

# **THE ECONOMICS OF LABOR ADJUSTMENT: MIND THE GAP**

**Russell Cooper and Jonathan L. Willis**

**AUGUST 2001**

**LAST REVISED: JULY 2003**

**RWP 03-05**

Research Division  
Federal Reserve Bank of Kansas City

Russell Cooper is a professor of economics at Boston University. Jonathan L. Willis is an economist at the Federal Reserve Bank of Kansas City. The authors are grateful to seminar participants at Boston University, Emory University, the University of British Columbia, the Federal Reserve Bank of Boston, the University of Haifa, the University of Iowa, the University of Michigan, the University of Pennsylvania, the University of Texas at Austin, the 2002 NBER EF Meeting and the 2000 CMSG conference at McMaster University for comments and suggestions. Discussions with Eduardo Engel, John Haltiwanger, Daniel Hamermesh, Peter Klenow and Christopher Ragan were much appreciated. The authors thank the NSF for financial support. The authors also appreciate comments from the referees and the editor. Russell Cooper thanks the Federal Reserve Bank of Minneapolis for its hospitality during revisions of this manuscript. The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Kansas City, the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Cooper email: [rcooper@bu.edu](mailto:rcooper@bu.edu)

Willis email: [jonathan.willis@kc.frb.org](mailto:jonathan.willis@kc.frb.org)

## **Abstract**

We study inferences about the dynamics of labor adjustment obtained by the "gap methodology" of Caballero and Engel [1993] and Caballero, Engel and Haltiwanger [1997]. In that approach, the policy function for employment growth is assumed to depend on an unobservable gap between the target and current levels of employment. Using time series observations, these studies reject the partial adjustment model and find that aggregate employment dynamics depend on the cross-sectional distribution of employment gaps. Thus, nonlinear adjustment at the plant level appears to have aggregate implications. We argue that this conclusion is not justified: these findings of nonlinearities in time series data may reflect mismeasurement of the gaps rather than the aggregation of plant-level nonlinearities.

JEL classification: E24, J23, J6

Keywords: Aggregate Employment, Employment, Adjustment Costs

# 1 Introduction

In recent contributions, Caballero and Engel [1993], hereafter CE, and Caballero, Engel and Haltiwanger [1997], hereafter CEH, investigate labor dynamics using a methodology which postulates that employment changes depend on a gap between the actual and target levels of employment.<sup>1</sup> Both studies find evidence of nonlinearities in aggregate time-series data: employment growth depends on the cross-sectional distribution of employment gaps. This finding is taken as evidence that nonlinear adjustment at the microeconomic level “matters” for aggregate time series. This is important for business cycle and policy analysis as it implies macroeconomics must take the cross sectional distribution of employment gaps into account. This paper questions the methodology and thus the conclusions of these studies.<sup>2</sup> We argue that these reported aggregate nonlinearities may be the consequence of mismeasurement of the gap rather than nonlinearities in plant-level adjustment.<sup>3</sup>

Both CE and CEH rely upon a hypothesis that employment growth ( $\Delta\tilde{e}$ ) respond to a gap ( $z$ ) between the desired and actual number of workers at a plant. The advantage to the gap approach is that the choice of employment, an inherently difficult dynamic optimization problem, is characterized through a nonlinear relationship between ( $\Delta\tilde{e}$ ) and ( $z$ ). That is, the adjustment rate,  $\Delta\tilde{e}/z$ , is postulated to be a nonlinear function of  $z$ . However, there is no “free lunch”: the desired number of workers, and hence the employment gap, is unobservable. Thus, in order to confront data, this approach needs an auxiliary theory to infer  $z$  from observed variables. Both CE and CEH use observed hours variations to infer the employment gap: this inference is one element of our critique of the gap methodology.

To assess this methodology, we simulate data from the solution of a dynamic model of labor adjustment **assuming** quadratic adjustment costs. We evaluate the approaches of CE

---

<sup>1</sup>Hamermesh [1989] uses a gap methodology as well but does not adopt the approach of estimating a nonlinear hazard function (explained below) to infer the nature of adjustment costs. Hence we focus on CE and CEH in this discussion of methodology.

<sup>2</sup>We do not contest the general view of nonlinear employment adjustment at the plant-level. This finding is consistent with other evidence that points to inactivity as well as bursts of employment adjustment at the plant-level. For example, Hamermesh [1989] provides a revealing discussion of lumpy labor adjustment at a set of manufacturing plants. Davis and Haltiwanger [1992] document large employment changes at the plant level. CEH also report evidence of inactivity in plant-level employment adjustment. There seems little doubt that an explanation of plant level employment dynamics requires a model of adjustment that is richer than the quadratic adjustment cost structure and includes some forms of non-differentiability and/or nonconvexity.

<sup>3</sup>As the gap approach is used in numerous other applications (capital adjustment, durable adjustment, price adjustment) to approximate the solution to dynamic optimization problems, our concerns may be relevant for those exercises as well.

and CEH by characterizing the conclusions those studies would reach from the simulated data.

