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Abstract: This paper assesses the implications for optimal discretionary monetary policy if the slope of the Phillips curve changes. The paper first derives a Phillips curve from the optimal pricing decision of a monopolistic firm that faces a changing cost of price adjustment. The second aspect of the paper constructs a utility-based welfare criterion. A novel feature of this criterion is that is has a relative weight on output gap deviations that changes synchronously with changes in the cost of price adjustment. The systematic component of the targeting rule that implements the optimal discretionary policy under the utility-based criteria is constant. In contrast, the systematic component of the targeting rule under an ad-hoc criteria changes along with changes in the slope of the Phillips curve.

Keywords: Optimal monetary policy, Phillips curve, regime-switching

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1. Introduction

The slope of the Phillips curve is an important parameter in the minds of policymakers. Empirical evidence suggests a 'flattening' of the Phillips curve in recent decades, indicating inflation has become less responsive to movements in measures of aggregate economic activity, such as the output gap.[1] Although this phenomenon appears using reduced-form estimation procedures, as in Atkeson and Ohanian (2001), it also appears using structural approaches to estimation, as in Smets and Wouters (2007). In the New Keynesian framework, the slope of the Phillips curve appears in targeting rules describing optimal monetary policy. Given these observations, this paper addresses two issues: (1) the derivation of a Phillips curve with a changing slope, driven by changes in the cost of price adjustment and (2) the implications for optimal discretionary monetary policy confronting this type of structural change.

The channel generating the change in the slope of the Phillips curve is a shift in the price setting friction for monopolistically competitive firms.[2] The microfoundations of the firm's price-setting problem are similar to Rotemberg (1982), except the term governing the cost of price adjustment is subject to change over time. The equation describing the optimal price-setting behavior of the firm is similar to a standard forward-looking New Keynesian Phillips curve, except the coefficients on expected inflation and the output gap are subject

[1] For example, Atkeson and Ohanian (2001), Roberts (2006) and Williams (2006) document the flattening of the Phillips curve for the U.S., whereas Iakova (2007) does so for the U.K. and De Veirman (2007) for Japan.

[2] Several competing explanations for the change in the slope of the Phillips exist. For example, improvements in the conduct of monetary policy and globalization present additional rationale for a decline in the slope of the Phillips curve. However, this paper does not address these potential causes and focuses only on changes arising from the price setting friction. See Mishkin (2007) for an overview.

to change.

In the presence of markup shocks, a central bank trying to stabilize inflation and output faces the Phillips curve as the constraint on achieving these objectives. Under discretion, the optimal targeting rule balances policy objectives by prescribing adjustments to the output gap in response to movements in inflation. The central bank adjusts the output gap aggressively if the relative weight on output gap fluctuations in its objective function is small or the slope coefficient on the output gap in the Phillips curve is large. If the slope of the Phillips curve changes, then the optimal targeting rule will also change under a loss function that has a constant relative weight on output gap deviations, such as a common ad-hoc loss function in squared deviations of inflation and the output gap.

A benefit, however, of deriving the Phillips curve under the potential for structural change is that it makes possible the derivation of a utility-based welfare criterion. In contrast to an ad-hoc loss function with a constant relative weight on output gap deviations, a central feature of the utility-based measure is that this weight depends on the slope of the Phillips curve, so changes when the slope of the Phillips curve shifts. The changing weight reflects that higher losses arise due to inflation in states with relatively high costs of price adjustment. Since inflation imposes higher costs on firms in states with relatively sticky prices, it is precisely in these states that monetary policy increases the relative weight on inflation stabilization. In contrast to the ad-hoc rule, the optimal targeting rule under the utility-based welfare criterion directs the central bank to have a *constant* systematic response to inflation. That is, the optimal targeting rule advocates a policy that consistently adjusts

the output gap to the same extent in response to inflation, regardless of the slope of the Phillips curve.

Blake and Zampolli (2006), Moessner (2006), Zampolli (2006) and Svensson and Williams (2007) also demonstrate how shifts in parameters governing private sector relations generate shifts in the central bank's targeting rule. This paper differs from this previous work in two respects. First, the Phillips curve relation with changing coefficients arises from a representative firm's optimal pricing problem. Moessner (2006), Zampolli (2006) and Svensson and Williams (2007) study macroeconomic relations with changing parameters, but do not incorporate the potential for parameter change into the original optimization problems of households and firms. In this paper, the potential for structural change is built into the primitive optimization problem of the firm. The different approaches, however, stems partially from the different focus. For example, Svensson and Williams (2007) are specifically interested in model uncertainty and not with the mechanics generating shifts in the private sector relations. Second, this paper constructs a utility-based welfare criterion, instead of using an ad-hoc loss, to evaluate different monetary policies confronting shifts in the slope of the Phillips curve. Deriving the utility-based metric is possible because the microfoundations of the firm's pricing problem are made explicit.

2. The NK Phillips Curve Under Changing Costs of Price Adjustment

This section embeds state-dependent parameters into the optimal pricing problem of a monopolistically competitive firm. As in Rotemberg (1982), the firm faces quadratic costs of price adjustment, except the term governing the magnitude of the cost is subject to change. Introducing these changing costs into the pricing problem results in a Phillips curve relation with coefficients on the output gap and expected inflation that change over time.

2.1 Changing Costs of Price Adjustment

The Rotemberg (1982) formulation imposes a cost on monopolistic intermediate-goods producing firms for adjusting their price, given by

$$ac_{jt} = \frac{\varphi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2 Y_t, \tag{1}$$

where $\varphi \geq 0$ governs the magnitude of the price adjustment cost, Π denotes the gross steady-state rate of inflation and $P_t(j)$ denotes the nominal price set by firm $j \in [0,1]$.[3] The cost is measured in terms of the final good Y_t. The assumption of quadratic adjustment costs implies that firms change their price every period in the presence of shocks, but will adjust only partially towards the optimal price the firm would set in the absence of such costs. As with any type of quadratic adjustment cost, a firm prefers a sequence of small adjustments to very large adjustments in a given period. Alternatively, these costs may vary according

[3] See Ireland (2004) for a detailed treatment of quadratic costs of price adjustment in a DSGE model.

to a state, s_t, such as

$$ac_{jt}(s_t) = \frac{\varphi(s_t)}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2 Y_t, \tag{2}$$

where firms face a state-dependent cost of price adjustment. For $s_t \in \{1, 2\}$, the state evolves according to a two-state Markov chain with transition probabilities given by $p_{mn} = \Pr[s_t = n | s_{t-1} = m]$ for $m, n = 1, 2$.[4] Both private agents and the central bank observe s_t.

