Inflation During and After the Zero Lower Bound *

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I Introduction

The zero lower bound (ZLB) for nominal interest rates constrains monetary policy responses to adverse shocks. This inability to stabilize the economy is a major concern of central bankers. Because Japan experienced a long period of zero interest rates accompanied by falling prices from the late 1990s to the present, central bankers are also concerned about the possibility of deflation. This paper studies inflation dynamics at the ZLB and during an exit from the ZLB. In particular, we examine the following four broad questions: First, what is the inflation outlook for Japan, the United States, and the Euro Area, the three largest

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economies for which the ZLB has been a constraint in recent years? Second, what inflation dynamics should one expect before and after nominal interest rates lift off from the ZLB? Third, does the fact that both Japan and U.S. have experienced near zero interest rates for more than five years mean that these countries have entered a new, persistent regime in which inflation rates will remain below the value targeted by the central bank? Finally, we ask the questions what would have been different had the U.S. adopted a higher inflation target over the past decade and what would be the effect of raising the inflation target now?

To generate inflation forecasts we estimate an unobserved components model that decomposes inflation into a low-frequency trend component and high-frequency fluctuations around this trend. This model is based on work by Stock and Watson (2007). According to our estimates, trend inflation has remained positive in the U.S. and the Euro Area, whereas it has been negative in Japan since the late 1990s. Looking into the future, the time series model predicts a substantial probability of deflation for Japan over the next five years, while for the U.S. and Europe these probabilities are no more than 20%.

Our answers to the remaining three questions are based on a textbook-style New Keynesian dynamic stochastic general equilibrium (DSGE) model with ZLB constraint. Although DSGE models abstract from the complexities of modern-day economies, they provide a useful framework to analyze the dynamics of output, inflation, and interest rates as well as the potential effects of monetary and fiscal policy interventions. Unfortunately, the predictions coming out of typical DSGE models with a ZLB constraint are ambiguous: the model generates a set of different economic outcomes conditional on the same set of fundamentals or, in more technical terms, the model has multiple equilibria.[1]

Multiplicity of equilibria is both a blessing and a curse. It is a blessing for empirical researchers who are trying to explain very different macroeconomic experiences, say in the U.S. and Japan, with a single economic model. It is a curse for policy makers, because the same monetary policy action of, say, changing interest rates or making announcements about targeted inflation rates, may have very different effects, depending on the equilibrium. However, there is also an opportunity for policy making: actions and statements of central banks may influence the coordination of beliefs among private sector agents and lead to the selection of a desirable equilibrium. Moreover, one can attempt to design policies that make some of the equilibria, preferably the undesirable ones, unsustainable. While the model considered in this paper is not rich enough to provide a formal analysis of equilibrium selection through central bank actions, we will offer an informal assessment.

Considering perfect foresight dynamics we use the small-scale DSGE model to illustrate that there is a multiplicity of inflation and real activity paths around the lift-off from the ZLB. By choosing a desired inflation path and an interest rate feedback rule that implements this path, the central bank can have control over the severity of the liquidity trap caused by an adverse real interest rate shock that pushes the economy to the ZLB. The analysis closely follows recent work by Cochrane (2015).

Equilibrium multiplicity also manifests itself in the existence of two steady states, one in which interest rates are positive and inflation equals the value targeted by the central bank and one in which interest rates are zero and the economy experiences deflation. This fundamental feature of a wide class of DSGE models has led to concern among policy makers whether Japan or the U.S. have transitioned to a persistent regime in which inflation rates are low (or negative) and interest rates are zero. The theoretical mechanism behind such a transition has been studied by Benhabib, Schmitt-Grohé, and Uribe (2001b). For the U.S. the concern that such a transition is underway since 2009 has been prominently voiced by Bullard (2010).

Some authors, for example Christiano and Eichenbaum (2012) have challenged the relevance of deflation equilibrium or a sunspot equilibrium that contains a deflation regime on the basis of learnability. However, Mertens and Ravn (2014) show that with recursive learning, as long as expectational errors are not very large, learning dynamics do not matter much for the key results. Evans (2013) builds a different equilibrium, one in which the economy falls in to a stagnation regime with a large adverse shock and exits this regime with a large fiscal shock, and shows this is robust to learning. We take a more agnostic approach in this paper. We simply would like to investigate the empirical relevance of other equilibria, other than the standard targeted-inflation equilibrium.

Based on our work in Aruoba, Cuba-Borda, and Schorfheide (2014), henceforth ACS, we construct a stochastic two-regime equilibrium in which the economy may alternate between a targeted-inflation and a deflation regime. This equilibrium features an exogenous sunspot shock that serves as a placeholder for a more complete theory of how firms and households coordinate their beliefs and actions. We confront this quantitative model with data from the three economies. Looking at inflation and interest rates, we cannot rule out the possibility that Japan and the U.S. have transitioned to a deflation regime. While too early to tell, so far the European experience appears also to be consistent with the targeted-inflation regime.

Finally, we provide a quantitative assessment for the U.S. of an increase in the target inflation, which has been advocated by several prominent policy makers and scholars, e.g., Blanchard, DellAriccia, and Mauro (2010), Ball and Mazumder (2011), and Krugman (2014). First, we discuss the implications of a historical counterfactual where the Federal Reserve adopted a 4% inflation target after the Volcker disinflation period. In this scenario there could be some improvements in welfare, especially if the Federal Reserve acts even more aggressively to cut the policy rates. Our results show that the recovery from the Great Recession would have been about a year shorter. Second, we have the Federal Reserve change their target abruptly in 2014, during the ZLB episode in the U.S., which is of course the more realistic experiment. Our findings show that this policy change does not generate clear short- to medium-run benefits. The long-run benefits (or costs) strongly depend on the likelihood of adverse shocks that push the economy to the ZLB yet again.

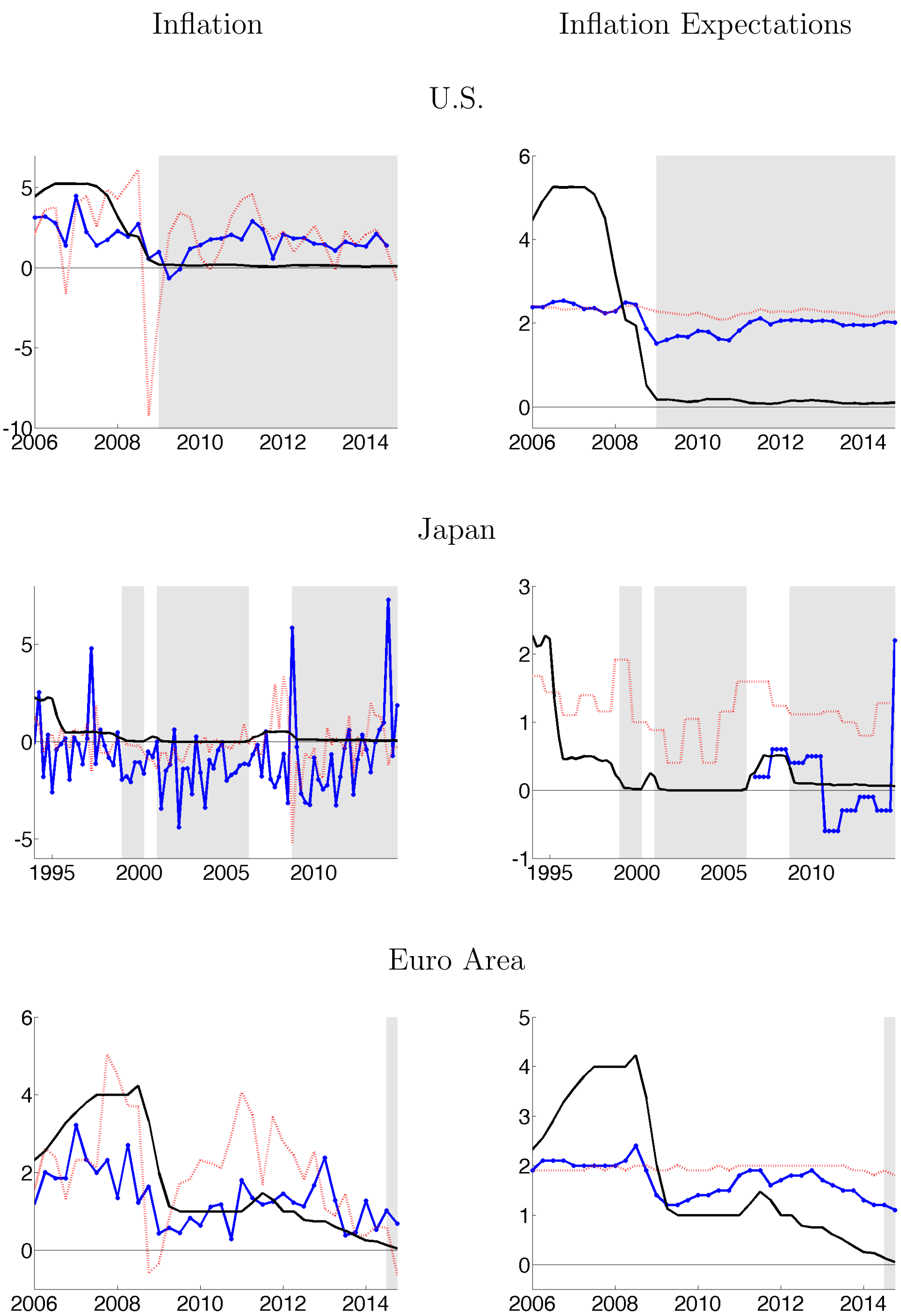
The remainder of this paper is organized as follows. In Section II we compare interest rate, inflation, and inflation expectations data from the U.S., Japan, and the Euro Area, we estimate the unobserved component model, and generate inflation forecasts. Section III starts by reviewing the main building blocks of New Keynesian DSGE models: the consumption Euler equation, the New Keynesian Phillips curve (NKPC), and the monetary policy rule. We then discuss the multiplicity of equilibria in this model, focusing on the two steady states, the model's implied perfect foresight dynamics. Finally we construct a stochastic equilibrium that features a targeted-inflation and a deflation regime. In Section IV we assess the likelihood that the three economies have transitioned to the deflation regime. The potential macroeconomic costs of low inflation rates are discussed in Section V. In Section VI we examine the consequences of adopting to a 4% target inflation rate and discuss monetary and fiscal policies designed to eliminate the multiplicity of equilibria and hence the ambiguity for policy makers among the relationship between policy interventions and macroeconomic outcomes. Finally, Section VII provides a brief conclusion. Data definitions, detailed model specifications, parameter estimates, and analytical derivations are relegated to the Appendix.

II Inflation in the U.S., Japan, and the Euro Area

The empirical analysis in this paper focuses on the recent experiences of the U.S., Japan, and the Euro Area. Interest rates in the U.S. reached the ZLB in 2009. The policy rate of the Bank of Japan has been essentially zero since 1999, with the exception of a short period in 2000-2001 and 2007-08 when it increased to roughly 50 basis points (bp). Interest rates in the Euro Area have been below 50 bp since 2012:Q2 and effectively reached zero in 2014:Q3. Figure 1 depicts monetary policy rates, inflation rates, and inflation expectations for these three economies.[2]

Two observations from Figure 1 stand out. First, while in the U.S. the ZLB episode is associated with positive inflation, GDP deflator inflation rates in Japan have been negative, with the exception of two short spikes.[3] The verdict on the Euro Area is still out: inflation rates have been falling toward the end of the sample as the policy rate has approached zero. Second, long-run (5-year-ahead) inflation expectations have been remarkably stable in the U.S. and the Euro Area, despite falling policy rates. Even more remarkable, 10-year-ahead inflation expectations in Japan have stayed around 1% even the average inflation rate over the past 15 years was negative. Short-run inflation expectations appear to be more sensitive

Figure 1: Inflation and Inflation Expectations



Notes: Left panels: monetary policy interest rate (solid), CPI inflation (dotted), GDP deflator inflation (solid-dotted), where the latter two are annualized quarterly rates. Right panels: monetary policy interest rate (solid), 5-year-ahead (10-year-ahead for Japan) inflation expectations (dotted), 1-year-ahead inflation expectations (solid-dotted). The shaded intervals characterize the ZLB episodes.

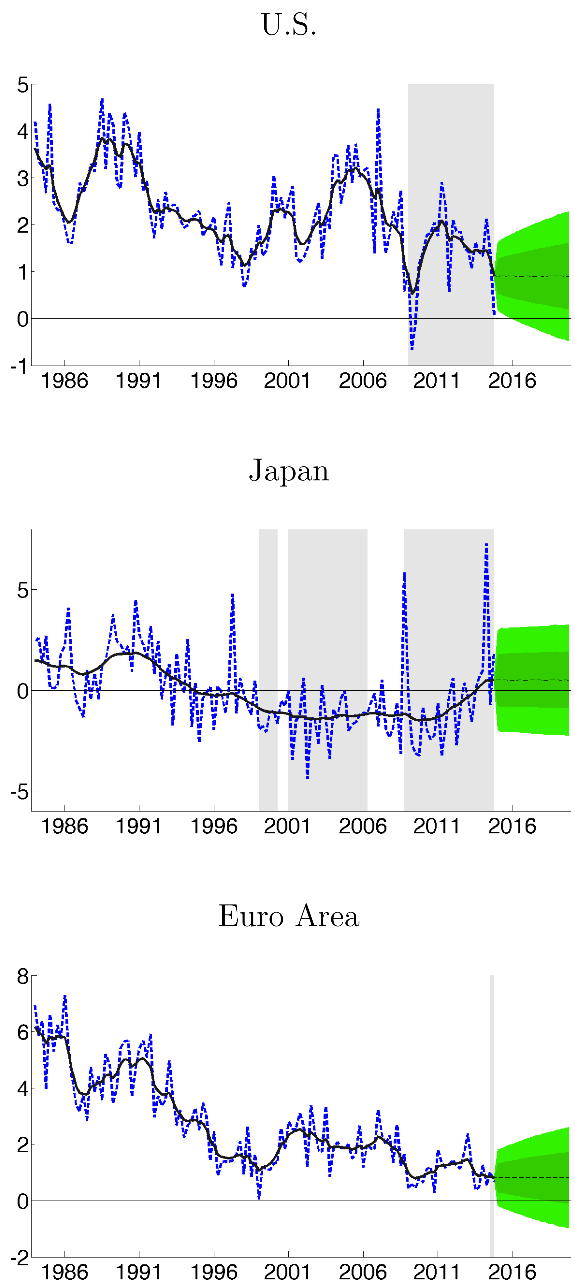
to economic conditions. In the U.S. they started to fall in 2008:Q4 as the economy was experiencing a major disruption in the financial sector. However, at quarterly frequency they never dropped below 1.5% and climbed to 2% by 2011:Q1, which is consistent with the evolution of actual inflation. In the Euro Area, prolonged drops in the policy rate are associated with a fall in the 1-year-ahead inflation expectations but at the end of 2014, short-run inflation expectations are still above 1%.