The quadratic adjustment cost model is a useful benchmark for two reasons. First, it has served as the primary model for the study of aggregate employment dynamics. To quote CE (p. 365),

Since the latter [representative-agent framework with quadratic adjustment costs] is a specification often used by macroeconomists to characterize aggregate dynamics, it constitutes a convenient benchmark for a discussion of the more realistic increasing-hazard [adjustment rate] models.

Second, the quadratic adjustment cost model is nested in the employment gap approach. As a matter of theory, if adjustment costs are quadratic, shocks follow a random walk and the gap is correctly measured, then the adjustment rate is constant, implying that aggregate employment is independent of the cross-sectional distribution of employment gaps.<sup>4</sup> From our simulations of a model with quadratic costs of adjustment, this result holds when the gap is properly measured, even if shocks do not follow a random walk.<sup>5</sup> Thus, if the CE and CEH procedures uncover aggregate nonlinearities from a data set created from a model with quadratic adjustment costs and stationary shocks, then this is a consequence of mismeasurement of the gap rather than economic fundamentals.

In our quadratic adjustment cost model we find the following:

- If the gap is correctly measured, the adjustment rate is essentially constant and the cross-sectional distribution of employment gaps is irrelevant for aggregate employment dynamics.
- If the employment gap is **mismeasured**, then
  1. A quadratic cost of adjustment model can generate a nonlinear adjustment rate ( $\Delta\tilde{e}/z$  depends nonlinearly on  $z$ ).
  2. Aggregate employment dynamics can depend on the cross-sectional distribution of the employment gap.

---

<sup>4</sup>If the adjustment rate is independent of the gap, then the cross-sectional distribution of the gap is irrelevant for aggregate behavior. The fact that the partial adjustment model implies constant adjustment is essentially by construction. The link between the quadratic cost of adjustment structure and the partial adjustment model is more subtle and is discussed in Cooper and Willis [2002a].

<sup>5</sup>The issue of the correlation of the shocks is important and one that we return to below. We are grateful to a referee for stressing this point. Further, our model includes additional nonlinearities through the compensation and profit functions.

- The gap is mismeasured using either the CE or the CEH approaches.

Problems measuring the gap may be severe enough to create nonlinearities that otherwise would not be present. This is true for the simulated data and may well lie behind the results reported by CE and CEH for actual data.

We thus conclude that the time-series evidence of nonlinear hazards reported by CE and CEH does not necessarily imply that nonlinear adjustment at the plant-level has aggregate effects. A methodology which is unbiased under the null hypothesis of quadratic adjustment costs is needed to assess the aggregate implications of that model relative to a competing model with non-convex adjustment costs.

## 2 The Gap Approach: An Overview

As background for our analysis, we begin with a summary of the methodology employed by CE and CEH as well as a more precise statement of their findings.

### 2.1 Gap Methodology

We follow the notation and presentation in CEH. The gap between the desired employment and the actual employment (in logs) in period  $t$  for plant  $i$  is defined as

$$z_{i,t} \equiv \tilde{e}_{i,t}^* - \tilde{e}_{i,t-1}. \quad (1)$$

Here  $\tilde{e}_{i,t}^*$  is the (log of the) desired level of employment given the realization of all period  $t$  random variables and  $\tilde{e}_{i,t-1}$  is the (log of the) level of employment prior to any period  $t$  adjustments.<sup>6</sup> Thus  $z_{i,t}$  represents a gap between the state of the plant at the beginning of the period and the level of employment it would choose if it could “costlessly” alter employment.

CEH hypothesize a relationship between employment growth  $\Delta\tilde{e}_{i,t}$  and  $z_{i,t}$  given by

$$\Delta\tilde{e}_{i,t} = \phi(z_{i,t}). \quad (2)$$

A key issue is characterizing the policy function,  $\phi(z_{i,t})$ , and inferring properties of adjustment costs from it. In some cases, it is convenient to refer to an adjustment rate or hazard

---

<sup>6</sup>Throughout, with  $\tilde{x}$  represents the log of variable  $x$ , lower case variables are plant-level and upper case variables are aggregate measures.

function:  $\Phi(z_{i,t}) \equiv \phi(z_{i,t})/z_{i,t}$ .<sup>7</sup> Specifying that employment adjustment depends only on the gap is an assumption: the validity of this approximation to the optimal policy function of the plant can be evaluated using our structural model.

Because the gap is central to this analysis, it is important to be very precise about how it is defined and measured. The key is the meaning of “costlessly adjusting employment.” In fact, there are two ways to characterize the target, and as we demonstrate in our quantitative analysis, the results depend on the definition.

First, one could define the target as the level of employment that would arise if there were **never** any costs of adjustment.<sup>8</sup> This version of the target is quite easy to characterize since it is the solution of a static optimization problem. This is termed the **static target** in the discussion that follows.

Second, one could construct a target measure in which the adjustment costs are removed for a single period. This is termed the **frictionless target**.

This hypothesized relationship between employment changes and the gap cannot be analyzed directly since  $z_{i,t}$  is a theoretical construct that is not observed: there exists no data set which includes  $z_{i,t}$ . In the literature, various approaches have been pursued.