Changes in φ capture shifts in the myriad of costs facing firms when they adjust their price. Blinder, Canetti, Lebow, and Rudd (1998) and Zbaracki, Ritson, Levy, Dutta, and Bergen (2004) provide in-depth reviews of actual pricing decisions at firms and emphasize the complexity of the price adjustment process. Empirical evidence at the firm level, however, on how these costs change over time is lacking.

At the macroeconomic level, estimation of DSGE models suggest a change in the parameters governing price setting frictions in the early 1980s. For example, Boivin and Giannoni (2006) and Smets and Wouters (2007) split samples and estimate a lower slope coefficient on the output gap in the New Keynesian Phillips curve after about 1980. They discuss the change in slope as potentially arising from less frequent price adjustment under the low and stable aggregate inflation of the past few decades. Using the Rotemberg (1982) framework, this interpretation implies firms face higher costs of price adjustment in the post-Volcker period. Under the specification of adjustment costs in (2), the exogenously evolving cost of price adjustment captures these shifts in price setting frictions, but does not incorporate the possible linkages between aggregate conditions and firm-level pricing behavior.

[4] The assumption of two states is made for convenience and tractability, it can be replaced with an assumption concerning any finite number of states.

2.2 The Optimal Pricing Problem

Each of the monopolistically competitive intermediate-goods producing firms seek to maximize the expected present-value of profits,

$$E_t \sum_{s=0}^{\infty} \beta^s \Delta_{t+s} \frac{D_{t+s}(j)}{P_{t+s}}, \tag{3}$$

where Δ_{t+s} is the representative household's stochastic discount factor, $D_t(j)$ are nominal profits of firm j, and P_t is the nominal aggregate price level. Also, firm j produces good j. For given s_t, real profits are

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} y_t(j) - \Psi_t y_t(j) - \frac{\varphi(s_t)}{2} \left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1 \right)^2 Y_t, \tag{4}$$

where Ψ_t denotes real marginal cost and $y_t(j) = n_t(j)$ is the production of intermediate goods by firm j using labor input $n_t(j)$.

There exists a final-goods producing firm that purchases the intermediate inputs at nominal prices $P_t(j)$ and combines them into a final good using the following constant-returns-to-scale technology

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\theta_t - 1}{\theta_t}} dj \right]^{\frac{\theta_t}{\theta_t - 1}}, \tag{5}$$

where $\theta_t > 1 \ \forall \ t$ is the elasticity of substitution between goods. Variations in θ_t translate into markup shocks of the monopolistic firm's price over its marginal cost. The profit-maximization problem for the final-goods producing firm yields a demand for each intermediate good given by

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta_t} Y_t. \tag{6}$$

For a given s_t, substituting (4) and (6) into (3) then differentiating with respect to $P_t(j)$ yields the first-order condition

$$\begin{aligned} 0 &= (1-\theta_t)\Delta_t\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t}\left(\frac{Y_t}{P_t}\right)+\theta_t\Delta_t\Psi_t\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t-1}\left(\frac{Y_t}{P_t}\right)- \\ &\quad \varphi(s_t)\Delta_t\left(\frac{P_t(j)}{\Pi P_{t-1}(j)}-1\right)\left(\frac{Y_t}{\Pi P_{t-1}(j)}\right)+ \\ &\quad \beta E_t\left[\varphi(s_{t+1})\Delta_{t+1}\left(\frac{P_{t+1}(j)}{\Pi P_t(j)}-1\right)\left(\frac{P_{t+1}(j)Y_{t+1}}{\Pi P_t(j)^2}\right)\right], \end{aligned} \tag{7}$$

where an analogous condition exists for each s_t.

In a symmetric equilibrium, every firm faces the same Ψ_t and Y_t, so the pricing decision is the same for all firms, implying $P_t(j) = P_t$. Also, steady-state inflation and output are constant across states. Steady-state marginal costs are given by

$$\Psi = \frac{\theta-1}{\theta}, \tag{8}$$

and $\Psi^{-1} = \mu$, where μ is the steady-state markup of price over marginal cost. In the flexible-price case, where $\varphi(1) = \varphi(2) = 0$, marginal cost is $\Psi_t = \theta_t^{-1}(\theta_t - 1)$ and the markup is $\mu_t = \theta_t^{-1}(\theta_t - 1) > 1$.

To obtain a linear system the captures the firm's pricing decision, (7) is log-linearized conditional on each s_t and expectations are written using the approach in Gordon and St-Amour (1999), Bansal and Zhou (2002), and Davig and Leeper (2007). This approach requires defining a smaller information set that excludes the current state, Ω_t^{-s}, where $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$ and Ω_t represents the full information set available at time t. For example, conditional expectations of inflation using Ω_t are $E_t\pi_{t+1} = E[\pi_{t+1}|\Omega_t]$, where $\pi_t = \log(\Pi_t/\Pi)$. Using Ω_t^{-s},

distributing probability mass over states at $t+1$ yields

$$E_t\pi_{t+1} = E[\pi_{t+1} \left| s_t = i, \Omega_t^{-s}\right.] = p_{i1}E[\pi_{1t+1} \left|\Omega_t^{-s}\right.] + p_{i2}E[\pi_{2t+1} \left|\Omega_t^{-s}\right.], \tag{9}$$

which uses the state-contingent notation that defines $\pi_t = \pi_{it} \Leftrightarrow s_t = i$ for $i = 1, 2$. This notation simply indicates that inflation at t depends on the state at t, and not directly on past states (i.e. s_{t-i} for $i > 0$). Similar notation applies to all other endogenous variables. When taking expectations of variables written in state-contingent notation, let $E_t\pi_{it+1} \equiv E[\pi_{it+1} \left|\Omega_t^{-s}\right.]$.