In the remainder of this section, we fit a univariate unobserved-components model with stochastic volatility (UC-SV) to the GDP deflator inflation series plotted in Figure 1. This model serves three purposes: we use it to extract a low-frequency trend component from the inflation series, we generate probability density forecasts conditional on data until 2014:Q4, and we produce one-quarter-ahead inflation expectations to compute ex-ante real interest rates. The UC-SV model was proposed by Stock and Watson (2007) and takes the following form:

$$
\begin{aligned}
\pi_t &= \tau_t + \sigma \exp(h_{\epsilon,t})\epsilon_t, \\
\tau_t &= \tau_{t-1} + (\varphi\sigma)\exp(h_{\eta,t})\eta_t \\
h_{j,t} &= \rho_j h_{j,t-1} + \sqrt{1-\rho_j^2}\sigma_{v_j} v_{j,t}, \quad j \in \{\epsilon, \eta\}.
\end{aligned}
\tag{1}
$$

The model decomposes the inflation series into trend inflation, τ_t, and serially uncorrelated short-run fluctuations, ϵ_t. The innovations associated with trend inflation and the short-run fluctuations exhibit stochastic volatility to account for the fact that the degree of time variation in the low frequency component and the importance of the short-run fluctuations for the inflation dynamics may change over time. As a consequence, the model is also able to capture time-variation in the persistence of inflation.

Figure 2: Trend Inflation and Inflation Density Forecasts



Notes: Each panel depicts GDP deflator inflation (dashed) and filtered estimates (solid) of the low frequency component of inflation as measured by the local-level component τ_t in (1). The local-level models are estimated based on data from 1984:Q1-2014:Q4. The shaded bands characterizes 20-step-ahead predictive distribution, using 2014:Q4 as forecast origin (median, 60%, and 90% predictive intervals). The shaded intervals characterize the ZLB episodes.

The solid lines in Figure 2 depict the estimated trend-inflation processes $\hat{\tau}_{t|t} = \mathbb{E}[\tau_t|\pi_{1:t}]$ for the three economies. Here $\pi_{1:t}$ denotes the sequence of past observations $\{\pi_1, \ldots, \pi_t\}$. As desired, $\hat{\tau}_{t|t}$ tracks the low frequency moments of inflation. For the U.S. and the Euro Area trend inflation clearly has been positive until 2014:Q4, whereas it has been negative in Japan since 1996. Figure 2 also shows density forecasts for the period 2015:Q1 to 2019:Q4. The shaded areas starting in 2015:Q1 indicate 60% and 90% predictive intervals obtained from the local-level model. Because trend inflation is assumed to evolve according to a random walk, the point prediction stays constant over time but the prediction intervals widen. As the forecast horizon increases, uncertainty about trend inflation dominates uncertainty about the short-run fluctuations. For the U.S. and Euro Area the short-run fluctuations have been fairly stable recently and the uncertainty about trend inflation is apparent in the widening interval predictions. For Japan, uncertainty about short-run fluctuations caused by a recent spike in inflation volatility is the main contributor to uncertainty about future inflation.

According to the forecasts from the UC-SV model the risk of experiencing deflation over the next five years remains close to 50% for Japan. For the U.S. it increases from essentially zero in the short-run to about 15% in five years from now. Finally, the Euro Area is in-between Japan and the U.S. In the short-run the risk of deflation is about 5% and it increases to about 20% over the next five years. Because the UC-SV model is univariate, it does not deliver any forecasts of the probability of leaving the ZLB. It simply extrapolates past inflation rates into the future in a way that is more accurate than many competing econometric models.[4] For the remainder of this paper, we turn to a multivariate structural model that allows us to interpret the inflation and interest rate data in light of modern macroeconomic theory and to examine the effect of monetary policy interventions on the projected path of interest and inflation rates.

III Inflation in New Keynesian DSGE Models

In the remainder of this paper we look at inflation dynamics through the lens of a small-scale New Keynesian DSGE model. Since the influential work of Smets and Wouters (2003) central banks around the world started to include estimated DSGE models into the suites of econometric models that are used to generate projections and support policy decisions. Although these models abstract from the complexities of modern-day economies, they provide a useful framework to understand the dynamics of output, inflation, and interest rates as well as the potential effects of monetary and fiscal policy interventions. While the Great Recession of 2007-09 has triggered a lot of research on how to incorporate financial and labor market frictions into DSGE models and how to model unconventional monetary policy, we work with a fairly rudimentary version of a New Keynesian DSGE model and focus on some fundamental mechanisms that are also part of richer DSGE models. We first review the key model elements (Section III.A) and then discuss various types of equilibria that can arise in these models (Section III.B). Each equilibrium is associated with distinct implications for inflation dynamics.

III.A Key Model Elements

New Keynesian DSGE model comprises three main elements: a consumption Euler equation that links interest rates to consumption and economic activity more generally; a New Keynesian Phillips curve (NKPC) that links inflation to expectations about current and future marginal costs, and hence real activity; and monetary and fiscal policy rules that determine interest rate and taxes conditional on the state of the economy. In turn, we will review each of these elements and examine the data from the perspective of these equilibrium relationships. A fully specified small-scale DSGE model that encompasses these elements is presented in the Appendix. We assume that time is discrete and that length of a period t is three months.

III.A.i Consumption Euler Equation and Fisher Equation

Households in DSGE models are assumed to derive utility from consumption and leisure and to be able to invest in a variety of financial assets, including a one-period nominal bond. The maximization of the expected sum of discounted future utility with respect to the choice of consumption leads to the following inter-temporal first-order condition:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\delta_{t+1}}{\delta_t} \right) Q_{t+1|t} \frac{R_t}{\pi_{t+1}} \right]. \tag{2}$$

Here β is the average discount factor, $Q_{t+1|t}$ is the ratio of the marginal utilities of consumption in periods $t+1$ and t, R_t is the gross nominal interest rate on a one-period nominal bond, and π_t is the gross inflation rate. The process δ_t captures exogenous fluctuations in the discount factor for period t utility.

The consumption Euler equation implies a tight relationship between the nominal interest rate, the real interest rate, and expected inflation. This relationship is called the Fisher equation and it implies that, holding real interest rates fixed, high inflation rates are associated with high nominal interest rates. Consider a risk-free asset that generates a real return r_t^f between period t and $t+1$. To make the household indifferent between holding the nominal bond and the risk-free asset, the return r_t^f has to satisfy

$$r_t^f = \frac{1}{\beta}\left\{\mathbb{E}_t\left[\left(\frac{\delta_{t+1}}{\delta_t}\right) Q_{t+1|t}\right]\right\}^{-1}. \tag{3}$$

Thus, ceteris paribus, a falling marginal utility of consumption is associated with a high real return r_t^f.

The Fisher equation is obtained by combining (2) and (3). Throughout this paper we often refer to steady states and log-linear approximations around steady states. In our notion of steady state, appropriately detrended model variables are constant over time (which we denote by replacing the t subscript with a $*$ subscript) and the economy is not perturbed by any exogenous stochastic shocks. A log-linearization around a steady state refers to an approximation of $f(x_t)$ through a first-order Taylor expansion in terms of $\ln x_t$ around $\ln x_*$. We use the notation $\widehat{x}_t = \ln(x_t/x_*)$. The steady-state version of the Fisher equation takes the following form:

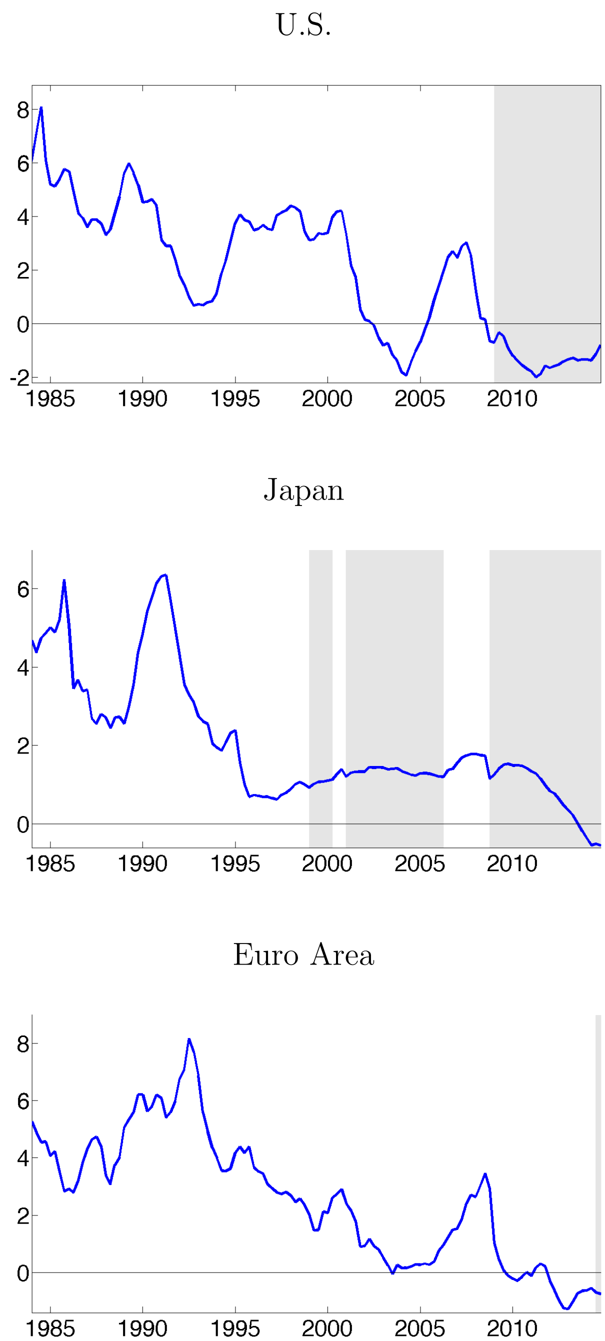
$$r_*^f = \frac{R_*}{\pi_*}. \tag{4}$$

Log-linearization approximations of (2) and (3) yield

$$\widehat{r}_t^f = \widehat{R}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}]. \tag{5}$$

Both (4) and (5) play a central role in the subsequent analysis.

Figure 3: Ex Ante Real Interest Rates



Notes: Each panel depicts ex-ante real interest rates computed as $400 \ln r_t^f = 400(\ln R_t - \mathbb{E}_t[\ln \pi_{t+1}])$. The inflation expectations are computed from the local-level model (1) and defined as the filtered estimates of τ_t. The shaded intervals characterize the ZLB episodes.

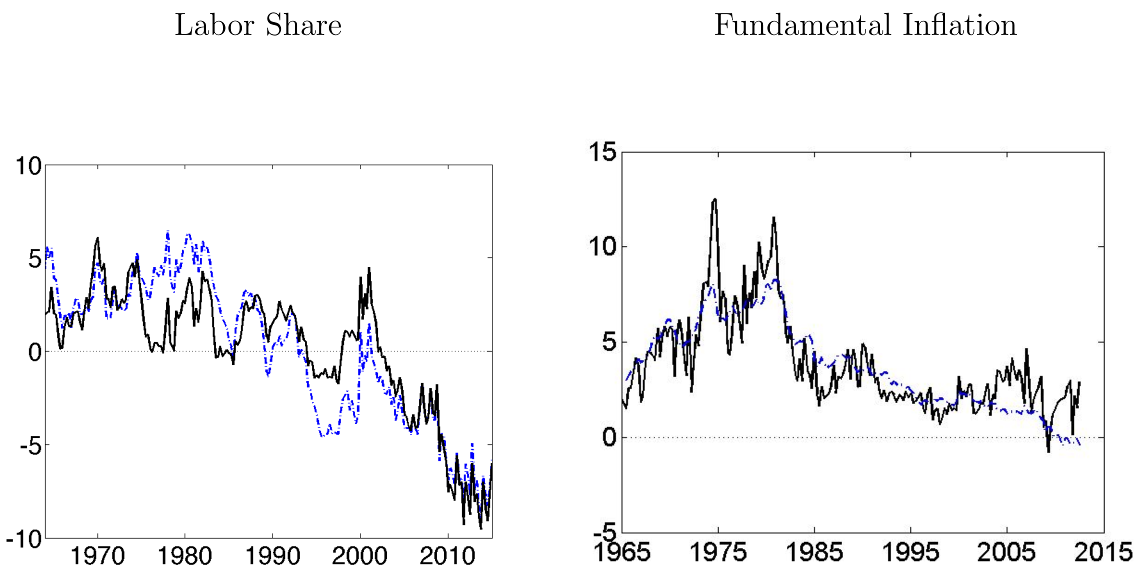
Figure 3 plots implied ex-ante real interest rates (in annualized percentages) based on (4) and (5). The one-step-ahead inflation forecasts $\mathbb{E}_t[\widehat{\pi}_{t+1}]$ are obtained from the UC-SV model (1) as the filtered estimates $\mathbb{E}[\tau_t|\pi_{1:t}]$. The most striking difference between the U.S. and the Euro Area on the one hand and Japan on the other hand is that the implied real interest rate in Japan has stayed positive throughout the ZLB episode until 2013:Q3, whereas it has been negative in the U.S. since 2008:Q4 and the Euro Area since 2009:Q4 (with the exception of 2011). According to (3), the discount factor shock δ_t is likely to play an important role in explaining the negative real rates and the zero nominal interest rates in the U.S.