## 2.2 Measurement of the Gap from Plant-Level Data

CEH construct a measure of the gap at the plant-level and use it to study aggregate employment growth. They hypothesize a second relationship between another (closely related) measure of the gap,  $(z_{i,t}^1)$ , and plant-specific deviations in hours:

$$z_{i,t}^1 = \theta(\tilde{h}_{i,t} - \bar{h}). \quad (3)$$

---

<sup>7</sup>From the discussion in CE and CEH, there are two interpretations of this function.  $\Phi(z)$  represents either the magnitude of adjustment (e.g., the fraction of a gap that is closed) or a probability of full adjustment to the target. The interpretation, of course, depends on the nature of adjustment costs. For the quadratic adjustment cost case, this function represents the rate at which the gap is closed. For a model with stochastic adjustment costs taking the values zero and infinity, this is the probability of full adjustment. We use the terminology of a hazard function throughout, as do CE and CEH.

<sup>8</sup>This approach to approximating the dynamic optimization problem is applied extensively but, from our perspective, places too much emphasis on static optimization. Nickell [1978, pp. 329–30] says,

the majority of existing dynamic models of factor demand simply analyze the optimal adjustment of the firm towards a static equilibrium and it is very difficult to deduce from this anything whatever about optimal behaviour when there is no “equilibrium” to aim at.

Here  $\bar{h}$  is the plant-level mean of log hours and  $z_{i,t}^1$  is the gap in period  $t$  **after** adjustments in the employment level have been made:  $z_{i,t}^1 = z_{i,t} - \Delta\tilde{e}_{i,t}$ .<sup>9</sup>

Intuition from the partial adjustment model suggests  $\theta$  should be positive. In response to a shock that causes profitability to rise, hours and the desired number of workers will both increase. The gap decreases as workers ( $e$ ) are added and hours fall closer to  $\bar{h}$ . Thus the supposed relationship between this measure of the gap and hours deviations seems reasonable, both in terms of the response of these variables to a shock and in terms of transition dynamics.

Rewriting this relationship in terms of the pre-adjustment gap leads to

$$z_{i,t} = \theta(\tilde{h}_{i,t} - \bar{h}) + \Delta\tilde{e}_{i,t}. \quad (4)$$

Hence, given an estimate of  $\theta$ , one can infer  $z_{i,t}$  from hours and employment observations.

The issue is estimating  $\theta$ . Using (1) in (4) and taking differences yields

$$\Delta\tilde{e}_{i,t} = -\theta\Delta\tilde{h}_{i,t} + \Delta\tilde{e}_{i,t}^*. \quad (5)$$

Adding a constant ( $\delta$ ) and noting that  $\Delta\tilde{e}_{i,t}^*$  is not observable, CEH estimate  $\theta$  from

$$\Delta\tilde{e}_{i,t} = \delta - \theta\Delta\tilde{h}_{i,t} + \varepsilon_{i,t}. \quad (6)$$

As CEH note, estimation of this equation may yield biased estimates of  $\theta$  since the error term (principally  $\Delta\tilde{e}_{i,t}^*$ ) is likely to be correlated with changes in hours. That is, a positive shock to profitability may induce the plant to increase hours (at least in the short run) and will generally cause the desired level of employment to increase as well. CEH argue that this problem is (partially) remedied by looking at periods of large adjustment since then the changes in hours and employment will overwhelm the error.<sup>10</sup>

CEH use their plant-level measures of the gap in two ways. First, they analyze the relationship between employment adjustment and employment gaps at the plant level. Second, they investigate aggregate implications by estimating a reduced-form hazard function from

---

<sup>9</sup>Implicitly this assumes that there is no lag between the decision to adjust employment and the actual adjustment. That is, unlike the time to build aspect of investment, employment adjustments take place immediately. We use this timing assumption in our structural model.

Further, we have removed the heterogeneity in  $\bar{h}$  and in  $\theta$  that is important for the empirical implementation in CEH. Finally, note that by assumption  $\bar{h}$  is independent of any shocks to the profitability of employment. We will argue below that this is an important restriction.

<sup>10</sup>They also note the presence of measurement error, which they address through the use of a reverse regression exercise. We have not included measurement error in our simulated environment.

time series. Letting  $f_t(z)$  be the period  $t$  probability density function of employment gaps across plants, the aggregate rate of employment growth is given by

$$\Delta \tilde{E}_t = \int_z z \Phi(z) f_t(z) dz. \quad (7)$$

As  $\Phi(z)$  is the adjustment rate or hazard function indicating the fraction of the gap that is closed by employment adjustment,  $z\Phi(z)$  is the size of the employment adjustment for plants with a gap of  $z$ . As in CEH [Section IV], simplification of (7) given the specification of a hazard function produces an aggregate relationship between employment changes and non-centered moments of the distribution of  $z$ .