Imposing symmetry and (8), a linear approximation to the firm's optimal price-setting equation can be written in terms of aggregate inflation using state-contingent notation as

$$\pi_{1t} = p_{11}\beta E_t\left[\pi_{1t+1}\right] + (1-p_{11})\frac{\varphi_2}{\varphi_1}\beta E_t\left[\pi_{2t+1}\right] + \frac{\theta-1}{\varphi_1}\left(\psi_{1t}+u_t\right), \tag{10}$$

and for $s_t = 2$ as

$$\pi_{2t} = p_{22}\beta E_t\left[\pi_{2t+1}\right] + (1-p_{22})\frac{\varphi_1}{\varphi_2}\beta E_t\left[\pi_{1t+1}\right] + \frac{\theta-1}{\varphi_2}\left(\psi_{2t}+u_t\right), \tag{11}$$

where $\varphi_i = \varphi(i)$, $\psi_{it} = \log\left(\Psi_{it}/\Psi\right)$, $u_t = \log\left(\mu_t/\mu\right) = -\left(\theta-1\right)^{-1}\widehat{\theta}_t$ is the markup shock and $\widehat{\theta}_t = \log(\theta_t/\theta)$.[5] More generally, these expressions can be rewritten as

$$\pi_{it} = \varphi_i^{-1}\beta E_t\left[\varphi\left(s_{t+1}\right)\pi_{t+1}\right] + \frac{\theta-1}{\varphi_i}\left(\psi_{it}+u_t\right), \tag{12}$$

for $i = 1, 2$, which reduces to the constant-parameter specification when either $\varphi_i = \varphi$ for all i or $p_{11} = p_{22} = 1$. Equation (12) illustrates how changing costs of price adjustment affect the coefficients on marginal cost and expected inflation. High costs of price adjustment results in

[5] See Appendix A for detailed derivations of (10) and (11).

a 'flat' Phillips curve, whereas lower costs results in a 'steep' Phillips curve. Thus, movements in real marginal cost in states with a flat Phillips curve have a relatively small effect on inflation, so equilibrium adjustments to shocks occur relatively more through quantities than prices.

3. Households

To analyze the implications of instability in the Phillips curve in a dynamic stochastic general equilibrium setting, this section gives the representative household's problem and optimality conditions. In a subsequent section, the households period-utility function forms the basis of the central bank's loss function.

The representative household chooses $\{C_t, N_t, B_t\}_{t=0}^{\infty}$ to maximize lifetime utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{H_t^{1+\eta}}{1+\eta} \right) \tag{13}$$

where C_t is the composite good, $H_t = \int_0^1 h_t(j)dj$ is time spent working, $\beta \in (0,1)$ is the discount factor and $\sigma > 0$ is the coefficient of relative risk aversion. Utility maximization is subject to the intertemporal budget constraint

$$P_t C_t + B_t = B_{t-1} + (1+\nu) W_t H_t + P_t X_t - P_t T_t, \tag{14}$$

where B_t are nominal bond holdings, X_t are real profits from ownership of firms, T_t are lump-sum taxes, P_t is the aggregate price level, W_t is the nominal wage and Q_t is the inverse of the gross nominal interest rate. Lump-sum taxes finance a constant employment subsidy, ν, which offsets the inefficiently low level of production in the steady-state arising from the

monopolistic distortion. The subsidy is set equal to $(1+\nu) = \mu$, which sets the flexible-price steady-state level of output equal to the level that would prevail in the absence of the monopolistic distortion (i.e. the efficient level).

The household's first-order conditions are

$$(1+\nu)\frac{W_t}{P_t} = \frac{H_t^{\eta}}{C_t^{-\sigma}}, \tag{15}$$

$$1 = \beta E_t\left[(Q_t\pi_{t+1})^{-1}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\right]. \tag{16}$$

In the previous section, the household discount factor is $\Delta_{t+s} = (C_{t+s}/C_t)^{-\sigma}$. In equilibrium, $H_t = N_t$ must also hold, where $N_t = \int_0^1 n_t(j)dj$. Also, there is no price dispersion in a symmetric equilibrium with quadratic costs of price adjustment, so aggregate output equals aggregate labor effort, so $Y_t = N_t$.[6]

The aggregate resource constraint is

$$Y_t = C_t + \frac{\varphi(s_t)}{2}(\Pi_t - 1)^2, \tag{17}$$

where steady-state inflation is set to zero.

4. Optimal Discretionary Policy Under an Ad-hoc Loss

Optimal policy under discretion in a standard New Keynesian framework, such as in Clarida, Gali, and Gertler (1999), instructs policy to contract aggregate demand when inflation rises. The extent of the response depends on two factors: the slope of the Phillips curve and the

[6] That is, $\int_0^1 n_t(j)dj = \int_0^1 y_t(j)dj$ is equivalent to $N_t = Y_t\int_0^1\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t}dj$, where $\int_0^1\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t}dj = 1$.

weight policymakers assign to output gap deviations in their loss function. A Phillips curve with a steep slope allows the central bank to exert considerable influence over inflation by adjusting aggregate demand. Concerns over output gap stability, however, tempers the use of the output gap to stabilize inflation.

Similar to Blake and Zampolli (2006), Moessner (2006), Zampolli (2006) and Svensson and Williams (2007), this section uses an ad-hoc loss function to determine how the central bank should respond to shifts in the slope of the Phillips curve. Specifically, the central bank's period ad-hoc loss function is

$$L_t = \pi_t^2 + \lambda x_t^2, \tag{18}$$

where λ is the relative weight on output deviations. The optimal discretionary policy takes private sector expectations as given and minimizes (18) subject to

$$\pi_{it} = \varphi_i^{-1} \beta E_t \left[\varphi \left(s_{t+1} \right) \pi_{t+1} \right] + \kappa_i x_{it} + e_t, \tag{19}$$

where $\kappa_i = \varphi_i^{-1} \left(\sigma + \eta \right) \left(\theta - 1 \right)$, $x_t = \log(Y_t / Y)$ and $e_t = \varphi_i^{-1} \widehat{\theta}_t$.[7] The disturbance e_t represents a transformed markup shock. Since the optimization problem is static, the central bank only needs to be concerned with setting policy based on the current state and disregards how the slope of the Phillips curve may change in the future.