III.A.ii New Keynesian Phillips Curve

The NKPC provides a link between inflation and real activity. It is typically derived under the assumption that production takes place in two stages. In the first stage, monopolistically competitive intermediate goods producers utilize labor and other factors of production, e.g., capital, to produce their goods. Each producer is facing a downward sloping demand curve and costs of adjusting nominal prices, which generates price stickiness. The intermediate goods are purchased by perfectly competitive final-goods-producing firms which simply turn the intermediate goods into an aggregate good that can be used for consumption, investment, or government spending.

The resulting equilibrium condition that describes the profit-maximizing prices set by the intermediate goods producers is called NKPC. A log-linear approximation around a level

Figure 4: Marginal Costs and Fundamental Inflation



Notes: The left panel depicts two labor share series in percentage deviations from their mean: solid line is nonfarm business sector labor share (Source: FRED); dashed line is the product of compensation per hour (nonfarm business sector), civilian employment (sixteen years and over), and average weekly hours (private industries) divided by GDP (Source: Haver Analytics). The right panel depicts GDP deflator inflation (solid line) and fundamental inflation (dashed line) from a medium-scale DSGE model with financial frictions (Source: Del Negro, Giannoni, and Schorfheide (2015)).

of inflation, assuming price adjustments at that rate are costless, takes the form:

$$\widehat{\pi}_t = \beta \mathbb{E}_t[\widehat{\pi}_{t+1}] + \kappa \widehat{mc}_t + \lambda_t, \tag{6}$$

where κ is the slope of the Phillips curve, $\widehat{mc}_t$ is marginal costs and λ_t is an exogenous price mark-up shock that sometimes is added to improve the empirical fit of the NKPC. The key feature of this version of the Phillips curve is that it is forward looking: current inflation depends on current real activity (through marginal costs) and expected inflation in the next period.

Many of the standard DSGE models, e.g., the widely-referenced Smets and Wouters (2007) model as well as the small-scale DSGE model used in this paper, imply that marginal costs are proportional to the labor share, which can be measured in the data. The left panel of Figure 4 depicts two measures of the labor share in the U.S in percentage deviations from a mean computed over the period 1964:Q1 to 2015:Q1 . The labor share has been fairly stable until 2002 and has exhibited a downward trend since then that continued during and after the Great Recession. It is apparent from (6) that, ceteris paribus, a drop in marginal costs generates deflationary pressure. How much depends on the details of the model. If the downward trend is generated by a shift of the steady state it may not affect inflation at all, because the NKPC in (6) characterizes fluctuations around a steady state or long-run trend. Most importantly, expectations about future marginal costs are very important, which we will discuss in more detail below.

The NKPC has been recently criticized by prominent macroeconomists, e.g., Ball and Mazumder (2011) and Hall (2011), because the absence of deflation in the U.S. in the aftermath of the Great Recession (see Figure 2) seems to be inconsistent with the drop in marginal costs in the left panel of Figure 4. For instance, Ball and Mazumder (2011) estimate a backward-looking Phillips curve (the term $\mathbb{E}_t[\widehat{\pi}_{t+1}]$ in (6) is replaced by lags of $\widehat{\pi}_t$) based on data from 1960 to 2007 and then predict inflation conditional on observed measures of economic slack for 2008-2010. Given the drop in marginal costs (and a measure of the output gap) the backward-looking Phillips curve predicts deflation as high as 4%, which did not happen. Thus, from the perspective of a backward-looking Phillips curve, there is a missing disinflation puzzle in the U.S.

However, the NKPC that underlies the current generation of DSGE models is forward-looking. Solving (6) forward under the assumption that the mark-up shock process is AR(1) with autoregressive parameter ρ_λ we obtain

$$\widehat{\pi}_t = \kappa \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t[\widehat{mc}_{t+j}] + \frac{1}{1+\rho_\lambda \beta}\lambda_t. \tag{7}$$

The first sum is called fundamental inflation. The right panel of Figure 4 shows the fundamental inflation series constructed by Del Negro, Giannoni, and Schorfheide (2015). It is based on an estimated version of the Smets and Wouters (2007) model with financial frictions and tracks the low frequency component of inflation well. Del Negro, Giannoni, and Schorfheide (2015) also document that their DSGE model is able to predict the observed path of inflation quite accurately from 2008:Q4 onward. Part of the reason is that despite the fall of the labor share toward the end of the sample, fundamental inflation does not become negative during and after the Great Recession because agents in the model expect marginal costs to rise again in the near future. Coibion and Gorodnichenko (2015) estimate forward-looking Phillips curves along the line of (6) by using survey expectations as proxies for expected inflation. They find that a deflation in 2009 - 2011 is avoided by high inflation expectations relative to current inflation due to, among other factors, an increase in energy prices and a preceding decline in inflation in early 2009.

III.A.iii Monetary Policy and Fiscal Policy

Monetary policy in DSGE models is typically described through an interest feedback rule. Because the ZLB constraint is an important part of our analysis we introduce it explicitly as follows:

$$R_t = \max\left\{1,\ \bar{R}_t e^{\epsilon_{R,t}}\right\}. \tag{8}$$

Here $\epsilon_{R,t}$ is an unanticipated monetary policy shock that captures deviations from the systematic part of the interest rate feedback rule, $\bar{R}_t$. $\bar{R}_t$ is determined as a function of the current state of the economy. We assume that

$$\bar{R}_t = \left(r_*^f \bar{\pi}_t \left(\frac{\pi_t}{\bar{\pi}_t} \right)^{\psi_1} \left(\frac{Y_t}{\bar{Y}_t} \right)^{\psi_2} \right)^{1-\rho_R} R_{t-1}^{\rho_R}, \tag{9}$$

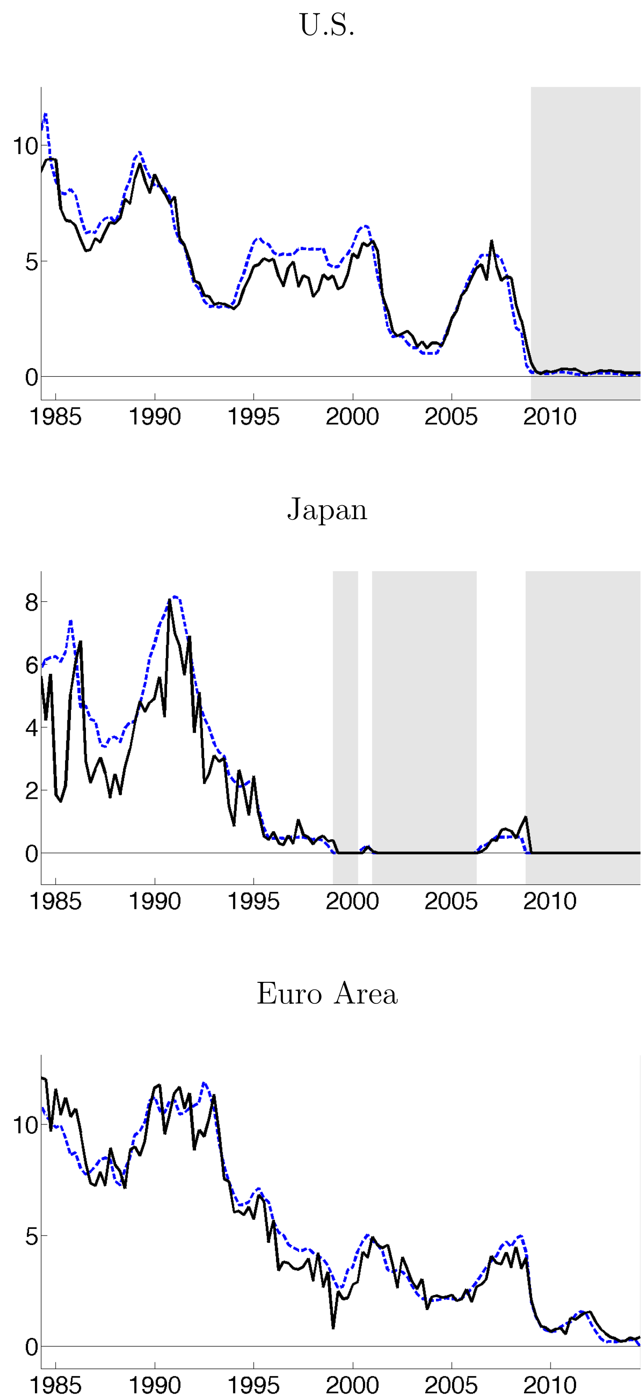
where $\bar{\pi}_t$ is the potentially time-varying target inflation rate and $\bar{Y}_t$ is the target level of output. In theoretical studies the targeted level of output often corresponds to the level of output in the absence of nominal rigidities and mark-up shocks because from an optimal policy perspective, this is the level of output around which the central bank should stabilize fluctuations. However, it appears that in reality the behavior of central banks is well described by trying to keep output close to official measures of potential output, which can be approximated by a slow-moving trend. Thus, throughout this paper we use exponential smoothing to construct $\bar{Y}_t$ directly from historical output data. It is given by

$$\ln \bar{Y}_t = \alpha \ln \bar{Y}_{t-1} + (1-\alpha) \ln Y_t + \alpha \ln \gamma. \tag{10}$$

The definition of $\bar{R}_t$ is such that conditional on the monetary policy rule coefficients, it can be directly computed from the data. We plot $\bar{R}_t$ in Figure 5. We calibrate α to match official measures of potential output and fix $\psi_1 = 1.5$ and $\psi_2 = 0.1$. These values are close to the classic Taylor rule coefficients. The interest rate smoothing coefficient is estimated along with other DSGE model coefficients in preparation for the analysis in the remaining sections of this paper. In general $\bar{R}_t$ tracks the actual interest rate fairly well, even during the ZLB episodes.

An important question for monetary policy analysis is whether an increase in interest rates is associated with a rise or a fall in inflation. The answer to this question depends on

Figure 5: MONETARY POLICY RATES



Notes: Each panel depicts the monetary policy interest rate (solid line, see Appendix A for data definition) and the systematic part of the desired interest rate $\bar{R}_t$ (dashed line), see (9) for definition. The shaded intervals characterize the ZLB episodes.

what generates the rising interest rates. Suppose that inflation is below its target value and the interest rate is below its steady state value, but the economy is in the process of returning to the steady state in which inflation equals the targeted value. In this case, by virtue of the monetary policy rule and the Fisher equation, interest rates will rise as inflation reverts back to its target. Alternatively, if the central bank surprises the public by setting the interest rate above $\bar{R}_t$, i.e., $\epsilon_{R,t} > 0$, then inflation will fall. The unanticipated contractionary monetary policy generates an increase in real rates, which triggers a fall in current consumption and output (Euler equation), and leads to falling prices (NKPC).

In addition to the monetary policy rule, we also need to specify a fiscal policy. We write the government budget constraint in real terms as

$$G_t + R_{t-1}\frac{1}{\pi_t}\frac{B_{t-1}}{P_{t-1}} = \frac{T_t}{P_t} + \frac{B_t}{P_t}, \tag{11}$$

where G_t is an exogenous spending process, B_t is nominal government debt, and T_t are nominal taxes or transfers. Government spending, debt, and taxes, may react to the state of the economy. In most monetary DSGE models it is assumed that government spending as a fraction of GDP is exogenous and that the government uses lump-sum taxes and transfers to balance the budget. Because the exact nature of the response of the fiscal authority to the state of the economy has important consequences for the multiplicity of equilibria, we will postpone a more detailed discussion.

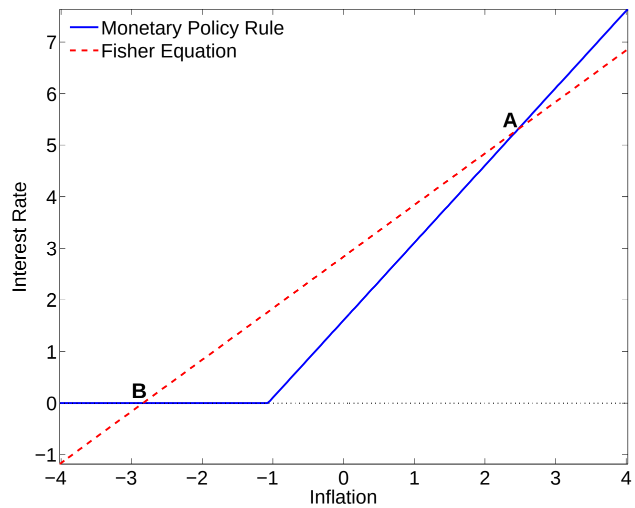
III.A.iv Small-Scale versus Large-Scale Models

In the preceding sections we sketched the key building blocks of New Keynesian DSGE models. Appendix C contains the remaining missing pieces to turn these building blocks into a coherent small-scale DSGE model. The literature has developed much richer medium- and large-scale DSGE models. To give a few examples, the models estimated by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) contain capital as a factor of production and feature habit formation in consumption, investment-adjustment costs, variable capital utilization and wage rigidity. The models of Christiano, Motto, and Rostagno (2003) and Gertler and Kiyotaki (2010) prominently feature financial frictions. The models of Gertler, Sala, and Trigari (2008) and Christiano, Eichenbaum, and Trabandt (2013) include labor market frictions. The models of Chen, Curdia, and Ferrero (2012) and Gertler and Karadi (2011) are designed to study the effects of unconventional monetary policies. In the remainder of this paper we will proceed with a small-scale DSGE model because many of the calculations are more transparent, while it is still sufficiently rich to be used to track output, consumption, inflation, and interest rates from the U.S., Japan, and the Euro Area.

III.B ZLB and Multiplicity of Equilibria

DSGE models are well-suited to assess the effects of interest rate policies on inflation and output dynamics. We will subsequently examine why interest rates may fall to zero, what happens to inflation while interest rates are zero, and how inflation evolves during a lift-off from the ZLB. Unfortunately, the presence of multiple equilibria complicates the analysis and implies that a DSGE model may predict a wide range of inflation and real activity outcomes. The quantitative illustrations are based on a version of the DSGE model described in the Appendix.[5]

Figure 6: Two Steady States



Notes: The dashed line depicts the Fisher equation (4) and the solid line depicts the monetary policy rule (12). The intersections (A) and (B) correspond to the target-inflation and deflation steady states, respectively.