The findings by CEH are summarized as follows:

- Using (6), CEH report a mean (across 2-digit industries) estimate of  $\theta = 1.26$ . Their estimate comes from using observations in which percent changes in both employment and hours exceed one standard deviation of the respective series.
- Using their estimates of  $\theta$  to construct a gap measure ( $z_{i,t}$ ), CEH (Figure 1a, p. 122) find a nonlinear relationship between the average adjustment rate and the employment gap.
- CEH specify that

$$\Lambda(z) = \begin{cases} \lambda_0 + \lambda_1^- z & \text{for } z < 0 \\ \lambda_0 + \lambda_1^+ z & \text{for } z > 0 \end{cases} \quad (8)$$

and estimate  $\lambda_1^- = 1.30$  and  $\lambda_1^+ = 1.32$ .<sup>11</sup> Hence, employment growth, expressed by (7), depends on the second moment of the distribution of employment gaps.

### 2.3 Measurement of the Gap from Aggregate Data

CE construct a measure of the gap using aggregated data and estimate an aggregate adjustment hazard. In contrast to CEH, CE do not estimate  $\theta$  but instead calibrate it from a structural model of static optimization by a plant with market power.<sup>12</sup> Because CE do not have plant-level data, their estimation uses aggregate observations on net and gross

---

<sup>11</sup>This aggregate hazard is a reduced-form function that links employment growth to the gap. This relationship is imposed on the aggregate data. Optimal decisions at the plant-level will not necessarily obey this simple specification.

<sup>12</sup>Though CE do not have any microeconomic data, CEH work with plant-level data, and so we refer to these microeconomic units as *plants*. The appendix of Cooper and Willis [2002a] characterizes the mapping from the structural parameters of the quadratic adjustment cost model to  $\theta$ .



flows for U.S. manufacturing employment to estimate a hazard function. They create the following measure of the growth of the aggregate target using the calibrated value for  $\theta$ :

$$\Delta \tilde{E}_t^* = \Delta \tilde{E}_t + \theta \Delta \tilde{H}_t. \quad (9)$$

This growth of the target, which is an aggregate version of (5), is used in a specification of employment growth:

$$\Delta \tilde{E}_{t+1} = \int_{-\infty}^{\infty} (\Delta \tilde{E}_{t+1}^* - \hat{z}) \Lambda(\hat{z} - \Delta \tilde{E}_{t+1}^*) f_t(\hat{z}) d\hat{z} \quad (10)$$

which is similar to (7) with  $\Lambda(z - \Delta \tilde{E}_{t+1}^*)$  representing the aggregate hazard function and  $\hat{z}$  representing the negative of the employment gap after the realization of the idiosyncratic shock but before the aggregate shock. In terms of CEH notation,  $z_{i,t+1} = \Delta \tilde{E}_{t+1}^* - \hat{z}_{i,t+1}$ .

CE estimate a quadratic hazard:

$$\Lambda(z) = \tilde{\lambda}_0 + \tilde{\lambda}_2(z - z_0)^2 \quad (11)$$

where  $z_0$  is a constant. This nests a constant and a quadratic specification for  $\Lambda(\cdot)$ . To obtain parameter estimates, they calculate the growth rate of the employment target from (9) using observations on employment and hours growth. This measure is used in (10).<sup>13</sup> CE find parameter values for the hazard that minimize the sum of squared differences between the actual and predicted employment growth. CE (p. 375, Table I, BLS) report the following estimates:  $\tilde{\lambda}_0 = 0.02$ ;  $\tilde{\lambda}_2 = 0.53$ ;  $z_0 = -0.82$ . CE conclude that a quadratic hazard specification fits the data better than the flat hazard.

### 3 A Dynamic Optimization Framework

Our analysis builds from the specification of a dynamic optimization problem at the plant level. Our structure is purposely close to that outlined in CE.<sup>14</sup> We use the model as a data-generating mechanism to evaluate the CE and CEH methodologies.

Letting  $A$  represent the profitability of a production unit (e.g., a plant),  $h$  the hours per worker and  $e$  the number of workers, we consider the following dynamic programming problem:

---

<sup>13</sup>This procedure, which also includes generating a sequence of cross sectional distributions of the gap through plant-specific shocks to the gap, is described in detail by CE. We follow this procedure in our exercises.

<sup>14</sup>For example, we have not added stochastic adjustment costs since these would drive an immediate wedge between employment changes and any gap measure.

$$V(A, e_{-1}) = \max_{e, h} R(A, e, h) - \omega(e, h) - \frac{\nu}{2} \left( \frac{e - e_{-1}}{e_{-1}} \right)^2 e_{-1} + \beta E_{A'|A} V(A', e) \quad (12)$$

for all  $(A, e_{-1})$  where  $e_{-1}$  is the inherited stock of workers.<sup>15</sup> Note the timing assumption of the model: workers hired in a given period become productive immediately.

For our analysis we use a Cobb-Douglas production function in which the labor input is simply the product of employment and hours,  $eh$ . Allowing for market power of the plant, the revenue function is specified as

$$R(A, e, h) = A(eh)^\alpha \quad (13)$$

where the parameter  $\alpha$  is determined by the shares of capital and labor in the production function and by the elasticity of demand.

The costs of adjustment are assumed to be a quadratic function of the percent change in the stock of employed workers multiplied by the initial stock of employees.<sup>16</sup> That is, the adjustment cost arises for net, *not* gross, hires. In (12),  $\nu$  parameterizes the level of the adjustment cost function.