Combing the first-order conditions for each state yields the optimal state-contingent targeting rules

$$x_{it} = -\frac{\kappa_i}{\lambda} \pi_{it}, \tag{20}$$

[7] The relationship between the output gap and marginal cost term is given by $\psi_t = (\sigma + \eta) x_t$.

for $i = 1, 2$. This set of targeting rules indicates that the central bank should optimally vary how aggressively it acts to offset aggregate supply disturbances depending on the slope of the Phillips curve. In states with relatively low costs of price adjustment, say in $s_t = 1$, the Phillips curve is steep and implies that inflation is relatively responsive to changes in the output gap. In this case, the optimal targeting rule instructs policy to use this leverage and adjust the output gap more aggressively in response to inflation. So with $\kappa_1 > \kappa_2$, the central bank adjusts aggregate demand more aggressively when $s_t = 1$ than when $s_t = 2$.

5. The Utility-Based Welfare Criterion

The ad-hoc loss function in squared deviations of the output gap and inflation from their steady-state values is a common specification and reflects the objectives of many central banks. Woodford (2003) shows how a second-order approximation to the expected utility of the consumer under the assumption of staggered price-setting, as in Calvo (1983), gives rise to a loss function of the ad-hoc form. However, the weight on the output gap term is a function of the frequency of price adjustment, among other structural parameters. Eusepi (2005) derives the utility-based welfare function for price adjustment subject to quadratic costs, as in Rotemberg (1982), and shows how the weight on the output gap term depends on the parameter governing the cost of price adjustment. In a setting where this cost can change, this section shows that the weight on the output gap also changes with the cost of price adjustment and how this affects the optimal policy under discretion.

The appendix derives the following period loss function from an approximation to the

period utility function of the representative household

$$L_{it}^{ub} = \Omega_i \left[\pi_t^2 + \lambda_i x_t^2\right], \tag{21}$$

where $\Omega_i = .5\varphi_i$ scales the loss according to the cost of price adjustment and

$$\lambda_i = \frac{\eta + \sigma}{\varphi_i}, \tag{22}$$

indicating that the weight on output gap deviations depends on the state governing the cost of price adjustment. If the utility function has log consumption and is linear in labor, so $\sigma = 1$ and $\eta = 0$, then (22) is simply $\lambda_i = \varphi_i^{-1}$.

In a state with a relatively low cost of price adjustment, deviations in inflation create a small loss, so the weight on the output gap is relatively high. Conversely, in a state with a high cost of price adjustment, deviations in inflation are costly, so the central bank should place less emphasis on output stabilization. This intuition is similar to that from the utility-based welfare criteria derived under the Calvo mechanism of price adjustment, as in Woodford (2003). When the price adjustment is infrequent, losses arise from price dispersion, so the central bank should place low weight on output stabilization relative to the case when price adjustment occurs more frequently.

Minimizing the central bank's utility-based loss function subject to the Phillips curve under the assumption that policy actions do not affect private agents' expectations yields

$$x_{it} = -\frac{\kappa_i}{\lambda_i}\pi_{it}, \tag{23}$$

or after substituting for λ_i and κ_i,

$$x_{it} = (1 - \theta)\,\pi_{it}, \tag{24}$$

indicating the central bank should *not* optimally vary how aggressively it acts to offset aggregate supply disturbances. The optimal targeting rule is a constant relation between output and inflation, independent of the state, and depends only upon the elasticity of substitution between goods. This result differs from the optimal discretionary policy under an ad-hoc loss, where the optimal discretionary policy instructs the central bank to switch policies in accordance with the slope of the Phillips curve.

In the state with relatively high costs of price adjustment, both the weight attached to output gap stabilization and the slope coefficient in the Phillips curve are relatively small. Under an ad-hoc loss, a high cost of price adjustment (i.e. flat Phillips curve) directs policy to reduce the systematic output gap response to inflation deviations precisely because such movements are less effective at stabilizing inflation. However, inflation volatility is more costly to firms in states with high costs of price adjustment, an aspect that the ad-hoc loss function neglects. The utility-based welfare criterion captures this higher cost of inflation volatility by reducing the weight on output gap stabilization in the states with a high cost of price adjustment.

Thus, in the high-cost state, two opposing forces exactly offset to bring about the invariant policy response : 1) a lower slope of the Phillips curve, which directs policy to reduce output gap movements to stabilize inflation and 2) a lower weight on the output gap, which directs policy to increase output gap movements to stabilize inflation.[8] The difference in comparison to the optimal policy under the ad-hoc loss function is that it only accounts for the first factor,

[8] Analogous reasoning applies to the low-cost state.

the change in the slope of the Phillips curve, and ignores the welfare implications of inflation in the different states.

6. Welfare Analysis

In this section, I compute the loss due to a central bank that sets policy to minimize the ad-hoc loss function instead of the utility-based metric. Clearly, loss arises if the weight on output gap deviations in the ad-hoc loss differs from the utility-based value. This weight, however, can align for one particular state, but a change in the slope of the Phillips curve will result in a welfare loss. To compute this loss, I derive the solutions for inflation and output using the targeting rules that minimize the ad-hoc loss function, then evaluate the welfare implications using the utility-based metric.

Using the method of undetermined coefficients on the minimum set of state variables, which are s_t and e_t, solutions under the ad-hoc loss have the form

$$x_{it} = a_i e_t, \tag{25}$$

$$\pi_{it} = -\frac{\lambda}{\kappa_i} a_i e_t \tag{26}$$

for $i = 1, 2$. The appendix illustrates how to find the unknown state-contingent coefficients, a_i for $i = 1, 2$, which have the following relationship to the structural parameters

$$\begin{bmatrix} (1 - p_{11}\gamma_1\rho) & -(1 - p_{11})\,\gamma_1 \frac{\kappa_1\varphi_2}{\kappa_2\varphi_1}\rho \\ -(1 - p_{22})\,\gamma_2 \frac{\kappa_2\varphi_1}{\kappa_1\varphi_2}\rho & (1 - p_{22}\gamma_2\rho) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -\frac{\kappa_1}{\lambda+\kappa_1^2} \\ -\frac{\kappa_2}{\lambda+\kappa_2^2} \end{bmatrix}. \tag{27}$$

In the case of i.i.d. shocks, solutions to (27) are $a_i = -\kappa_i \left(\lambda + \kappa_i^2\right)^{-1}$ for $i = 1, 2$.