III.B.i Steady States

The absence of stochastic shocks simplifies the analysis considerably. It is well known that in DSGE models in which monetary policy is active, meaning the central bank responds strongly to inflation deviations from the target ($\psi_1 > 1$ in our model), and fiscal policy is passive, meaning that fiscal policy responds only weakly to the level of government debt, the ZLB constraint generates a second steady state. The model predicts that two outcomes are possible: (A) inflation is equal to the value targeted by the central bank and nominal interest rates are positive; (B) inflation rates are negative and nominal interest rates are zero. We refer to (A) as the targeted-inflation outcome and (B) as the deflation outcome.

The existence of two steady states is illustrated in Figure 6 and can be easily seen by combining (4) with a steady state version of the simplified monetary policy rule:

$$R_* = \max \left\{ 1, \left(\frac{\pi_*}{\bar{\pi}} \right)^{\psi_1} \right\}. \tag{12}$$

There exist two solutions to this system of equations. The targeted-inflation steady state is given by

$$R_* = r_*^f \bar{\pi}, \quad \pi_* = \bar{\pi} \tag{13}$$

and the deflation steady state takes the form:

$$R_* = 1, \quad \pi_* = \frac{1}{r_*^f}. \tag{14}$$

In both steady states the real interest rate is given by $r_*^f = \gamma/\beta$. The model is not rich enough to predict whether agents coordinate on steady state (A) or (B). A casual look at the data in Figures 1 and 3 suggest that Japan's experience of zero nominal interest rates, deflation, and positive real rates is consistent with the deflation steady state. The U.S. experience of negative real rates does not seem to be consistent with either steady state.

III.B.ii Perfect Foresight Dynamics

The analysis of steady states does not provide any insights into how the economy reached the ZLB and how it might exit from the ZLB. We proceed by exploring some of the dynamic properties of our DSGE model. For now, we abstract from uncertainty about the realization of exogenous shock processes and assume that agents have perfect foresight. In a perfect foresight setting, the economy can reach the ZLB either by transitioning from the targeted-inflation steady state to the deflation steady state, as emphasized in the work of Benhabib, Schmitt-Grohé, and Uribe (2001b), or through an adverse real rate shock that is sufficiently strong to push the nominal interest rate against the ZLB. We provide numerical illustrations for both scenarios. Assuming that the adverse real rate shock is temporary, we also study the escape from the ZLB under the second scenario.

The subsequent analysis is based on a log-linear approximation of the three key model equations around the targeted-inflation steady state. We impose the ZLB constraint on the log-linearized monetary policy rule. The consumption Euler equation and NKPC curve can be written as

$$\begin{aligned} \widehat{c}_t &= \widehat{c}_{t+1} - (\widehat{R}_t - \widehat{r}_t - \pi_{t+1}) \\ \widehat{\pi}_t &= \beta \widehat{\pi}_{t+1} + \kappa \widehat{c}_t, \end{aligned} \tag{15}$$

where $\widehat{r}_t$ can be interpreted as a real rate shock.[6] Note that under perfect foresight we can drop the expectations $\mathbb{E}_t[\cdot]$. The log-linearization of the monetary policy rule yields

$$\widehat{R}_t = \max \big\{ -\ln(r_*^f \bar{\pi}),\ \psi_1 \widehat{\pi}_t \big\}. \tag{16}$$

Throughout this section we assume that monetary policy is active and $\psi_1 > 1$.

The dynamics of consumption, inflation, and interest rates have to satisfy the set of difference equations in (15) and (16). Notice that the multiplicity of steady states is still present in (15) and (16). Suppose that $\widehat{r}_t = 0$, then one time invariant solution is $\widehat{c}_t = \widehat{R}_t = \widehat{\pi}_t = 0$. The second time invariant solution is

$$\widehat{R}_t = \widehat{\pi}_t = -\ln(r_*^f \bar{\pi}), \quad \widehat{c}_t = -\frac{1-\beta}{\kappa} \ln(r_*^f \bar{\pi}), \quad \text{for all } t.$$

We can call the second solution the deflation steady state of the linearized system. The literature typically focuses on solutions to these difference equations that are non-explosive, because explosive dynamics tend to violate transversality conditions associated with the underlying dynamic programming problem.[7]

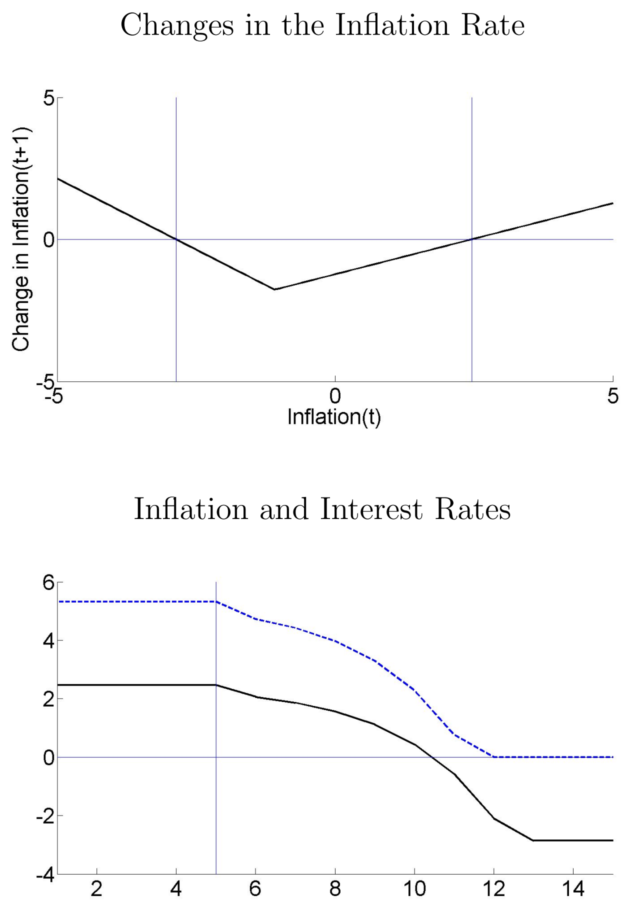
Scenario 1: Transition from Targeted-Inflation to Deflation Steady State. Benhabib, Schmitt-Grohé, and Uribe (2001a) and Benhabib, Schmitt-Grohé, and Uribe (2001b) discuss various equilibria that can arise in the nonlinear version of a three-equation New Keynesian DSGE model. The equilibrium that has drawn a lot of attention and is of concern to policy makers is one in which the economy transitions from the targeted-inflation steady state to the deflation steady state. A casual look at the data suggests that this might describe the Japanese experience. We can illustrate these dynamics easily in the context of our linearized model. We start by assuming that prices are flexible, which implies that $\kappa = \infty$ and $\widehat{c}_t = 0$. Combining the consumption Euler equation with the monetary policy rule yields the following nonlinear difference equation for inflation

$$\widehat{\pi}_{t+1} = \max \Big\{ -\ln(r_*^f \bar{\pi}),\ \psi_1 \widehat{\pi}_t \Big\}. \tag{17}$$

The dynamics associated with this difference equation are depicted in Figure 7. The top panel depicts $\Delta\widehat{\pi}_{t+1}$ as a function of $\widehat{\pi}_t$. If $\Delta\widehat{\pi}_{t+1} = 0$, the system is in a steady state. The figure shows that any perturbation away from the targeted-inflation steady state will move the system away from that steady state. In particular, if inflation drops below the targeted inflation steady state, it will continue to fall and eventually settle on the deflation steady state. The bottom panel shows the time path of inflation and interest rate, assuming that the system is in the targeted-inflation steady state from $t = 1$ to $t = 5$. In period $t = 6$ inflation falls and triggers the transitions to the deflation steady state.[8]

Scenario 2: Exit from the ZLB after an Adverse Real Rate Shock. According to

Figure 7: TRANSITION TO THE DEFLATION STEADY STATE



Notes: Top panel: the vertical lines indicate the two steady states. Formally, the plot depicts $400\ln(\pi_{t+1}/\pi_t)$ versus $400\ln\pi_t$. Bottom panel: interest rate (dashed) and inflation rate (solid) during a transition from the targeted-inflation to the deflation steady state.

our benchmark calibration, the real interest rate and the inflation rate are 2.9% and 2.5%, respectively, in the targeted-inflation steady state. Suppose that there is an adverse real rate shock that sends the economy in the liquidy trap: $\widehat{r}_t = -7.4\%$. Simultaneously the nominal interest rate drops to the ZLB: $\widehat{R}_t = -5.4\%$. This situation is depicted in the top panel of Figure 8. Our subsequent analysis examines exit paths from the ZLB. To keep the analysis as simple as possible, we assume that agents know the exit date $t = T$. In period $t = T + 1$,

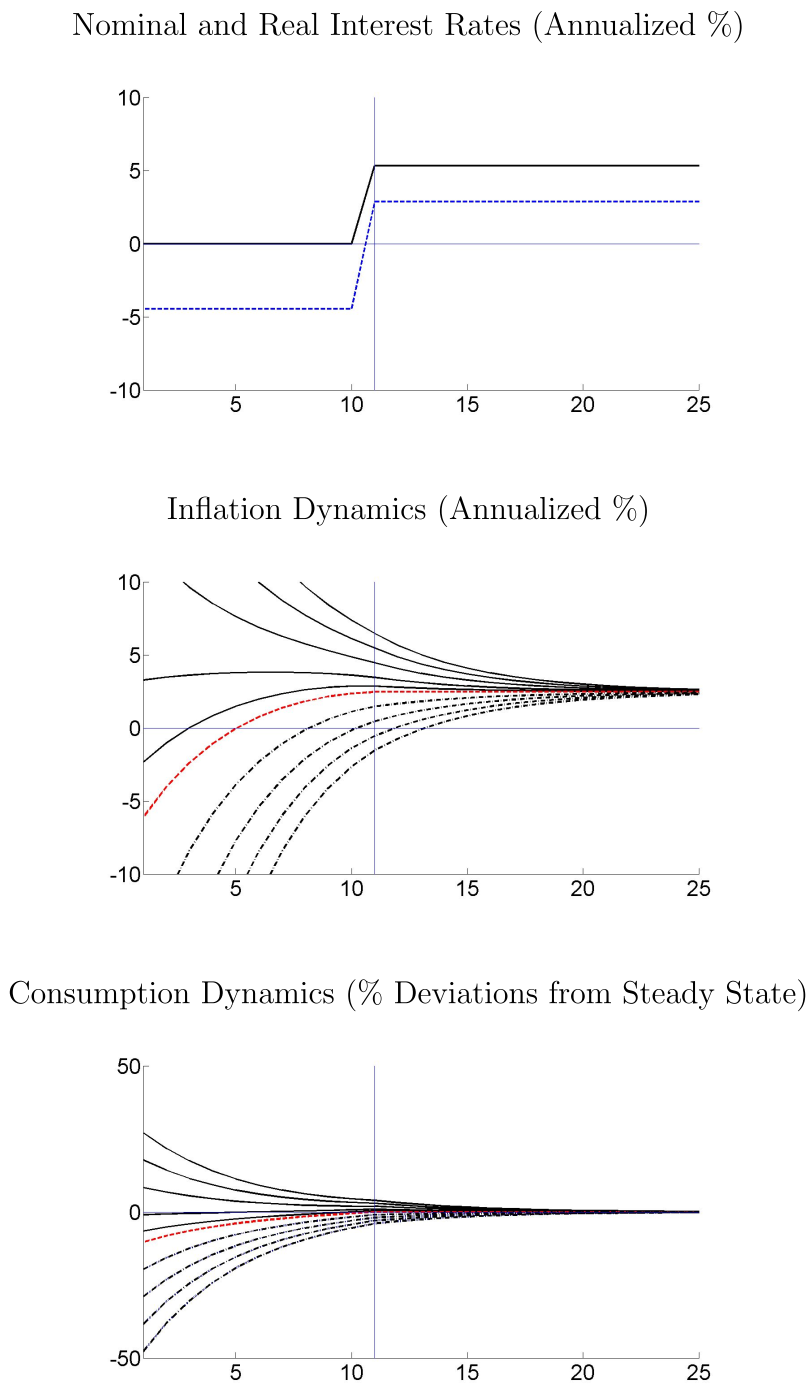
$\widehat{r}_t$ and $\widehat{R}_t$ revert back to their steady state values.

If we impose the Taylor rule (16) after $t = T$, then under the assumption that $\psi_1 > 1$ the only path that is non-explosive is one in which the economy reverts instantaneously to the targeted-inflation steady state, which determines $\widehat{R}_t$, $\widehat{\pi}_t$, and $\widehat{c}_t$ in periods $t > T$. For $t \leq T$ nominal interest rates are zero and output and consumption have to satisfy (15). The solution can be easily found by backward iteration: solve for time t variables as a function of time $t+1$ variables. The resulting inflation and consumption dynamics are depicted by the uppermost dashed lines in the center and bottom panel of Figure 8. The economy starts in a liquidity trap with deflation and low consumption caused by a negative real rate shock. Then inflation and consumption rise and eventually revert back to the targeted-inflation steady state. The longer the spell of an adverse real rate shock and zero nominal interest rates, the deeper the liquidity trap.

Mechanically, the potentially disastrous outcomes during the liquidity trap are due to the fact that the bivariate system (15) has one stable and one unstable root. Thus, the root that is stable during forward iterations turns unstable during backward iterations. This can generate deep contractions, but also large stimulative effects of keeping interest rate at zero for an extended period of time as discussed, for instance, in Carlstrom, Fuerst, and Paustian (2012) and Del Negro and Schorfheide (2013). Because the interest rate increase in period $T = T + 1$ is expected, inflation starts to rise well before the date of the interest rate and essentially reaches the target value prior to period T. The large deflation in the initial period looks very different from the actual ZLB experience of the U.S.