The function  $\omega(e, h)$  represents total compensation to workers as a function of the number of workers and the average number of hours per worker. This compensation function is critical for generating movements in both hours and the number of workers.<sup>17</sup> For our analysis, the compensation function is given by:  $\omega(e, h) = e(w_0 + w_1 h^\zeta)$ .<sup>18</sup>

Using the reduced-form profit function and assuming quadratic costs of adjustment,

---

<sup>15</sup>Other inputs into the production function, such as capital and energy, are assumed, for simplicity, to be flexible. Maximization over these factors is thus subsumed by  $R(A, e, h)$ , and variations in inputs costs are part of  $A$ . We assume that  $A$  is a stationary first-order Markov process.

<sup>16</sup>The literature uses both a quadratic specification in which the cost is in terms of percent differences (Bils [1987]) and specifications in which adjustment costs are in terms of employment changes alone (Hamermesh [1989]). Cooper and Willis [2002a] finds that these results are robust to this alternative specification.

<sup>17</sup>A simpler model with a production function, a fixed wage rate and an employment adjustment cost is not sufficient as there is no “penalty” for overworking employees. Thus, as long as there is no cost to adjusting hours, firms will only modify hours in reaction to shocks. There will be no need to adjust employees.

<sup>18</sup>In contrast to Sargent [1978] there is no exogenous component to wage variation. In Sargent’s study, variations in productivity were much larger than variations in wages. Further we follow CE and consider a wage function rather than a model with overtime hours as in Sargent. Cooper and Willis [2002a] also study a model in which the compensation function follows Bils [1987] and Shapiro [1986] with similar findings.

we solve the dynamic programming problem using value function iteration.<sup>19</sup> Let  $e = \psi_e(A, e_{-1})$  be the policy function for employment. Employment is determined by a stochastic difference equation from the policy function. Let  $h = \psi_h(A, e_{-1})$  be the policy function for hours.

The frictionless target,  $e^*(A)$ , is the solution to the optimization problem when  $\nu = 0$  for one period. For this model, the frictionless target is equivalent to the solution to  $e = \psi_e(A, e)$ . The adjustment process, defined by iterations of  $e = \psi_e(A, e_{-1})$  given  $A$ , converges to the frictionless target,  $e^*(A)$ . The frictionless hours target is denoted by  $h^*(A) = \psi_h(A, e^*(A))$  and will generally depend on  $A$ .

The static target, used by CE, is defined as the solution to (12) when  $\nu = 0$  in **all** periods. Thus employment and hours satisfy static first-order conditions.

The top two panels of Figure 1 illustrate the policy functions and employment targets for two realizations of  $A$ . Both the frictionless and the static employment targets are indicated in the figure. Since plants take future adjustment costs into account in determining the frictionless target, this target is not as responsive as the static target to changes in the productivity shock. In general, the frictionless target is less than the static target for above average productivity shocks and vice versa for below average shocks.<sup>20</sup>

As a result, the frictionless hours target for a given shock,  $h^*(A)$ , differs from the static hours target, as shown in the bottom panel of Figure 1.<sup>21</sup> If the frictionless employment target is below the static employment target for a given shock, then the frictionless hours target is above the static hours target to compensate for the lower level of employment.

The characterization of the policy functions for (12) do not relate employment or hours to any measure of the gap. As presented in Cooper and Willis [2002a], if one replaces (12) with a dynamic optimization problem in which the plant is minimizing the sum of deviations from an exogenously given employment target and facing adjustment costs, then a policy function relating employment adjustment to the target will be created.<sup>22</sup> Cooper and Willis

---

<sup>19</sup>Instead of value function iteration, one could characterize the policy functions from the Euler equations or consider a linear-quadratic approximation. We chose the value function iteration over the linear quadratic approximation so that we could explicitly include curvature in the profit and compensation functions. Sargent [1978] discusses the linear-quadratic specification and the solution methodology for finding the path of employment adjustment.

<sup>20</sup>In an model with only one shock, this will result in a narrower cross-sectional distribution of employment gaps defined using the frictionless target than employment gaps defined using the static target.

<sup>21</sup>See the appendix of Cooper and Willis [2002a] for a discussion of the static hours target. It is determined from the first-order condition for hours if employment is set at its static target. As discussed in the appendix, the static hours target is not state dependent.

<sup>22</sup>In that exercise, there was no variation in hours worked.

[2002a] argue that if shocks follow a random walk, then the partial adjustment model and the prediction of a flat hazard function,  $\Phi(z)$  independent of  $z$ , will hold.

## 4 Empirical Approach

Our goal is to consider the empirical implications of the quadratic adjustment cost model. To do so, we use our model to directly measure the employment gap at the plant level. We call this the **observed gap**. Corresponding to the **frictionless** and **static** targets are two measures of the observed gap: the **frictionless gap** and the **static gap**. We measure these directly using our model as a data-generating mechanism.

Of course, neither CE nor CEH directly observe these gaps. Thus, we follow CEH and infer the employment gap from observed hours variations, using (4) where  $\theta$  is estimated from (6). We term this the **CEH gap**. Following CEH, we provide two measures of this gap based upon two estimates of  $\theta$ . The first uses the full simulated panel, and the second uses a subsample comprised of observations entailing large changes in employment and hours, where large changes are defined as those greater than one standard deviation. Similarly, we use the CE procedure of estimating a hazard function from (9) and (10) with time-series data produced by our model.