The utility-based metric implies that the average welfare loss for $s_t = i$ each period is

$$\overline{L}_{it}^{ub} = .5\varphi_i \left[var\left(\pi_t\right) + \frac{\eta + \sigma}{\varphi_i} var\left(x_t\right) \right]. \tag{28}$$

Let $\widehat{\theta}_t \sim$ i.i.d. $N\left(0,1\right)$, then recalling $e_t = \varphi_i^{-1}\widehat{\theta}_t$ and substituting the solutions for inflation and the output gap into (28) yields

$$\overline{L}_{it}^{ub} = .5\varphi_i^{-1} \left[\left(\frac{\lambda}{\kappa_i}\right)^2 + \frac{\eta + \sigma}{\varphi_i} \right] a_i^2, \tag{29}$$

where $\min_\lambda \overline{L}_{it}^{ub} = \frac{\eta+\sigma}{\varphi_i}$. However, the common ad-hoc specification holds λ constant, so $\min_\lambda \overline{L}_{1t}^{ub} \neq \min_\lambda \overline{L}_{2t}^{ub}$, which indicates that λ can maximize welfare in one state, but will result in a welfare loss when the slope of the Phillips curve changes.

For a quantitative illustration, I set $\sigma = 1$ and $\eta = 0$, which implies a utility function that has log consumption and is linear in labor, and take numerical estimates of the slope coefficients for the Phillips curve from Lubik and Schorfheide (2004). For the other parameters, $\beta = .99$ and $\mu = 1.1$. Using the mid-points of the credibility intervals from the pre- and post-Volcker samples, the slope coefficients in the Phillips curve are $\kappa_1 = .725$ (pre-Volcker) and $\kappa_2 = .58$ (post-Volcker). Figure 1 plots each utility-based loss function (29) as a function of λ. The loss is given as a percentage of the loss under the policy that minimizes the utility-based welfare metric. For κ_1, the minimizing point corresponds to $\lambda = \varphi_1^{-1}$, which is the weight on output gap deviations prescribed by the utility-based metric. Figure 1 shows that the policy minimizing the ad-hoc loss following a flattening of the Phillips curve no longer minimizes the utility-based metric (i.e. a change from φ_1^{-1} to φ_2^{-1}). In this case, the loss is the vertical distance from the minimum of the solid line to the dashed line. The loss

arising from a flatter Phillips curve under this numerical specification is quite modest, on the order of just a few percent.

As another example, Figure 2 uses the boundaries of the 90-percent credibility intervals from Lubik and Schorfheide (2004) for the slope of the Phillips curve, where $\kappa_1 = 1.07$ (pre-Volcker) and $\kappa_2 = .27$ (post-Volcker). In this case, an ad-hoc λ that minimizes the utility-based loss in one state can generate a substantial welfare loss in the event that the slope of the Phillips curve changes. If $\lambda = \varphi_1^{-1}$ and the Phillips curve flattens, then the suboptimal policy results in a relatively large welfare decline, equal to roughly 50 percent of the utility-based value. However, Figure 2 also highlights an asymmetry. If $\lambda = \varphi_2^{-1}$ and the Phillips curve steepens, the welfare decline is relatively modest.

7. Conclusion

This paper shows that a change in the cost of price adjustment can generate instability in a forward-looking Phillips curve relation. In particular, the coefficients on both expected inflation and marginal cost, or the output gap, are subject to change in coordination with changes in the state governing the cost of adjusting prices.

In addition, Phillips curve instability has implications for optimal monetary policy. Under an ad-hoc welfare criterion, the coefficient in the optimal targeting rule changes when the slope of the Phillips curve changes. However, since the microfoundations of the firm's optimization problem are made explicit, it is possible to derive a utility-based welfare metric. A novel feature of this metric is that it has a state-dependent weight on the output

gap term. The weight depends inversely on the cost of price adjustment, so in the low cost state, relatively more weight is placed on output stabilization. The implication for optimal monetary policy under discretion is that the optimal targeting rule should *not* vary the systematic component of policy, standing in contrast to the prescription coming from the ad-hoc criterion.

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APPENDIX

A. Deriving the Phillips Curve Under Changing Costs of Price Adjustment

For $s_t = 1$, the conditional first-order condition after distributing the $\varphi(s_{t+1})$ term is

$$\begin{aligned} 0 &= (1-\theta_t)\,\Delta_t \left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} \left(\frac{Y_t}{P_t}\right) + \theta_t \Delta_t \Psi_t \left(\frac{P_t(j)}{P_t}\right)^{-\theta_t - 1} \left(\frac{Y_t}{P_t}\right) - \\ & \varphi(1)\,\Delta_t \left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)\left(\frac{Y_t}{\Pi P_{t-1}(j)}\right) + \\ & \beta p_{11} \varphi(1)\, E_t \left[\Delta_{t+1}\left(\frac{P_{t+1}(1,j)}{\Pi P_t(j)} - 1\right)\left(\frac{P_{t+1}(1,j)\,Y_{t+1}(1)}{\Pi P_t(j)^2}\right)\right] + \\ & \beta(1-p_{11})\,\varphi(2)\, E_t \left[\Delta_{t+1}\left(\frac{P_{t+1}(2,j)}{\Pi P_t(j)} - 1\right)\left(\frac{P_{t+1}(2,j)\,Y_{t+1}(2)}{\Pi P_t(j)^2}\right)\right], \end{aligned} \tag{A-1}$$