Cochrane (2015) points out that the "standard" equilibrium generated by the interest

Figure 8: Perfect Foresight Dynamics in Response to a Real Rate Shock



Notes: Top panel: solid line is $\widehat{R}_t$; dashed line is $\widehat{r}_t$. Center and bottom panels: the uppermost dashed response is obtained by imposing the Taylor rule for $t > T$. The solid lines correspond to $\widehat{\pi}_{T+1} > 0$ whereas the dashed-dotted lines correspond to $\widehat{\pi}_{T+1} < 0$. The vertical line indicates $t = T + 1$.

rate rule in (16) and depicted by the uppermost dashed lines in the center and bottom panels of Figure 8 is not the only one. He constructs alternative paths for inflation and consumption, depicted with the solid and dashed-dotted lines, by solving the bivariate system (15) forward from $T+1$ onward, imposing stability. The stability restriction determines consumption as a function of inflation in period $T+1$, which means that each equilibrium path can be indexed by $\widehat{\pi}_{T+1}$. In our perfect foresight environment $\widehat{\pi}_{T+1} = \mathbb{E}_0[\widehat{\pi}_{T+1}]$, which can be interpreted as expectations of the inflation rate during the exit from the ZLB determine inflation and real activity outcomes.

Cochrane's point has a positive and a normative dimension. On the positive side, the solid path along which inflation starts out at approximately 3% and then slightly rises and subsequently converges to its long-run target describes the current U.S. ZLB episode better than the dashed path which exhibits substantial deflation.[9]

On the normative side, monetary policy has the potential to put the economy on a path in which inflation is positive and fairly stable and consumption does not collapse. With regard to implementation, Cochrane (2015) points out that for $t > T$, the solid paths could be implemented using a policy rule of the form

$$\widehat{R}_t = \psi_1(\widehat{\pi}_t - \widehat{\overline{\pi}}_t), \tag{18}$$

where $\widehat{\overline{\pi}}_t$ is the central bank's desired inflation path. By announcing and enforcing the time-varying target path $\widehat{\overline{\pi}}_t$ the central bank conducts an equilibrium selection policy to choose one among the equilibria that are consistent with $\widehat{R}_t = 0$. Thus, ultimately the central bank's equilibrium-selection policy determines whether the liquidity trap is benign or disastrous.

III.B.iii A Stochastic Two-Regime Equilibrium

While the analysis of steady states and perfect foresight equilibria can deliver important theoretical and qualitative insights, it is not suitable for confronting the model with actual data, because it abstracts from the shocks that constantly hit the economy. Broadly speaking, these shocks capture agents' uncertainty about future fundamentals. In our small-scale DSGE model we consider a shock to the growth rate of total factor productivity, a shock to the discount factor which generates exogenous fluctuations in the real rate, a shock to aggregate demand, and a monetary policy shock that reflects unanticipated deviations from the systematic part of the interest rate feedback rule.

To capture the possibility that an economy experiences zero interest rates and low inflation rates either because of a shift from a targeted-inflation to a deflation regime (as in Scenario 1 above) or because of adverse fundamental shocks (as in Scenario 2), we introduce a binary sunspot shock that serves as a coordination device for agents' expectations. Depending on the realization of the sunspot shock the economy either fluctuates around the targeted-inflation steady state or around the deflation steady state. We refer to these two outcomes as targeted-inflation and deflation regime, respectively. As we will explore in more detail below the two regimes have different implications about the likelihood of an exit from the ZLB, about inflation dynamics, and about the effect of monetary policy interventions.

In order to keep the numerical solution of the two-regime equilibrium tractable, we make the simplifying assumption that the sunspot shock $s_t \in \{0, 1\}$ evolves according to an exogenous two-state Markov-switching process. In our formal model, the transition probabilities are time-invariant. In particular, the probability of transitioning from the targeted-inflation to the deflation regime is independent of the realization of the fundamental shocks and the level of inflation and nominal interest rates. Likewise, the probability of staying in the deflation regime is independent of the duration of that regime.[10] Solving a model in which the transition probabilities depend on the level of interest rates or on announcements of the central is beyond the scope of this paper. In turn, all statements that we make subsequently about central bank actions influencing the coordination of beliefs are based on reasoning outside of the realm of the formal model.

IV Did the U.S., Japan, or the Euro Area Shift to a Deflation Regime?

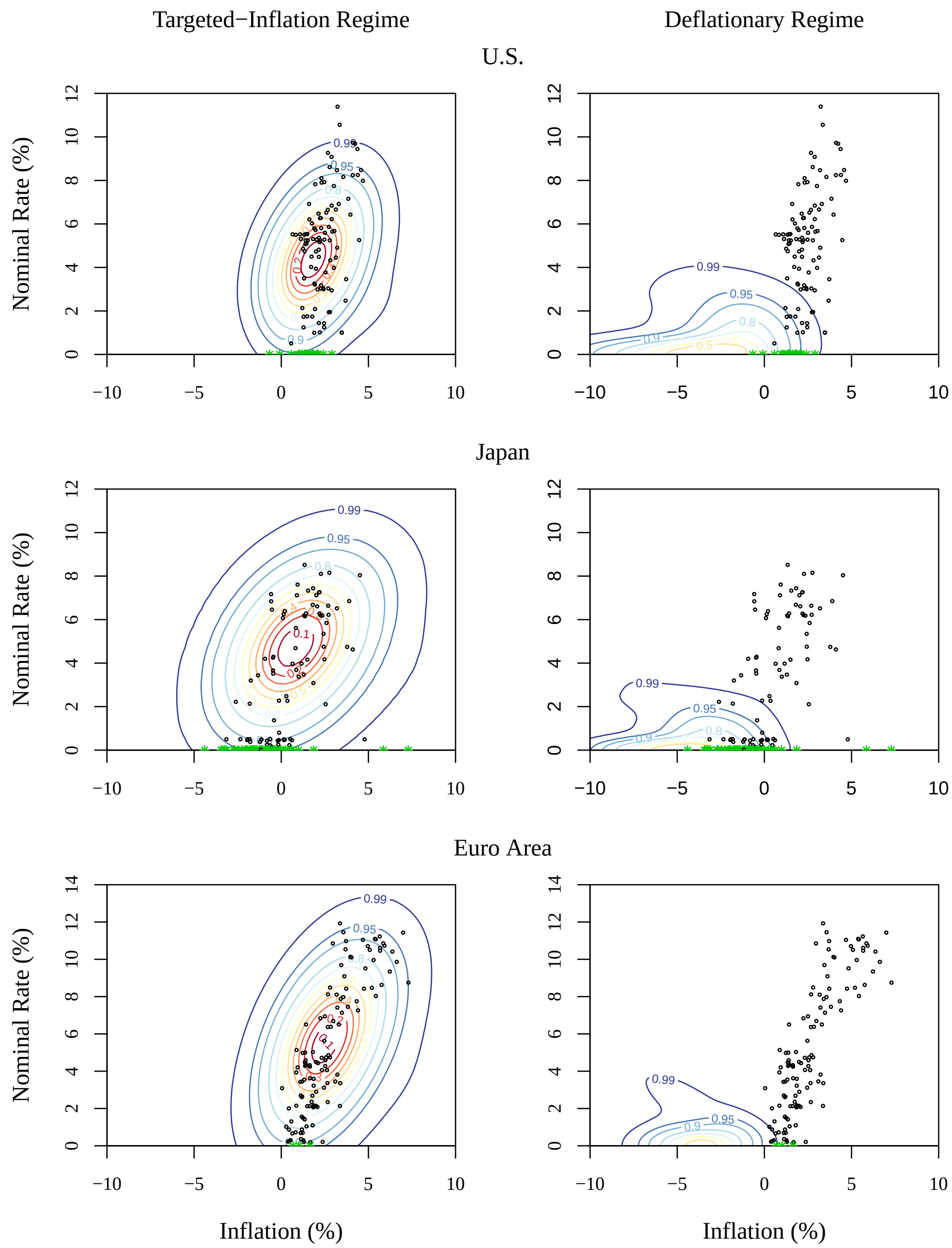
An extended period of zero interest rates and low inflation rates is reason for concern that the economy has transitioned to a deflation regime. For the U.S., this concern has been prominently voiced by James Bullard, President of the Federal Reserve Bank of St. Louis, in Bullard (2010, 2015). Based on the stochastic two-regime equilibrium, we can formally assess the likelihood of a shift to the deflation regime. In ACS we estimated a small-scale DSGE model for the U.S. and Japan using data that pre-date the ZLB episodes for these two countries. The estimation was conducted under the assumption that the economies were in the targeted-inflation regime. In this paper we repeat the estimation for the DSGE model presented in the Appendix and also generate estimates for the Euro Area. To assess whether we have observed a shift to a deflation regime in any of the three economies, we conduct the following experiment: we simulate data from the DSGE models to characterize the joint distribution of interest rates and inflation conditional on the two regimes. We then overlay the observed data to assess whether they appear to be more likely under one of the two regimes.

Results are presented in Figure 9. The depicted contours in the figure can be interpreted as coverage sets: for instance, the probability that interest rates and inflation fall into the region delimited by the contour labeled 0.95 is 95%. Under the targeted-inflation regime reaching the ZLB is a rare event because it requires an (unlikely) sequence of exogenous shocks. The probabilities of reaching the ZLB are 0.1%, 0.2%, and 0.2% for the U.S., Japan, and Europe, respectively. A switch to the deflation regime makes it much more likely that the nominal interest rates drop to zero and that we observe negative inflation rates. However, note that especially for the U.S. and Japan, and to some extent for Europe, there is considerable overlap in the regime-conditional distributions: under both regimes it is possible to observe low interest and inflation rates.

The dots in Figure 9 represent non-ZLB observations for the three economies most of which have been used to estimate the DSGE model parameters. Not surprisingly, they mostly fall within the contours associated with the targeted-inflation regime. More interesting are the stars, which correspond to near-zero interest rate periods and are excluded from the estimation. It is difficult to infer whether these interest rate and inflation observations are more likely conditional on the deflation regime or the targeted-inflation regime for the U.S. and Japan, whereas for the Euro Area a shift to the deflation regime at the current stage looks unlikely to have occurred.

The examination of the contour plots ignores the model's predictions for output and

Figure 9: Ergodic Distribution and Data



Notes: In each panel we report the joint probability density function (kernel density estimate) of annualized net interest rate and inflation, represented by the contours. Dots represent non-ZLB observations: 1984:Q1 - 2008:Q4 (U.S.), 1981:Q1 - 1998:Q4, 2000:Q2-2001:Q1, 2006:Q3-2008:Q4 (Japan), 1984:Q1 - 2014:Q2 (Euro Area). Stars represent the remaining observations, all which feature the ZLB.

consumption and the information from dynamic correlations. It is no substitute for the formal econometrics analysis conducted in ACS. In ACS we concluded (using a slightly different model without discount factor shocks) that the U.S. did not enter a deflation regime in 2009, whereas Japan did, starting in 1999. While too early to tell (due to limited number of observations) so far Europe seems to stay in the targeted inflation regime as well. In ACS we linked a switch in the sunspot regime to a change in expectations, which Mertens and Ravn (2014) call a confidence shock. We concluded that the actions of Bank of Japan following adverse shocks in the late 1990s made the public doubt the central bank's commitment to a positive inflation target and caused a switch in inflation expectations. This lower (and negative) expectations then meant that the economy started fluctuating around the $s = 0$ (deflation) steady state. In contrast, the actions of the Fed following the 2008 financial crisis reassured the public that the positive inflation target is alive and well, and the economy continued to fluctuate around the targeted-inflation steady state.

In the two-regime equilibrium, inflation expectations are sensitive to the regimes. For instance, if the targeted-inflation regime is very persistent, as is assumed in our numerical analysis, then conditional on being in that regime, long-run inflation expectations are close to the target value. If the economy transitions from one regime to another, then inflation expectations should also adjust. A casual look at Figure 1 reveals that for the U.S. and for Europe long-run inflation expectations remain remarkably stable during each country's ZLB episode, which is consistent with the economies staying in the targeted-inflation regime. In Japan, on the other hand, long-run inflation expectations started to fall throughout the 1990s which, although not consistent with a sudden switch, is consistent with a more gradual transition to the deflation regime that our model approximates by a regime switch.

V Low Inflation and Economic Outcomes

Thus far, we have documented that the zero-interest-rate episodes in the U.S., Japan, and the Euro Area are associated with low inflation and, in the case of Japan, with disinflation. Moreover, looking at the data through the lens of a nonlinear New Keynesian DSGE model, we find some evidence that Japan may have shifted to a what we call deflation regime for an extended period of time. Historically, periods of zero or negative inflation have been associated with low output and high unemployment. The Great Depression of the 1930s and the recent Global Financial Crisis are prominent examples. In the context of DSGE models these crisis are generated by adverse shocks to productivity, aggregate demand, or financial intermediation. Thus, to a large extent, deflation is merely a symptom, but not the cause of poor economic conditions.

Central bankers generally do not like deflation, in part because of concern that deflation might amplify the effects of adverse shocks to the economy, and because if deflation is associated with near zero interest rates, the ZLB constrains conventional expansionary monetary policies. Many central banks implicitly or explicitly target an inflation rate of about two percent:

> The Federal Open Market Committee (FOMC) judges that inflation at the rate of 2 percent (as measured by the annual change in the price index for personal consumption expenditures, or PCE) is most consistent over the longer run with

> the Federal Reserve's mandate for price stability and maximum employment. Over time, a higher inflation rate would reduce the public's ability to make accurate longer-term economic and financial decisions. On the other hand, a lower inflation rate would be associated with an elevated probability of falling into deflation, which means prices and perhaps wages, on average, are falling–a phenomenon associated with very weak economic conditions. Having at least a small level of inflation makes it less likely that the economy will experience harmful deflation if economic conditions weaken. The FOMC implements monetary policy to help maintain an inflation rate of 2 percent over the medium term. (Source: *www.federalreserve.gov/faqs/economy_14400.htm.*)

Even though there is no theoretical justification for an inflation target as high as two percent, see Schmitt-Grohé and Uribe (2011), our model embodies the notion that an inflation rate of approximately two percent is important for the public to be able to make accurate longer-term economic and financial decisions.[11] Formally, we assume in the model that it is costly for firms to adjust prices at a rate which differs from the targeted inflation rate. This cost leads to a loss of output in the aggregate, which we call the New Keynesian distortion. While the output loss is not directly observable in the data, in the model it is linked to the slope of the NKPC, which can be estimated. The flatter the NKPC, the larger the output loss. The New Keynesian distortion makes deflation undesirable. For instance, in the simplified version of our DSGE model discussed in Section III.B, welfare in the deflation steady state (in which prices fall at the gross rate of $1/r_*^f$) is substantially lower than in the targeted inflation steady state: one would have to raise consumption in the former by approximately

2.7% to achieve the same level of welfare as in the latter. Of course, if firms would adjust their price-setting technology to the presence of prolonged deflation, the welfare loss would be smaller.