To solve the dynamic programming problem given in (12), we need to specify functional forms and calibrate the parameters. We assume the following:

- The production function is Cobb-Douglas, where hours and workers are perfectly substitutable. Labor’s share is 0.65, and the markup is set at 25%.
- For the compensation function, the parameters  $w_0$  and  $w_1$  are chosen so that steady state hours are 40 and steady state employment at each plant is 600.<sup>23</sup> The wage elasticity,  $\zeta$ , is set at 2.03 based upon estimates from Cooper and Willis [2002b] using the reduced-form results of CE.<sup>24</sup>
- We set  $\nu = 2$ .<sup>25</sup>

---

<sup>23</sup>This is the average number of workers per plant in the balanced panel that underlies CEH. This figure comes from conversations with John Haltiwanger.

<sup>24</sup>This estimate is also close to the value of 1.9 assumed by CE.

<sup>25</sup>We are grateful to Daniel Hamermesh for suggestions on this parameterization. Cooper and Willis [2002a] displayed results for  $\nu = 1$  and  $\nu = 10$  and found nonlinearities for the former but not the latter value of  $\nu$ . In more recent work, Cooper and Willis [2002b] use the reduced-form results of CE to estimate adjustment cost parameters. They estimate an asymmetric quadratic adjustment cost function in which the midpoint of the estimates corresponds to  $\nu = 2$ .

- We assume that the profitability shock consists of two multiplicative exogenous components, an aggregate shock ( $A_{agg}$ ) and an idiosyncratic shock ( $A_{idio}$ ), such that the profitability shock to plant  $j$  in period  $t$  is given by  $A_{j,t} = A_{agg,t} \times A_{idio,j,t}$ . We also assume that both exogenous components follow log-normal AR(1) processes with serial correlation of  $\rho$  and innovations have a standard deviation of  $\sigma$ . For our baseline specification, we set  $\rho = 0.73$  and  $\sigma = 0.013$ .

Of the components of the basic parameterization, the last assumption is the most controversial for the evaluation of the CE and CEH results.<sup>26</sup> In a complete empirical exercise, the stochastic process for the shocks would be jointly estimated with the adjustment cost process.<sup>27</sup> However, the methodologies of CE and CEH do not include estimation of the driving processes. Therefore, outside evidence must be used to calibrate the parameters ( $\rho, \sigma$ ). With this specification of the process for the shocks, we are able to match the time series representation of aggregate employment from the BLS sample used by CE.<sup>28</sup>

Given this parameterization of the basic functions, the optimization problem given in (12) is solved using value function iteration to obtain policy functions. The state space of employment is discretized into a fine grid with 250 points in the relevant portion of the state space. For the given values of the serial correlation and standard deviation for both the aggregate and idiosyncratic shocks, we use the procedure outlined in Tauchen [1986] to create a discrete state space representation for the shocks.<sup>29</sup> Using these policy functions, we create a simulated panel data set where the number of plants equals 1000 and the number of time periods is 1000.<sup>30</sup>

---

<sup>26</sup>We are grateful to a referee for highlighting this important point. Cooper and Willis [2002a] provides a lengthy discussion of the robustness of the results to alternative parameterizations.

<sup>27</sup>See Sargent [1978] on this and the identification problems in distinguishing adjustment costs from the serial correlation of shocks.

<sup>28</sup>This parameterization is based on results from Cooper and Willis [2002b] where aggregate employment moments and the reduced-form results of CE are used to estimate the main parameters of the model.

<sup>29</sup>The aggregate and idiosyncratic profitability shocks are each represented by an 11-point state space equally divided between two standard deviations of their respective means. Given the mean, standard deviation, and serial correlation parameters, an  $11 \times 11$  transition matrix is created for each process. In a simulated sample of 1000 plants over 1000 periods, the serial correlation and variance properties of these generated series are very close to the parameters used to designate the state space and transition matrix. Adding points to the state space does not meaningfully change the results.

<sup>30</sup>CEH have a panel with 36 quarters and 10,000 plants. Our results are robust to adding more plants. We analyze only 1000 plants to reduce computation time. The number of time periods is set at 1000 to minimize simulation error.

## 5 Aggregate Implications

Given that both CE and CEH present quantitative results on the estimation of hazard functions from time series data, we begin by analyzing the aggregate implications of the quadratic adjustment model. We create a time series by aggregating across the plants in our simulated panel data set. Following CE and CEH, we investigate aggregate implications by looking at the relationship between aggregate employment changes and the cross-sectional distribution of the employment gap.

