “forecast targeting” procedure only because it is assumed that, if that procedure is also expected to be followed in the future, it leads to a determinate result. If, instead, one recognizes the possibility of alternative possible equilibria in the future, they result in alternative equilibria in the present as well.

<sup>19</sup> As shown in Woodford (1999a, 1999c), this depends upon the existence of sufficiently strong feedback, of the right sort, from the endogenous goal variables to the interest rate. Sufficient conditions for determinacy are for the interest rate to be increased more than one-for-one with increases in inflation, and for it to be a non-decreasing function of output, both of which are true of Taylor's (1993) proposed coefficients.

<sup>20</sup> For examples of such analyses, see Coletti et al. (1996), Brayton et al. (1997), Black et al. (1998), Drew and Hunt (1998), Levin et al. (1999), and Williams (1999).

<sup>21</sup> See footnote 16.

<sup>22</sup> This seems to be what McCallum (1999, sec. 2) has in mind in advocating a “systematic” decision-making process. His footnote 6 states: “My meaning of systematic implies that the same actions are specified each time the same conditions are faced, so the response pattern cannot be different for the ‘first’ or ‘first few’ periods. Basically, the optimization calculation must be made from the perspective of a dynamic stochastic steady state.” However, McCallum also states that “systematic” behavior requires that the central bank “optimize once, not each period.” This formulation is not helpful, in my view; what is important is not that the central bank never reconsider its pattern of conduct, but that it adopt a “timeless perspective” when considering it.

<sup>23</sup> The fictitious prior “commitment” that justifies one's actions under this approach to policy-making is somewhat analogous to the fictitious “contract” that is referred to in “social contract” theories of justice. John Rawls' (1971) proposal that the fair terms of social cooperation are those that would be chosen in an “original position” is the best-known modern example of such a theory.

<sup>24</sup> This last temptation exists only insofar as  $\chi^* > 0$ . We now allow for this case as it sharpens the contrast between a once-and-for-all commitment and the commitment chosen from the “timeless perspective,” just as it does the contrast between optimal commit-

ment and discretionary optimization. Note that the value of  $\chi^*$  is irrelevant for the form of commitment that one chooses from the “timeless perspective”; thus, under the procedure recommended here, there is no need to insist upon assigning the central bank a loss function with  $\chi^*=0$ .

<sup>25</sup> In the case that the nominal interest rate, rather than the output gap, is the central bank’s instrument, we could derive a similar expression for the desired evolution of the interest rate as a function of the history of exogenous shocks. But such an expression would not be suitable as a policy rule, since commitment to such a rule would lead to price-level indeterminacy, as mentioned earlier. Hence, an alternative representation of the policy rule, involving feedback from the endogenous goal variables to the instrument, would be essential. See Woodford (1999c).

<sup>26</sup> See also McCallum (1999, sec. 2) for a forceful defense of the view that central banks are capable of rule-like behavior.

<sup>27</sup> In particular, I shall not address the possibility of a self-fulfilling deflationary trap—a rational expectations equilibrium in which deflation is expected, and the zero bound is expected to bind, forever, even though there exists another equally possible rational expectations equilibrium in which prices do not fall and the zero bound does not bind. This possibility, and the role of fiscal policy commitments in excluding undesirable equilibria of that kind, are treated in Woodford (1999a).

<sup>28</sup> As Krugman (1998) points out, the most plausible interpretation of the current situation in Japan implies that the natural rate has, in fact, become significantly negative there. Whether this occurred for reasons independent of monetary policy, or as a result of previous policy mistakes that exacerbated the effect of the asset market crash, will not be addressed here.

<sup>29</sup> The need for a history-dependent policy of this kind is an important theme in Reifschneider and Williams (1999), which considers the properties of several simple history-dependent rules when the long-run target rate of inflation is low enough to cause the zero bound to bind with some frequency.

<sup>30</sup> The advantages of policies that stabilize the price level rather than the inflation rate in mitigating the consequences of the zero bound are analyzed quantitatively by Wolman (1998), in the context of two types of simple forward-looking models.

<sup>31</sup> To take account of the possibility that the zero bound binds,  $i_t$  actually would have to be set equal to the maximum of the right-hand side of (9) and zero.

<sup>32</sup> A similar characterization of the optimal response to an inefficient supply shock is presented in Clarida et al. (1999) and in Vestin (1999), though in these references  $\chi^*=0$  is assumed, and in the first of them the specification of (1) replaces  $\beta$  by one.

<sup>33</sup> Woodford (1999b) shows that if (11) is derived as a quadratic approximation to the level of expected utility of the representative household in a Calvo-type model of staggered price-setting, the weight  $\lambda$  should equal  $\kappa$  divided by the elasticity of demand for an individual product. In this case,  $\theta c$  can be interpreted as this elasticity of demand, which is necessarily greater than one.

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