In addition to this New Keynesian channel, downward nominal wage rigidity is often cited as an important reason why deflation is undesirable. While this mechanism is not incorporated into the model that is used in our paper, it is prominently featured in Schmitt-Grohé and Uribe (2012)'s making of a great contraction with a liquidity trap and a jobless recovery. In the presence of downward nominal wage rigidity, deflation leads to increasing real wages, which depresses employment and output during a recession. While downward rigidity is a well-documented feature of nominal wage changes at the micro level, e.g., Gottschalk (2005), Barattieri, Basu, and Gottschalk (2014), and Daly, Hobijn, and Lucking (2012), making it quantitatively important at the aggregate level is more difficult, because aggregate downward nominal wage rigidity is difficult to measure. The estimates reported in Aruoba, Bocola, and Schorfheide (2013) of the amount of wage rigidity and the asymmetry in the wage adjustment costs are relatively small.

A prominent mechanism that favors low or negative inflation rates is the "Friedman channel," according to which positive nominal interest rates serve as a tax on cash balances, or, more generally, liquid assets that bear negligible interest, and lead agents to economize on transactions involving such assets. Many monetary models without a strong New Keynesian friction prescribe the Friedman rule as the optimal policy. At the steady state, this entails deflation at the rate of time preference. The magnitude of welfare effects depends on how the benefit to consumers and firms of holding cash balances is modeled and how the interest-rate elasticity of money demand is measured.

In the presence of long-term nominal contracts, e.g., mortgages, deflation leads to a transfer from borrowers to lenders by increasing the real debt burden of the borrowers. If borrowers and lenders respond differently in terms of consumption and labor supply to changes in wealth, then changes in inflation can have aggregate effects through this debt channel. Doepke and Schneider (2006) study the net nominal asset positions of U.S. households. Meh, Ríos-Rull, and Terajima (2010) find that in an overlapping generations model an increase in inflation under an inflation targeting regime can have negative net effects on aggregate output. Herman and Pugsley (2014) study the welfare costs associated with a disinflation in an incomplete markets model. Even though low inflation in their model is desirable in the long-run, they find significant welfare losses for poor households with nominal debt contracts. While the debt channel may be potentially important, we leave a careful quantitative assessment in the context of an optimal target inflation rate for future research.

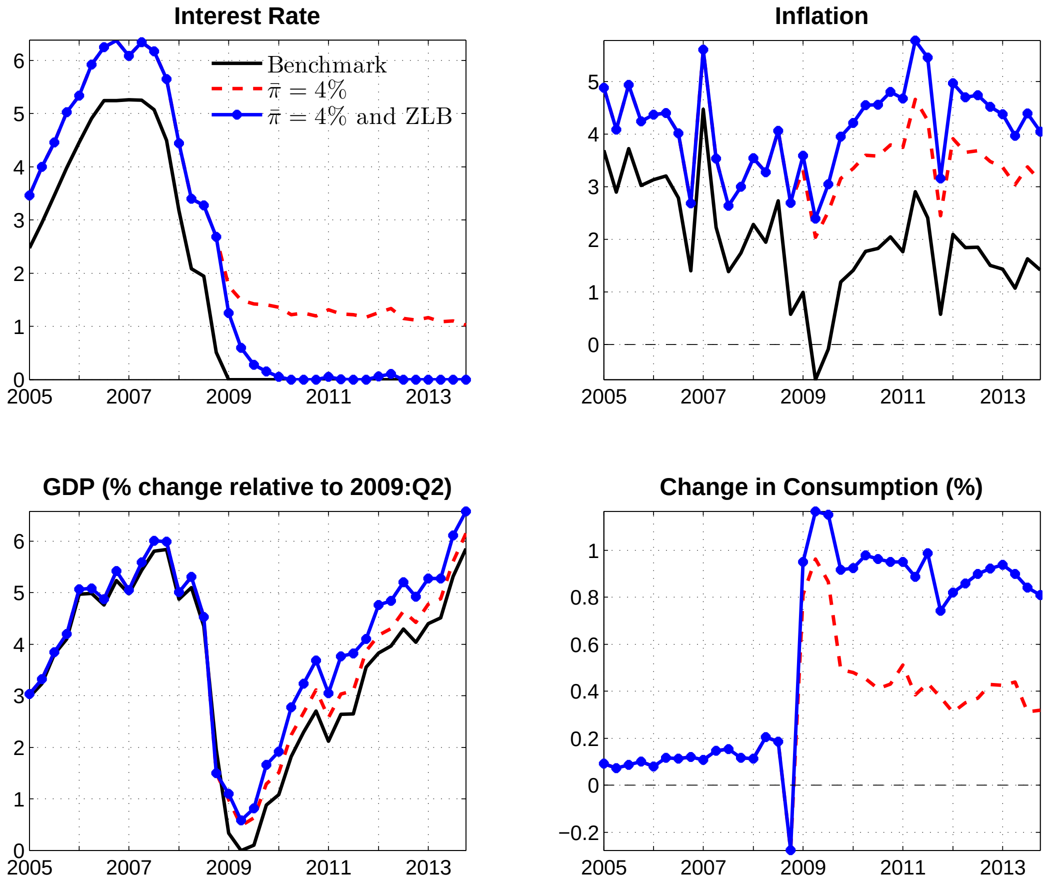
VI Policy Questions

Several prominent economists, e.g., Blanchard, DellAriccia, and Mauro (2010) and Ball (2013), have proposed to raise the inflation target to, for instance, four percent in order to reduce the probability of reaching the ZLB during a period of large adverse shocks. A reappraisal of the targeted inflation rate has remained part of the monetary policy discussions, see, for instance, Krugman (2014) and Appelbaum (2015). We conduct two counterfactual experiments for the U.S. In the first experiment, we go back to the beginning of our estimation period, which is 1984, and set an inflation target of four percent instead of the estimated target of 2.5 percent (Section VI.A).[12] The model is then solved under the assumption that price changes at the rate of four percent are costless, that is, the public accepts this target, views it as credible, and internalizes it in its decisions. This assumption eliminates any welfare cost of the higher trend inflation associated with the New Keynesian channel and maximizes the possible gains of implementing this policy. To generate the counterfactual outcomes, we subject the economy to the same shocks that, according to our benchmark estimation, have occurred during the period from 1984 to 2013. In the second experiment, we change the target inflation rate to four percent at the end of 2013 in a way that is understood to be perfectly credible and will remain that way forever (Section VI.B). We then compare simulated trajectories of output, inflation, and interest rate under the two target rates.

VI.A What If... the U.S. Had Targeted 4% Inflation?

The solid lines in the two left panels and the top right panel of Figure 10 depict the path of output, inflation, and interest rates from 2005 to 2013 under the estimated benchmark model with a 2.5% target. By construction, we are able to recover the actual U.S. data (subject to some small measurement errors) using the estimated shocks. We consider two alternative scenarios. Under the first scenario, the Fed picked 4% as their inflation target at the beginning of our sample in 1984. The counterfactual path of the key variables is given by the dashed lines. A few observations are in order. First, prior to 2009 the main difference between the benchmark scenario and the counterfactual policy are an upward shift of interest and inflation rates by 1.5%, which is the difference in the target inflation rates. Because we

Figure 10: Counterfactual Policy: Long-run Inflation Target of 4%



Notes: Solid lines correspond to the benchmark policy and reproduce the actual data. Dashed lines correspond to a counterfactual policy with a target inflation rate of 4% ($\bar{\pi} = 1.01$). Solid-dotted lines correspond to a counterfactual target of 4% and a sequence of expansionary monetary policy shocks $\epsilon_{R,t}$ that lower the interest rate to zero. The percentage change in consumption depicted in the bottom right panel is relative to the benchmark policy.

assume that firms adjust their price-setting technology to the new target, the path of output under the two scenarios is virtually identical up until the end of 2008. Second, after 2008 the ZLB never binds under this counterfactual policy. Third, inflation never drops below zero and promptly returns near the target of the Fed. Fourth, the recovery in GDP is somewhat faster.

A non-binding ZLB between 2009 and 2014 would have given the Fed the ability to conduct conventional expansionary monetary policy by lowering interest rates to zero. We consider such a policy in our second scenario. Using a sequence of unanticipated monetary policy shocks $\epsilon_{R,t}$, we reduce the nominal interest rate to zero under the 4% inflation target. The path of variables under this scenario is shown by circles in Figure 10. Here the return of inflation to average levels is even quicker and recovery of GDP takes about a year less than under the historical policy. The bottom right panel of Figure 10 shows that under both of the scenarios consumption is substantially higher relative to the benchmark after 2009.

Unfortunately, raising the inflation target is also associated with a cost that is not shown in Figure 10: if the public does not adjust their price- and wage-setting technology to the higher level of inflation and the 4% average inflation rate leads to increased price adjustment costs, then there will be an output and welfare loss associated with the New Keynesian channel. Moreover, there could be costs not captured in our model. As we explained above, the Friedman channel may contribute to additional welfare costs during "normal" times. For example, using the calculations in Aruoba and Schorfheide (2011), which account for both the New Keynesian and the Friedman channel, the welfare loss of changing the long-run inflation to 4% from 2.5% is about 0.6% of consumption.

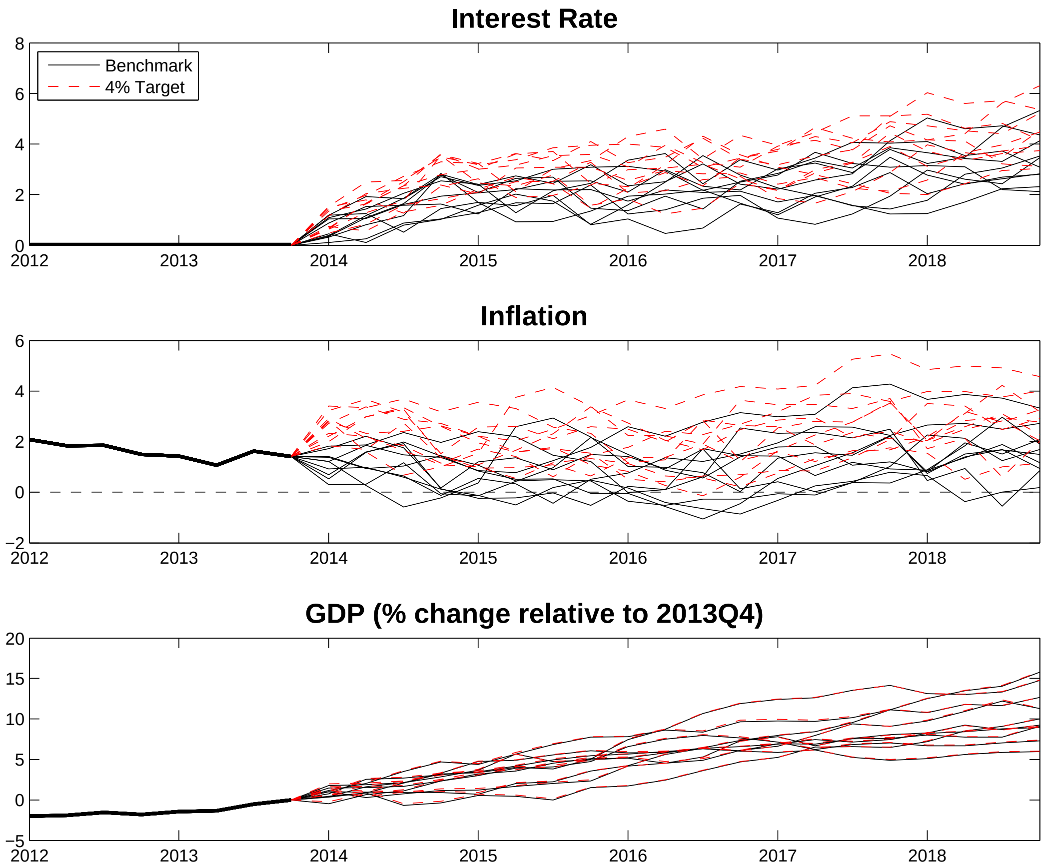
From an *ex ante* perspective, the costs and benefits of the high-inflation-target policy have to be weighted by the probability of reaching the ZLB. This probability is very small in our estimated model – less than 0.1% for the U.S – even under a 2.5% target rate. Thus, the costs are potentially incurred over a long period of time without reaping any benefits. Moreover, as Japan's experience illustrates, spending a considerable amount of time at the ZLB may be unrelated to the central bank's inflation target. Within the logic of our model, a switch to the deflation regime is equally likely to occur for targets of 2% and 4%, respectively.[13] We conclude that from an *ex ante* viewpoint the case for a higher inflation target is not particularly strong.

VI.B What If... the U.S. Switches to a 4% Target?

We now consider a hypothetical switch to a 4% target rate in 2014:Q1, conditioning on the state of the U.S. economy at the end of 2013:Q4. Results are depicted in Figure 11. We show 10 random trajectories under the two target inflation rates that share the same underlying structural innovations. First, notice that even under the benchmark target of 2.5%, the model predicts a lift-off from the ZLB. This prediction is common to many DSGE models, indicating that the current monetary policy is, by historical standards, unusually expansionary.[14] According to the estimated model, adverse shocks pushed the economy to the ZLB but, based on historical experience, these shocks tend to be mean reverting, which is consistent with the observed (albeit slower than expected) recovery.