Table 1 presents estimates of (7) for three specifications of an aggregate hazard function: constant, piecewise linear and quadratic. More precisely, we specify

$$\Lambda(z) = \begin{cases} \lambda_0 + \lambda_1^- z + \lambda_2 z^2 & \text{for } z < 0 \\ \lambda_0 + \lambda_1^+ z + \lambda_2 z^2 & \text{for } z > 0 \end{cases} \quad (14)$$

which nests the different specifications of the aggregate hazard function used in CE and CEH. As described in those papers, (14) is substituted into the aggregate growth equation (7) yielding the following equation in which aggregate employment growth depends on the parameters of (14) and the moments of the cross-sectional distribution of the gaps:

$$\Delta E_t = const + \lambda_0 m_{1,t} - \lambda_1^- F_t(0) m_{2-,t} + \lambda_1^+ (1 - F_t(0)) m_{2+,t} + \lambda_2 m_{3,t} + \varepsilon_t \quad (15)$$

where  $m_{i,t}$  is the  $i^{th}$  uncentered moment of the cross-sectional distribution of the gap in period  $t$ , an index of  $+$  ( $-$ ) indicates that the moment only includes observations with a positive (negative) gap, and  $F_t(0)$  represents the fraction of plants with a negative gap at time  $t$ .

### 5.1 Frictionless Target

The results for the estimation of (15) with the frictionless target computed using the observed gap are reported in Table 1a. When the appropriate target is used, the estimated hazard is flat with an adjustment rate of 0.35. There is essentially no evidence of any economically significant nonlinearity. The  $R^2$  for this specification is virtually 1: the model with a constant hazard fits quite well.<sup>31</sup> So, even though our driving process is not a random walk, the flat hazard prediction of the quadratic adjustment cost model seems to work well **using the observed frictionless gap**.

---

<sup>31</sup>This high value of  $R^2$  partly reflects the limited nature of the model: there are no other factors of production with adjustment costs, no shocks to the adjustment costs directly, no measurement error, etc.

There are three deviations from this benchmark associated with three potential “errors” in measuring the gap. First, as in CE, the static target, which is easy to compute in our simulated environment, may be used instead of the frictionless target. Second, the estimation procedure used by CE relies on an artificial measure of the static target. The third is the CEH measure of the gap.

## 5.2 Static Target

Table 1a also shows the results obtained when (15) is estimated using the **observed static gap**. Using this measure, one would strongly **reject** the hypothesis that the hazard function is flat in favor of either the piecewise linear or the quadratic case. For example, in the quadratic specification, we find that  $\lambda_2$  is estimated at 0.69 with a standard error of 0.19. Further, the coefficients in the piecewise linear specification ( $\lambda_1^+ = 0.27, \lambda_1^- = 0.27$ ) are also statistically and economically significant.

The final two rows of the table present estimates of the quadratic specification given in (15), where  $\lambda_1^+ = \lambda_1^-$  has been imposed. The mapping between these estimates,  $\{\lambda_1, \lambda_1, \lambda_2\}$ , and those reported in CE,  $\{\tilde{\lambda}_0, \tilde{\lambda}_2\}$ , is given by  $\lambda_0 = \tilde{\lambda}_0 + \tilde{\lambda}_2 z_0^2$ ,  $\lambda_1 = 2z_0 \tilde{\lambda}_2$  and  $\lambda_2 = \tilde{\lambda}_2$ .

In this case, the estimate of  $\lambda_2$  equals 0.69 and is significantly different from zero. In fact, this estimate of the nonlinearity in the hazard is not far from the estimate of  $\tilde{\lambda}_2$ , 0.53, reported by CE. The constant terms ( $\lambda_0$ ) in the hazard functions are somewhat close as well: the CE specification yields a constant term of 0.38 while we report a constant of 0.19. However, CE find  $z_0$  equal to  $-0.82$  while our estimate of  $z_0$  is approximately 0.

The difference in results between using the frictionless and static targets to determine the employment gap can be viewed as the introduction of measurement error into the regression. If the static target is equal to the frictionless target, we should not see any change in results. Figure 1, however, illustrates the difference between the two targets. Switching to the static target is likely to lead to a bias in the estimate, as there is not a constant difference between these targets.

Using the hazard given in (14) with the restrictions accompanying the CE quadratic hazard, the aggregate employment growth equation, (15), becomes:

$$\Delta E_t = \lambda_0 m_{1,t}^s + \lambda_1 m_{2,t}^s + \lambda_2 m_{3,t}^s + \overbrace{\varepsilon_t + \lambda_0 (m_{1,t}^f - m_{1,t}^s) + \lambda_1 (m_{2,t}^f - m_{2,t}^s) + \lambda_2 (m_{3,t}^f - m_{3,t}^s)}^{\text{error term}}. \quad (16)$$

The error term contains three mismeasurement terms in addition to  $\varepsilon_t$ . If any of these measurement errors is correlated with the moments of the static employment gap, then a

bias in the estimates will be present.

To study this bias, we regress the measurement error in the first uncentered moment,  $m_{1,t}^f - m_{1,t}^s$ , on the three moments of the static gap  $\{m_{1,t}^s, m_{2,t}^s, m_{3,t}^s\}$  using simulated data.<sup>32</sup> We estimate  $\{-0.45, 0.03, 1.96\}$  as the coefficients on the three moments with standard errors of  $\{0.02, 0.07, 0.54\}$ . These results indicate that the error in (15) is related to the static gap in a nonlinear way, thus leading to the nonlinear estimates of the adjustment function.<sup>33</sup>

The intuition draws upon Figure 1. In periods of very low (high) aggregate shocks, the average static gap is significantly below (above) the average frictionless gap and, similarly, the non-centered third moment,  $m_{3,t}$ , of the static gap is significantly below (above) the frictionless counterpart. Therefore, the measurement error in the first uncentered moment is correlated with the first and third uncentered moments of the static gap distribution.