where $P_{t+1}(i,j)$ represents the nominal price for firm j when $s_{t+1} = i$ and $Y_{t+1}(i)$ represents final output when $s_{t+1} = i$. An analogous first-order condition exists for $s_t = 2$, except p_{11} is replaced with $(1 - p_{22})$ and $(1 - p_{11})$ is replaced with p_{22}. Using $(A - 1)$, the firm's optimal pricing condition for $s_t = 1$, after imposing $P_t(j) = P_t$, is given by

$$\begin{aligned} 0 &= (1-\theta)\,\Delta_t \left(\frac{Y_t}{P_t}\right) + \theta \Delta_t \Psi_t \left(\frac{Y_t}{P_t}\right) - \varphi(1)\,\Delta_t \left(\frac{P_t}{\Pi P_{t-1}} - 1\right)\left(\frac{Y_t}{\Pi P_{t-1}}\right) + \\ & \beta p_{11} \varphi(1)\, E_t \left[\Delta_{t+1}(1)\left(\frac{P_{t+1}(1)}{\Pi P_t} - 1\right)\left(\frac{P_{t+1}(1)\,Y_{t+1}(1)}{\Pi {P_t}^2}\right)\right] + \\ & \beta(1-p_{11})\,\varphi(2)\, E_t \left[\Delta_{t+1}(2)\left(\frac{P_{t+1}(2)}{\Pi P_t} - 1\right)\left(\frac{P_{t+1}(2)\,Y_{t+1}(2)}{\Pi {P_t}^2}\right)\right], \end{aligned} \tag{A-2}$$

where substituting in $P_t / P_{t-1} = \Pi_t$ yields

$$\begin{aligned} 0 &= (1-\theta_t)\,\Delta_t + \theta_t \Delta_t \Psi_t - \varphi(1)\,\Delta_t \left(\frac{\Pi_t}{\Pi} - 1\right)\left(\frac{\Pi_t}{\Pi}\right) + \\ & \beta p_{11} \varphi(1)\, E_t \left[\Delta_{t+1}(1)\left(\frac{\Pi_{t+1}(1)}{\Pi} - 1\right)\left(\frac{\Pi_{t+1}(1)\,Y_{t+1}(1)}{\Pi Y_t}\right)\right] + \\ & \beta(1-p_{11})\,\varphi(2)\, E_t \left[\Delta_{t+1}(2)\left(\frac{\Pi_{t+1}(2)}{\Pi} - 1\right)\left(\frac{\Pi_{t+1}(2)\,Y_{t+1}(2)}{\Pi Y_t}\right)\right]. \end{aligned} \tag{A-3}$$

Log-linearizing around the constant steady state yields

$$\begin{aligned} 0 &= \left(1-\theta\left(1+\widehat{\theta}_t\right)\right)\Delta\left(1+\widehat{\Delta}_t\right)+\theta\left(1+\widehat{\theta}_t\right)\Delta\left(1+\widehat{\Delta}_t\right)\Psi\left(1+\psi_t\right)- \\ &\quad \varphi\left(1\right)\Delta\left(1+\widehat{\Delta}_t\right)\pi_t\left(1+\pi_t\right)+ \\ &\quad \beta p_{11}\varphi\left(1\right)E_t\left[\Delta\left(1+\widehat{\Delta}_{t+1}\left(1\right)\right)\pi_{t+1}\left(1\right)\left(1+\pi_{t+1}\left(1\right)\right)\left(1+Y_{t+1}\left(1\right)\right)\left(1-Y_t\right)\right]+ \\ &\quad \beta\left(1-p_{11}\right)\varphi\left(2\right)E_t\left[\Delta\left(1+\widehat{\Delta}_{t+1}\left(2\right)\right)\pi_{t+1}\left(2\right)\left(1+\pi_{t+1}\left(2\right)\right)\left(1+Y_{t+1}\left(2\right)\right)\left(1-Y_t\right)\right], \end{aligned} \tag{A-4}$$

where $\pi_t = \log\left(\Pi_t/\Pi\right)$, $\psi_t = \log\left(\Psi_t/\Psi\right)$, $\widehat{\Delta}_t = \log\left(\Delta_t/\Delta\right)$, and $\widehat{\theta}_t = \log\left(\theta_t/\theta\right)$. Values without a time subscript are steady-state values. Eliminating higher-order terms and using $\Psi = \theta^{-1}\left(\theta-1\right)$ yields

$$\pi_t = \beta p_{11}E_t\left[\pi_{1,t+1}\right]+\left(1-p_{11}\right)\beta\frac{\varphi_2}{\varphi_1}E_t\left[\pi_{2,t+2}\right]+\frac{\left(\theta-1\right)}{\varphi_1}\left(\psi_{1t}+u_t\right), \tag{A-5}$$

where $u_t = -\left(\theta-1\right)^{-1}\widehat{\theta}_t$. The same approach is taken for $s_t = 2$, where the general representation can be rewritten as (12).

B. Deriving the Utility-Based Welfare Criterion

If prices are fully flexible, then monopolistic firms set prices using

$$\left(\frac{P_t\left(j\right)}{P_t}\right) = \frac{\theta_t}{\left(\theta_t-1\right)}\frac{W_t}{P_t},$$

where in a symmetric equilibrium

$$\left(W_t/P_t\right) = \mu_t^{-1}.$$

Substituting this expression into (15) and using $Y_t = C_t = H_t$ gives

$$\left(1+\nu\right)\frac{1}{\mu_t} = \frac{Y_t^{\eta}}{Y_t^{-\sigma}},$$

where $(1+\nu)\,\mu^{-1} = 1$ by construction, where then solving for the steady-state yields the efficient steady-state level for production of $Y^* = 1$. Under the labor subsidy, monetary policy focuses on stabilization policies, versus policies to undo the monopolistic distortion. Also, in the steady state, the monopolistic firm is not adjusting its price, so the changing parameter governing the costs of price adjustment does not create any distortions.