Second, the interest rate, output, and inflation forecasts reflect substantial uncertainty. Under the 2.5% target rate there remains a risk of deflation as late as 2017, meaning that some of the simulated inflation trajectories fall below zero prior to 2017, which is broadly in line with the forecasts presented from the univariate UC-SV model in Figure 2. Third, the lift-off from the ZLB is faster under the 4% percent target inflation rate and the deflation risk is reduced. Because the interest rate increase (relative to the benchmark policy) is generated by an increase in the inflation target, inflation is higher rather than lower under the alternative policy. Finally, while the raise in the target inflation rate affects interest rate

Figure 11: Counterfactual Policy: Inflation Target of 4% in 2014



Notes: The lines prior to 2014:Q4 represent actual U.S. data. The subsequent (solid) hairs correspond to simulated trajectories under the prevailing policy. The dashed lines correspond to simulated trajectories (based on the same sequence of stochastic disturbances) under the counterfactual 4% target.

and inflation dynamics, the path of GDP is largely unchanged.

This analysis suggests that if the central bank raises the inflation target now, even if it is able to communicate and convince the public about the credibility of this new policy, the expected real effects of this policy are essentially zero. As we saw from the first experiment, the only positive effect would be the ability to execute unanticipated expansionary monetary policy actions on trajectories along which adverse shocks push the economy back to the ZLB. In the case of Japan, which according to our analysis has a high likelihood of being in the deflation regime, raising the target inflation rate would also have no significant immediate effect, because raising the target does not eliminate the deflation regime. Finally, a target rate change may have an adverse effect on the credibility of the central bank. This concern has been prominently voiced by the head central bankers of Germany and Switzerland, see Weber and Hildebrand (2010), who have argued that changing the inflation target would destroy the credibility they built regarding their commitment to price stability.

VI.C Other Policies

Throughout this paper we have stressed multiplicity of equilibria in workhorse New Keynesian DSGE models. We now provide a brief discussion of policies that interact with these multiplicities.[15]

Managing Expectations. Our DSGE model features a belief shock that determines the inflation regime. It serves as a coordination device for agents in the model. We used the belief shock as a substitute for a theory of equilibrium selection. In reality it is conceivable that a central bank has considerable influence on this expectation coordination through its communication. In fact, in ACS we argue that the aggressive unconventional monetary policies in the U.S., in contrast to the more measured responses of the Bank of Japan, may have prevented a switch to the deflation regime in the U.S.

Eliminating the Deflation Steady State / Regime. Abstracting from fundamental shocks and focusing on steady states, Figure 6 suggests that the deflation steady state could be eliminated by (i) a policy rule that raises the nominal interest rate above the real rate once inflation becomes negative (see Benhabib, Schmitt-Grohé, and Uribe (2001b) and Bullard

(2010)); or (ii) by responding less strongly to the inflation gap such that the policy rule is flatter ($\psi_1 < 1$) than the Fisher equation in the graph. This policy is called a passive monetary policy.

Switching to a discontinuous monetary policy rule (i) does not seem to be a solution for the current ZLB episode in the U.S., because according to our empirical analysis in Section IV and in the ACS companion paper, the U.S. is still in the targeted-inflation regime, and, moreover, inflation is not low enough to have reached what would be a reasonable threshold for a jump in the interest rate. For Japan, the quantitative assessment of such a policy would be interesting and is indeed a topic of our ongoing research.

The downside the passive monetary policy (ii) is that, in combination with a passive fiscal policy in the terminology of Leeper (1991), that is, a fiscal policy that only responds weakly to the level of real government debt, it opens the door for undesirable belief-shocked induced fluctuations of output, inflation and interest rates around the targeted-inflation steady state.[16] A solution could be provided by either combining the passive monetary policy with a fiscal policy that is active in the sense that it responds strongly to the level of government debt, or by using a fiscal policy that responds to the nominal level of debt or directly to the level of inflation, signaling to the public that the deflation steady state is fiscally unsustainable. For recent studies on the interplay between monetary and fiscal policy and its effect on equilibrium determinacy in New Keynesian DSGE models see Davig and Leeper (2007), Cochrane (2011), and Del Negro and Sims (2015).

VII Conclusion

In this paper we tried to shed some light on how inflation dynamics may change when an economy hits the zero lower bound of interest rates. We considered the experiences of Japan, the U.S., and the Euro Area through the lens of a univariate time series model on the one hand and a New Keynesian DSGE model on the other hand. It turns out that the predictions of the workhorse DSGE model are ambiguous, because multiple equilibria can arise. The multiplicity is a blessing and a curse. It allows us to rationalize disparate cross-country experiences but it also generates a lot of uncertainty about the effect of economic policies. In this regard, it is desirable that monetary and fiscal policies are conducted in a way that prevents the coordination of private sector inflation expectations on a deflationary level and eliminates the possibility of a deflation regime altogether.

Appendix

A Data

A.1 United States

Real per capita GDP: We obtained real GDP (GDPC96) and converted into per capita terms using the Civilian Noninstitutional Population (CNP16OV). The population series is smoothed applying an eight-quarter backward-looking moving average filter. Source: FRB St. Louis FRED database.

Real per capita consumption: We obtained real personal consumption expenditures (PCECC96) and converted into per capita terms using the Civilian Noninstitutional Population (CNP16OV). Source: FRED.

GDP Deflator Inflation: computed as log difference of GDP deflator (GDPDEF), multiplied by 400 to convert it into annualized percentages. Source: FRED.

CPI Inflation: computed as log difference of CPI (CPIAUCSL), multiplied by 400 to convert it into annualized percentages. Source: FRED.

Interest Rate / Monetary Policy Rate: effective federal funds rate (FEDFUNDS) averaged over each quarter. Source: FRED.

Inflation Expectations: 1-year-ahead and 5-year-ahead inflation expectations from Aruoba (2014) averaged over each quarter.

A.2 Japan

Real per capita GDP: We collected real GDP (RGDP) from the Cabinet Office's National Accounts. We used the statistical release of benchmark year 2005 that covers the period 1994.Q1 - 2013.Q4. To extend the sample we collected RGDP figures from the benchmark year 2000 and constructed a series spanning the period 1981.Q1-2013.Q1 using the quarterly growth rate of the RGDP benchmark year 2000. Our measure of per-capita output is RGDP divided by the total population of 15 years and over. We smoothed the quarterly growth of the population series using an eight quarter backward-looking moving average filter. We obtained population data from the Statistics Bureau of the Ministry of Foreign Affairs Historical data Table b-1.

Real per capita consumption: We collected real Private Consumption data from the Cabinet Office's National Accounts and follow the same procedure as for real GDP to convert it into per capita terms.

GDP deflator inflation: For the price level we use the implicit GDP deflator index from the Cabinet Office. We also extend the benchmark year 2005 release using the growth rate of the index from the benchmark year 2000 figures.

Interest Rate / Monetary Policy Rate: For the nominal interest rate we use the Bank of Japan's uncollateralized call rate (STSTRACLUCON) from 1986:M7-2013:M12. To complete the series from 1981.M1 - 1985.M6 we use the monthly average of the collateralized overnight call rate (STSTRACLCOON). Finally the monthly figures are transformed using quarterly averages over the sample period.

Inflation Expectations: 10-year-ahead inflation expectations are obtained from iMFdirect March 4, 2014 post "Euro Area - Deflation versus Lowflation" by Moghadam, Teja, and Berkmen. As 1-year-ahead inflation expectations we use December Blue Chip forecasts for the following year. Both of these measures are observed at an annual frequency.

A.3 Euro Area

Real GDP: YER. Source: *Area Wide Model* database, see ECB Working Paper No. 42.

Real Consumption: PCR. Source: *Area Wide Model* database.

GDP Deflator Inflation: computed as log differences of YED, scaled by 400. Source: *Area Wide Model* database.

CPI Inflation: computed as log differences of HICP, scaled by 400. Source: *Area Wide Model* database.

Interest Rate: short-term interest rate (STN). Source: *Area Wide Model* database.

Monetary Policy Rate: interest rate on the main refinancing operations (MRO). Source: ECB.

Inflation Expectations: 1-year-ahead and 5-year-ahead inflation forecasts. Source: ECB Survey of Professional Forecasters.

B Estimation of the UV-SV Model

We estimate the UC-SV model (1) based on GDP deflator inflation data from the U.S., Japan, and the Euro Area over the period from 1984:Q1 to 2014:Q4 using Bayesian techniques designed for state-space models with stochastic volatility. The prior distribution is

Table A-1: Prior Distribution for UV-SV Model

	U.S.	Japan	Euro Area
φ	U[0,1]	U[0,1]	U[0,1]
σ	IG(3,2.5)	IG(3,5)	IG(3,4.5)
ν_η	N(0.9, 0.5)	N(0.9, 0.5)	N(0.9, 0.5)
ν_ϵ	N(0.9, 0.5)	N(0.9, 0.5)	N(0.9, 0.5)
σ_{ν_η}	IG(3, 0.1)	IG(3, 0.1)	IG(3, 0.1)
σ_{ν_ϵ}	IG(3, 0.01)	IG(3, 0.01)	IG(3, 0.01)

Table A-2: Posterior Medians and 90% Credible Intervals for UV-SV Model

	U.S.		Japan		Euro Area	
φ	0.59	[0.34, 0.88]	0.17	[0.09, 0.29]	0.53	[0.37, 0.77]
σ	0.53	[0.42, 0.66]	1.55	[1.55, 1.55]	0.65	[0.56, 0.77]
ν_η	0.43	[-0.42, 0.91]	0.44	[-0.34, 0.94]	0.48	[-0.40, 0.92]
ν_ϵ	0.74	[-0.23, 0.98]	0.51	[-0.98, 0.93]	0.56	[-0.32, 0.97]
σ_η	0.24	[0.14, 0.53]	0.24	[0.13, 0.54]	0.25	[0.14, 0.52]
σ_ϵ	0.12	[0.05, 0.39]	0.17	[0.056, 0.45]	0.09	[0.04, 0.27]

summarized in Table A-1. Note that Inverse Gamma distribution $IG(a, b)$ is parameterized as $p_{IG}(\sigma \mid a, b) \propto \sigma^{-a-1} \exp(b/\sigma)$. We use different priors for σ across countries. The median of the prior is chosen to match a pre-sample sample standard deviation of inflation.

C DSGE Model

C.1 Households, Firms, Government Policies, and Shocks

The empirical analysis in this paper is based on the following DSGE model:

Households: solve the following problem:

$$\max_{C_t,H_t,B_t} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \delta_t \left(\frac{(C_t/A_t)^{1-\tau} - 1}{1-\tau} - \frac{H_t^{1+1/\eta}}{1+1/\eta} \right) \right], \tag{A.1}$$

subject to:

$$P_t C_t + B_t + T_t = W_t H_t + R_{t-1} B_{t-1} + P_t D_t + P_t SC_t.$$

Here β is the discount factor, δ_t is a discount factor shock, C_t is consumption, which enters the utility functions relative to the level of technology A_t, H_t is hours worked. The budget constraint is written in nominal terms: P_t is the price of the final good, B_t are government bonds, T_t are taxes, W_t are nominal wages, R_t is the nominal interest rate, D_t are dividend payments from the firms, and SC_t net proceeds from trading state-contingent claims.

Firms: Perfectly competitive, final goods producing firms combine a continuum of intermediate goods indexed by $j \in [0, 1]$ using the technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}. \tag{A.2}$$

Here $1/\nu > 1$ represents the elasticity of demand for each intermediate good. The firm takes input prices $P_t(j)$ and output prices P_t as given. Profit maximization implies that the demand for intermediate goods is:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t. \tag{A.3}$$

Free entry implies that the relationship between intermediate goods prices and the price of the final good is:

$$P_t = \left(\int_0^1 P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \tag{A.4}$$

Intermediate good j is produced by a monopolist who has access to the following production technology:

$$Y_t(j) = A_t H_t(j), \tag{A.5}$$

where A_t is an exogenous productivity process that is common to all firms. Intermediate good producers buy labor services $H_t(j)$ at a nominal price of W_t. Moreover, they face nominal rigidities in terms of price adjustment costs. These adjustment costs, expressed as a fraction of the firm's output, are defined by the function

$$\Phi_p(x) = \phi(x - \bar{\pi})^2. \tag{A.6}$$

Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its labor inputs $H_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits:

$$\max_{\{H_{t+s}(j),P_{t+s}(j)\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \delta_{t+s} Q_{t+s|t} \left(\frac{P_{t+s}(j)}{P_{t+s}} A_{t+s} H_{t+s}(j) \right. \tag{A.7}$$
$$\left. -\Phi_p \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} \right) A_{t+s} H_{t+s}(j) - \frac{W_{t+s} H_{t+s}(j)}{P_{t+s}} \right),$$

subject to

$$A_t H_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t.$$

Monetary and Fiscal Policies: Monetary policy is described by the interest rate feedback rule defined in (8) and (9). The fiscal authority consumes a fraction ζ_t of aggregate output Y_t, where $\zeta_t \in [0, 1]$ follows an exogenous process. The government levies a lump-sum tax (subsidy) to finance any shortfalls in government revenues (or to rebate any surplus). The government's budget constraint is given by:

$$P_t G_t + R_{t-1} B_{t-1} = T_t + B_t, \tag{A.8}$$

where $G_t = \zeta_t Y_t$.

Exogenous Shock Processes: The model economy is perturbed by four fundamental exogenous processes:

$$\begin{aligned} \ln A_t &= \ln \gamma + \ln A_{t-1} + \ln z_t, \quad \text{where} \ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t} \\ \ln g_t &= (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t} \\ \ln \delta_t &= \rho_\delta \ln \delta_{t-1} + \epsilon_{\delta,t}, \end{aligned} \tag{A.9}$$

where $g_t = 1/(1-\zeta_t)$, and the monetary policy shock $\epsilon_{R,t}$ is assumed to be serially uncorrelated.

In addition, there is a two-state Markov switching process $s_t \in \{0,1\}$ that serves as a coordination device for agents' expectations. The transition probabilities of this process are $\mathbb{P}\{s_t = j | s_{t-1} = j\} = \rho_j$, $j = 1, 2$.

C.2 Equilibrium Conditions

We use the following stationarity inducing transformations: $y_t = Y_t/A_t$, and $c_t = C_t/A_t$. We also define the gross inflation rate $\pi_t = P_t/P_{t-1}$. The equilibrium conditions are given by

$$1 = \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \left(\frac{\delta_{t+1}}{\delta_t} \right) \frac{1}{\gamma z_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{A.10}$$

$$-1 + \frac{1}{\nu}\left(1 - c_t^\tau y_t^{1/\eta}\right) + \phi\left(\pi_t - \bar{\pi}\right)\left[\left(1 - \frac{1}{2\nu}\right)\pi_t + \frac{\bar{\pi}}{2\nu}\right] \tag{A.11}$$

$$= \phi \beta \mathbb{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \left(\frac{\delta_{t+1}}{\delta_t} \right) \pi_{t+1} \left(\pi_{t+1} - \bar{\pi} \right) \frac{y_{t+1}}{y_t} \right]$$

$$R_t = \left[r\pi^* \left(\frac{\pi_t}{\pi^*} \right)^{\psi_1} \left(\frac{Y_t}{\bar{Y}_t} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} \exp(\epsilon_{R,t}) \tag{A.12}$$

$$c_t = \left(\frac{1}{g_t} - \frac{\phi}{2} \left(\pi_t - \bar{\pi} \right)^2 \right) y_t \tag{A.13}$$

C.3 A Simplified Version of the Model

In the main text we refer to a simplified version of the DSGE model which is obtained by setting $\tau = 1$, $\eta = \infty$, $\psi_2 = 0$, $\rho_R = 0$. In the targeted-inflation steady state we have

$$\pi_* = \bar{\pi}, \quad r_*^f = \gamma/\beta, \quad R_* = r_*^f \bar{\pi}, \quad c_* = 1 - \nu, \quad y_* = g_* c_*. \tag{A.14}$$

The deflation steady state is given by

$$\begin{aligned}
\pi_* &= 1/r_*^f, \quad r_*^f = \gamma/\beta, \quad R_* = 1 \quad &(A.15)\\
c_* &= 1 - \upsilon - \frac{\phi}{2}(1-2\lambda)\left(\pi_* - \frac{1-\lambda}{1-2\lambda}\bar{\pi}\right)^2 + \frac{\phi}{2}\frac{\lambda^2}{1-2\lambda}\bar{\pi}^2 \\
y_* &= \frac{c_*}{\frac{1}{g_*} - \frac{\phi}{2}(\pi_* - \bar{\pi})^2},
\end{aligned}$$

where $\lambda = \nu(1-\beta)$.

We also refer to the log-linearized equilibrium conditions (around the targeted-inflation steady state), which are given by

$$\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}] - (\widehat{R}_t - \mathbb{E}_t[\widehat{\pi}_{t+1} + \widehat{z}_{t+1} - \widehat{\delta}_{t+1} + \widehat{\delta}_t]) \tag{A.16}$$

$$\widehat{\pi}_t = \beta \mathbb{E}_t[\widehat{\pi}_{t+1}] + \kappa \widehat{mc}_t, \quad \text{where} \quad \widehat{mc}_t = \widehat{c}_t, \; \kappa = c_*/(\nu\phi\bar{\pi}^2) \tag{A.17}$$

$$\widehat{R}_t = \max\left\{ -\ln R_*, \; \psi_1 \widehat{\pi}_t + \epsilon_{R,t} \right\} \tag{A.18}$$

$$\widehat{y}_t = \widehat{c}_t + \widehat{g}_t. \tag{A.19}$$

Here $\widehat{x}_t = \ln(x_t/x_*)$.

C.4 Derivations for Section III.B

This is a discrete-time version of the calculations in Cochrane (2015). To simplify the notation we omit hats from the variables. Let $Rr_t = R_t - r_t$. Then the perfect foresight system can be written as

$$\begin{aligned} c_t &= \mathbb{E}[c_{t+1}] - (Rr_t - E[\pi_{t+1}]) \\ \pi_t &= \beta E[\pi_{t+1}] + \kappa c_t. \end{aligned}$$

To iterate the system forward, we express time $t+1$ variables as functions of time t variables.

In matrix form, the system becomes: This leads to

$$\begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} c_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} Rr_t.$$

Solving for (c_{t+1}, π_{t+1}) we obtain

$$\begin{bmatrix} c_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\kappa/\beta & -1/\beta \\ -\kappa/\beta & 1/\beta \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} Rr_t = \Gamma_* \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} Rr_t. \quad \text{(A.20)}$$

We proceed by calculating the eigenvalues of the autoregressive matrix Γ_*. Define $\rho = 1/\beta$. This amounts to solving the quadratic equation

$$\begin{aligned} 0 &= (1+\kappa\rho-\lambda)(\rho-\lambda) - \kappa\rho^2 \\ &= \lambda^2 - \lambda(1+\kappa\rho+\rho) + (1+\kappa\rho)\rho - \kappa\rho^2 \\ &= \lambda^2 - \lambda(1+\rho(1+\kappa)) + \rho. \end{aligned}$$

The solutions are

$$\begin{aligned} \lambda_1 &= \frac{1+\rho(1+\kappa)}{2} + \sqrt{\frac{(1+\rho(1+\kappa))^2}{4} - \rho} \\ \lambda_2 &= \frac{1+\rho(1+\kappa)}{2} - \sqrt{\frac{(1+\rho(1+\kappa))^2}{4} - \rho}. \end{aligned}$$

Note that $\rho > 1$ and $\kappa > 0$, which implies that

$$\frac{1+\rho(1+\kappa)}{2} > 1.$$

Moreover

$$\begin{aligned}(1+\rho(1+\kappa))^2-4\rho &= 1+2\rho+2\rho\kappa+\rho^2(1+\kappa)^2-4\rho \\ &= (\rho-1)^2+\kappa\rho(2+\rho+\kappa\rho) \\ &> 0.\end{aligned}$$

We conclude that λ_1 is an unstable eigenvalue.

Now note that

$$\sqrt{\frac{(1+\rho(1+\kappa))^2}{4}-\rho} < \sqrt{\frac{(1+\rho(1+\kappa))^2}{4}} = \frac{1+\rho(1+\kappa)}{2},$$

which implies that $\lambda_2 > 0$. In order to show that $\lambda_1 < 1$, we need to show that

$$\frac{\rho-1}{2}+\frac{\rho\kappa}{2} \leq \frac{1}{2}\sqrt{(\rho-1)^2+\rho^2\kappa^2+2\rho\kappa(1+\rho)}.$$

Multiplying by 2 and squaring both sides of the equation yields

$$(\rho-1)^2+\rho^2\kappa^2+2\rho\kappa(\rho-1) < (\rho-1)^2+\rho^2\kappa^2+2\rho\kappa(\rho+1).$$

Thus, we verified that $0 \leq \lambda_2 < 1$.

Now consider the eigenvalue decomposition of the matrix Γ_*, which we write as as $\Gamma_* J\Lambda J^{-1}$. We can now define $w_{t+1} = J^{-1}[c_{t+1}, \pi_{t+1}]'$. Let $(J^{-1})_{1.}$ be the first row of J^{-1}, which corresponds to the eigenvector associated with the unstable root λ_1. To ensure that the system is stable for $t > T$ conditional on $Rr_t = 0$, it has to be the case that

$$J^{-1}_{1.}\begin{bmatrix} c_{T+1} \\ \pi_{T+1} \end{bmatrix} = 0, \tag{A.21}$$

which determines c_{T+1} as a function of π_{T+1}. Figure 8 is generated as follows (i) choose π_{T+1}; (ii) solve (A.21) for c_{T+1}; (iii) iterate (A.20) forward for $t > T + 1$; (iv) iterate backward using

$$\begin{bmatrix} c_t \\ \pi_t \end{bmatrix} = \Gamma_*^{-1} \begin{bmatrix} c_{t+1} \\ \pi_{t+1} \end{bmatrix} - \Gamma_*^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} Rr_t. \tag{A.22}$$

for $t \leq T$.

C.5 Parameterization of DSGE Models

The parameters for the DSGE model-based analysis are obtained as follows: (1) We calibrate γ, β, $\bar{\pi}$, g_*, η, ψ_1, ψ_2, ν, p_{00}, and p_{11}. The steady-state related parameters are calibrated based on long-run averages. (2) We use Bayesian techniques to estimate the remaining parameters. The estimation periods are: 1984:Q1 - 2007:Q4 (U.S.); 1981:Q1 - 1994:Q4 (Japan); 1984:Q1 - 2007:Q4 (Euro Area). The parameter values are summarized in Table A-3.

Table A-3: DSGE Model Parameters

Parameters	Description	U.S.	Japan	Euro Area
$100 \ln \gamma$	Quarterly growth rate of technology	0.496	0.565	0.574
$400(1/\beta - 1)$	Annualized discount rate	0.861	1.878	0.930
$400 \ln \bar{\pi}$	Annualized inflation rate	2.465	1.278	3.102
$(C/Y)_*$	SS consumption/output ratio	0.647	0.579	0.567
τ	Inverse IES	1.993	1.641	2.119
η	Frisch elasticity	0.720	0.850	0.791
ν	EOS intermediate inputs	0.100	0.100	0.100
κ	Slope (linearized) Phillips curve	0.101	0.425	0.525
ψ_1	Taylor rule: weight on inflation	1.500	1.500	1.500
ψ_2	Taylor rule: weight on output growth	0.100	0.100	0.100
α	Smoothing coeff. for trend output	0.900	0.850	0.630
ρ_R	Interest rate smoothing	0.799	0.745	0.737
ρ_d	Persistence: discount shock	0.954	0.906	0.957
ρ_g	Persistence: demand shock	0.955	0.928	0.981
ρ_z	Persistence: technology shock	0.188	0.086	0.098
$100\sigma_R$	Std dev: monetary policy shock	0.160	0.190	0.160
$100\sigma_d$	Std dev: discount shock	1.880	1.180	1.620
$100\sigma_g$	Std dev: demand shock	0.530	0.770	0.400
$100\sigma_z$	Std dev: technology shock	0.500	1.090	0.450
p_{00}	Prob of staying in deflation regime	0.975	0.975	0.975
p_{11}	Prob of staying in targeted-inflation regime	0.990	0.990	0.990

Notes: Note that $g_* = 1/(C/Y)_*$.

Endnotes

[1]Models with multiple equilibria are common in many areas of economics. For instance, an important example in the industrial organization literature is an entry game model with two potential entrants. For markets that can support a profitable monopoly but not a profitable duopoly the model tends to be silent about which firm enters the market.

[2]We show both GDP deflator inflation and CPI inflation. The DSGE model-based analysis is based on GDP deflator inflation. We include CPI inflation because the inflation expectations data refer to changes in the CPI.

[3]The first of these spikes is in 2008:Q4 and corresponds to a massive decline in imports during the global financial crisis that skews GDP deflator up. The second one is in 2014:Q2 and it corresponds to a one-time increase in value-added tax. Neither of these spikes show up in CPI inflation.

[4]See, for instance Stock and Watson (2007) and Faust and Wright (2013). It captures two features that are important for inflation forecasting: time-variation in trend inflation through τ_t and time variation in the persistence of inflation through the relative magnitude of the log volatilities $h_{\epsilon,t}$ and $h_{\eta,t}$.

[5]Formally, we set $\tau = 1$, $\eta = \infty$, $\psi_2 = \rho_R = 0$, and choose the remaining parameters according to Table A-3.

[6]In the context of the model described in Appendix C the real rate process is given by $\widehat{r}_t = \rho_z z_t - (\rho_d - 1)\delta_t$, where ρ_z and ρ_d are the autocorrelations of the technology growth and the discount factor process.

[7]There is some disagreement how to handle dynamics under which inflation is explosive but real consumption and output are not. Cochrane (2011) argues that such paths should not be ruled out, while other researchers tend to rule them out.

[8]A similar analysis can be conducted for the case of $1/\kappa > 0$. However, in this case the transition is instantaneous because the system given by (15) and (16) has two unstable roots and one root that is equal to zero. Once one sets the linear combinations of interest rates, inflation, and consumption associated with

the unstable eigenvalues to zero, the linear combination given by the third eigenvector adjusts instantaneously because the third eigenvalue is zero.

[9]However, this path does not capture the drop in real activity observed during the Great Recession.

[10]Bullard (2010) voiced concern that the probability of transitioning to the deflation regime increases the longer an economy experiences low interest and inflation rates.

[11]For U.S. data we set the target inflation rate in our model to 2.5% instead of 2% because the former number corresponds to the average GDP deflator inflation rate over our estimation sample.

[12]This estimate corresponds to the average inflation between 1984 to 2007.

[13]In our model the sunspot shock evolves exogenously. In a richer model in which the probability of transitioning to a deflation regime would increase as interest or inflation rates fall, raising the target inflation rate could potentially lower the probability of transitioning to the deflation regime.

[14]See also the discussion in Del Negro and Schorfheide (2013) of this phenomenon and how external interest rate forecasts in combination with anticipated monetary policy shocks are needed to forecast a prolonged period of zero interest rates with standard DSGE models.

[15]The literature has discussed many other policies in the context of the ZLB, in particular unconventional monetary policies such as large-scale asset purchases, often called quantitative easing, the effects of forward guidance signaling an extended period of low interest rates, and a switch from inflation to price level targeting as a way of creating a commitment to expansionary monetary policy after a period of low, possibly negative, inflation rates. Because our model abstracts from frictions that interact with these policies, e.g., limited asset market participation of some households and firms, or informational frictions that affect the credibility of central bank announcements, we do not analyze the effect of these policies here.

[16]Clarida, Gali, and Gertler (2000) and Lubik and Schorfheide (2004) document that passive monetary policy may have been one of the culprits behind high inflation rates and high macroeconomic volatility in the 1970s.

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