Substituting (17) into (13) yields

$$U\left(Y_t, \Pi_t, s_t\right) = \frac{\left(Y_t - \frac{\varphi(s_t)}{2}\left(\Pi_t - 1\right)^2\right)^{1-\sigma}}{1-\sigma} - \frac{Y_t^{1+\eta}}{1+\eta}. \tag{A-6}$$

The second-order approximation to the first term of the representative agent's period utility function is given by

$$\frac{\left(Y_t - \frac{\varphi(s_t)}{2}\left(\Pi_t - 1\right)^2\right)^{1-\sigma}}{1-\sigma} \approx \frac{Y^{1-\sigma}}{1-\sigma} + Y^{1-\sigma}\left(\frac{Y_t - Y}{Y}\right) - \frac{1}{2}\sigma Y^{1-\sigma}\left(\frac{Y_t - Y}{Y}\right)^2 \tag{A-7}$$

$$-\frac{\varphi\left(s_t\right)}{2} Y^{-\sigma}\left(\Pi_t - 1\right)^2. \tag{A-8}$$

Second-order approximations of the relative deviations in terms of the log deviations are

$$\frac{Y_t - Y}{Y} \approx \widehat{y}_t + \frac{1}{2}\widehat{y}_t^2 \tag{A-9}$$

$$\Pi_t - 1 \approx \widehat{\pi}_t + \frac{1}{2}\widehat{\pi}_t^2 \tag{A-10}$$

which yields

$$\frac{\left(Y_t - \frac{\varphi(s_t)}{2}\left(\Pi_t - 1\right)^2\right)^{1-\sigma}}{1-\sigma} \approx \frac{Y^{1-\sigma}}{1-\sigma} + Y^{1-\sigma}\left(\widehat{y}_t + \frac{1}{2}\widehat{y}_t^2\right) - \frac{1}{2}\sigma Y^{1-\sigma}\left(\widehat{y}_t + \frac{1}{2}\widehat{y}_t^2\right)^2 - \frac{\varphi\left(s_t\right)}{2} Y^{-\sigma}\left(\pi_t + \frac{1}{2}\pi_t^2\right)^2.$$

Removing terms higher than second order and denoting terms independent of policy as $t.i.p.$

yields

$$\frac{\left(Y_t - \frac{\varphi(s_t)}{2}\left(\pi_t - 1\right)^2\right)^{1-\sigma}}{1-\sigma} \approx Y^{1-\sigma}Y_t + \frac{1}{2}Y^{1-\sigma}\widehat{y}_t^2 - \frac{1}{2}\sigma Y^{1-\sigma}\widehat{y}_t^2 \tag{A-11}$$

$$-\frac{\varphi\left(s_t\right)}{2}Y^{-\sigma}\pi_t^2 + t.i.p. \tag{A-12}$$

The second-order approximation to the second term of the utility function is

$$\frac{Y_t^{1+\eta}}{1+\eta} \approx Y^{1+\eta}\left(\frac{Y_t - Y}{Y}\right) + \frac{1}{2}\eta Y^{1+\eta}\left(\frac{Y_t - Y}{Y}\right)^2 + t.i.p. \tag{A-13}$$

Using second-order approximations in terms of log deviations yields

$$\frac{Y_t^{1+\eta}}{1+\eta} \approx Y^{1+\eta}\left(\widehat{y}_t + \frac{1}{2}\widehat{y}_t^2\right) + \frac{1}{2}\eta Y^{1+\eta}\left(\widehat{y}_t + \frac{1}{2}\widehat{y}_t^2\right)^2 + t.i.p. \tag{A-14}$$

$$= Y^{1+\eta}\widehat{y}_t + \frac{1}{2}Y^{1+\eta}\widehat{y}_t^2 + \frac{1}{2}\eta Y^{1+\eta}\widehat{y}_t^2 + t.i.p. \tag{A-15}$$

Combining both components of the utility function and removing $t.i.p.$ yields

$$U\left(Y_t, \pi_t, s_t\right) \approx Y^{1-\sigma}\widehat{y}_t + \frac{1}{2}Y^{1-\sigma}\widehat{y}_t^2 - \frac{1}{2}\sigma Y^{1-\sigma}\widehat{y}_t^2 - \frac{1}{2}\varphi\left(s_t\right)Y^{-\sigma}\pi_t^2 - Y^{1+\eta}\left(\widehat{y}_t + \frac{1}{2}\left(1+\eta\right)\widehat{y}_t^2\right).$$

Rearranging terms and setting $Y = 1$ yields

$$\begin{aligned} U\left(Y_t, \pi_t, s_t\right) &\approx Y^{1-\sigma}\widehat{y}_t + \frac{1}{2}Y^{1-\sigma}\widehat{y}_t^2 - \frac{1}{2}\sigma Y^{1-\sigma}\widehat{y}_t^2 - \frac{1}{2}\varphi\left(s_t\right)Y^{-\sigma}\pi_t^2 - Y^{1+\eta}\left(\widehat{y}_t + \frac{1}{2}\left(1+\eta\right)\widehat{y}_t^2\right), \\ &= -\frac{1}{2}\varphi\left(s_t\right)\left(\pi_t^2 + \frac{(\sigma+\eta)}{\varphi\left(s_t\right)}\widehat{y}_t^2\right). \end{aligned}$$

C. Solutions for Inflation and Output

To solve for inflation and output under the ad-hoc metric, I first use the system of targeting rules in (20) , which in state-contingent notation is

$$\begin{bmatrix} \lambda p_{11} & \lambda\left(1-p_{11}\right) \\ \lambda\left(1-p_{22}\right) & \lambda p_{22} \end{bmatrix}\begin{bmatrix} E_t x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = \begin{bmatrix} -\kappa_1 p_{11} & -\kappa_2\left(1-p_{11}\right) \\ -\kappa_1\left(1-p_{22}\right) & -\kappa_2 p_{22} \end{bmatrix}\begin{bmatrix} E_t \pi_{1t+1} \\ E_t \pi_{2t+1} \end{bmatrix},$$

and solve in terms of the expected conditional output gaps, yielding

$$E_t x_{1t+1} = -\frac{\kappa_1}{\lambda} E_t \pi_{1t+1}, \tag{A-16}$$

$$E_t x_{2t+1} = -\frac{\kappa_2}{\lambda} E_t \pi_{2t+1}. \tag{A-17}$$

Substituting (20), $(A-16)$ and $(A-17)$ into the Phillips curve (19) yields a dynamic system in x_{1t} and x_{2t}

$$\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \begin{bmatrix} p_{11}\gamma_1 & (1-p_{11})\gamma_1 \frac{\kappa_1\varphi_2}{\kappa_2\varphi_1} \\ (1-p_{22})\gamma_2 \frac{\kappa_2\varphi_1}{\kappa_1\varphi_2} & p_{22}\gamma_2 \end{bmatrix} \begin{bmatrix} E_t x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} - \begin{bmatrix} \kappa_1 (\lambda+\kappa_1^2)^{-1} \\ \kappa_2 (\lambda+\kappa_2^2)^{-1} \end{bmatrix} e_t, \tag{A-18}$$

where $\gamma_i = \lambda\beta(\lambda+\kappa_i^2)^{-1}$ and $i = 1, 2$.

Using the method of undetermined coefficients on the minimum set of state variables, which are s_t and e_t, solutions to $(A-18)$ have the form

$$x_{it} = a_i e_t, \tag{A-19}$$

for $i = 1, 2$. Substituting these solutions into (20) yields the following solution for inflation

$$\pi_{it} = -\frac{\lambda a_i}{\kappa_i} e_t, \tag{A-20}$$

for $i = 1, 2$. To find the unknown coefficients, a_i for $i = 1, 2$, substitute the above solutions back into $(A-18)$ to obtain the following relationships between parameters

$$\begin{bmatrix} (1-p_{11}\gamma_1\rho) & -(1-p_{11})\gamma_1 \frac{\kappa_1\varphi_2}{\kappa_2\varphi_1}\rho \\ -(1-p_{22})\gamma_2 \frac{\kappa_2\varphi_1}{\kappa_1\varphi_2}\rho & (1-p_{22}\gamma_2\rho) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -\frac{\kappa_1}{\lambda+\kappa_1^2} \\ -\frac{\kappa_2}{\lambda+\kappa_2^2} \end{bmatrix}. \tag{A-21}$$

Under the utility-based metric, performing the same steps yields the following

$$\begin{bmatrix} (1-p_{11}\beta\delta_1\rho) & -(1-p_{11})\beta\delta_1 \frac{\varphi_2}{\varphi_1}\rho \\ -(1-p_{22})\beta\delta_2 \frac{\varphi_1}{\varphi_2}\rho & (1-p_{22}\beta\delta_2\rho) \end{bmatrix} \begin{bmatrix} a_1^{ub} \\ a_2^{ub} \end{bmatrix} = \begin{bmatrix} (1-\theta)\delta_1 \\ (1-\theta)\delta_2 \end{bmatrix}, \tag{A-22}$$

where $\delta_i = (1-(1-\theta)\kappa_i)^{-1}$ and $i = 1, 2$.

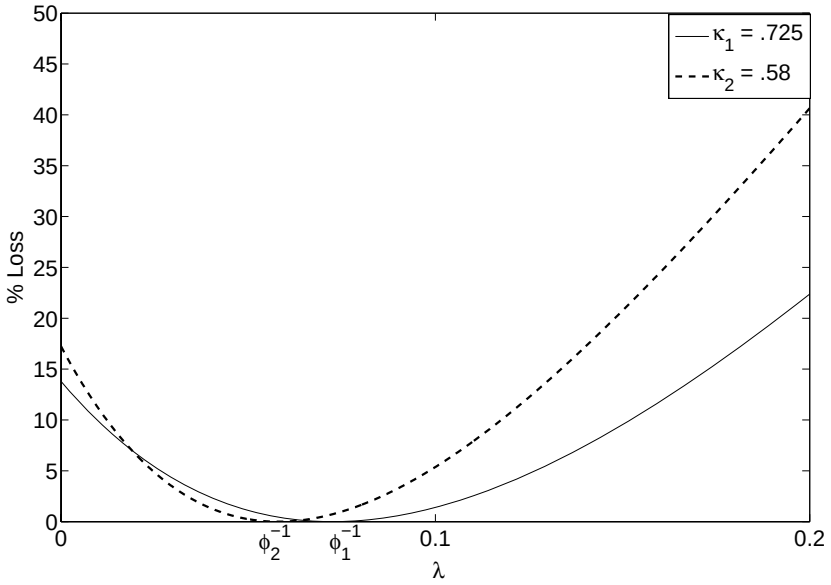

Figure 1: Welfare Loss Under Policy Based on an Ad-hoc Loss Function

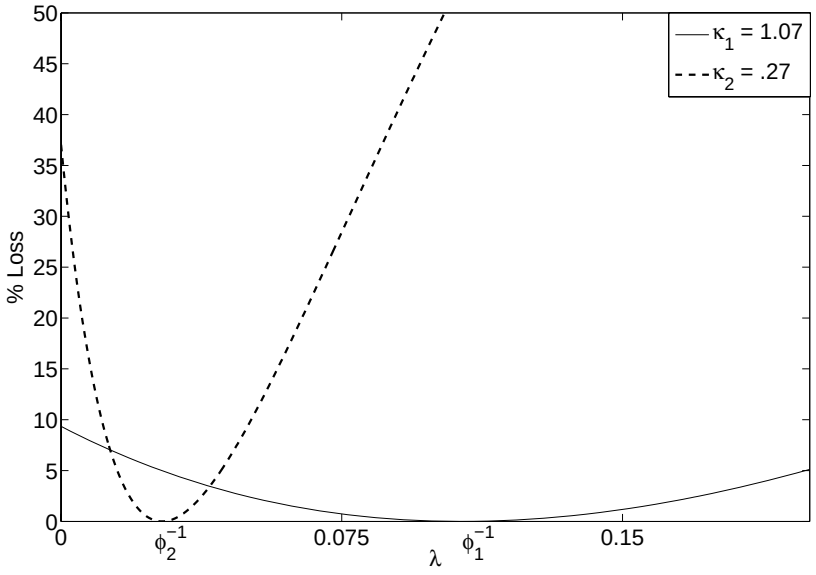

Figure 2: Welfare Loss Under Policy Based on an Ad-hoc Loss Function