### 5.3 CE Measure of the Static gap

The previous results assume the static target is observed. CE do not observe this and must instead infer the growth of the employment target using (9). To explore the CE methodology further, we used their procedure to estimate (11) from a simulated data set created by our parameterized model with quadratic adjustment costs.

The results of this experiment are summarized in Table 2. In this table, the parameters are those characterizing the aggregate hazard in (11) as well as  $\sigma_I$ , the parameter which controls the amount of plant-specific variability in the gap. Here we see that using the CE estimation procedure, the results from our model (the second row) are quite close to those reported by CE (the first row). In particular, the shape of the hazard is qualitatively the same. Moreover, the variability of the plant-specific shocks,  $\sigma_I$ , is higher in the simulated data than in the CE estimates from the BLS data. Thus we obtain both a nonlinearity in adjustment and a significant amount of variability in  $z$ .

### 5.4 CEH Measure of the Gap

Alternatively, the frictionless target could be inferred from variations in observed hours at the plant-level, as in CEH, opening the possibility of additional measurement error. The results for this case are in Table 1b. The different sections refer to alternative treatments of

---

<sup>32</sup>Thanks to Peter Klenow for discussions on this characterization of the effects of the measurement error.

<sup>33</sup>Regressions of the higher-order measurement error terms on the three moments of the static gap yield qualitatively similar results.



the data. “Full sample” means that we use the complete sample while “big change” refers to a sample constructed by including only observations in which the employment and hours changes exceed one standard deviation, as in the sample splits of CEH.

The flat hazard specification in the first row of Table 1b yields rather nonsensical results: the adjustment rate is in excess of 100%. The constant hazard hypothesis is rejected for the full sample but would not be rejected for the big change sample.<sup>34</sup> The source of the misspecification in this case is discussed in the next section.

## 5.5 Summary

Thus, from the aggregate estimation results, we find that the hazard function is essentially flat *iff* the gap is properly measured. When the static target is used to construct the gap, we reject the flat hazard model. Using either the CE or the CEH (full-sample) procedure for measuring the gap, one would reject the flat hazard specification and conclude that adjustment costs were not quadratic. Here we have seen that this conclusion is not valid: the measurement of the gap, not economic behavior, introduces nonlinearities.<sup>35</sup>

## 6 The Relationship between Hours and the Gap

A key element in the CE and CEH procedures is (3) and the calibration/estimation of  $\theta$ . This hypothesized relationship is the main link between the observable variable (hours) and the unobservable variable (the gap). Once this parameter is determined, CE calculate the aggregate targets from (9) and CEH use (4) to construct the plant-level gaps. The logic in both cases is to infer movements in the gap from variations in hours. Accordingly, the final step in our evaluation of the gap methodology is to explore (3), both the specification *per se* and the estimates of  $\theta$  using these two procedures.

For reference, Table 3 summarizes the estimates of  $\theta$  for a number of different specifications. The first two rows correspond to the value of  $\theta$  estimated from (3) using the actual gap that we construct in our simulated environment. Of these rows, the first measure uses the frictionless target to create the gap, and the second measure uses the static target. The other rows use the CEH approach to estimate  $\theta$ . Note that the CEH results do not depend on the definition of the target since it is not observable.

---

<sup>34</sup>From Cooper and Willis [2002a], this is not a robust finding: there are also parameterizations with significant nonlinearities in the big change sample.

<sup>35</sup>An earlier version of this paper included evidence on the plant-level hazard functions. We found the presence of nonlinearities when the gap was not measured properly there as well.

## 6.1 Specification

To evaluate the CE observed gap approach, consider the top part of Table 2. The estimate of  $\theta$  obtained using the frictionless and static gap measures differ. In fact, using the static target, as in CE, produces an estimate of  $\theta$  that is exactly equal to the one obtained analytically from the firm's optimal choice of hours worked per employee.<sup>36</sup>

Yet, from Table 1a, the gap measure produced by using this estimate of  $\theta$  **does not** correspond with the relevant measure for a dynamically optimizing firm, the frictionless gap. The difference is due to the dependence of the frictionless hours target on the productivity shock. As shown in Figure 1, the static hours target is independent of the shock.

This distinction between the two hours targets has important implications for the measurement of the gaps. From a log-linearization of the first-order conditions from (12), the relationship between the frictionless employment gap and hours deviations can be written as

$$z_{i,t} = \theta \left( \tilde{h}_{i,t} - h^*(A_{i,t}) \right). \quad (17)$$

Using the correct target for hours and the frictionless employment gap, we do obtain the analytically calculated value of  $\theta$  when we estimate (17).

The problem for the CE methodology is that there are two unobservables in (17). The hours target cannot be approximated by a constant mean, as is assumed in the construction of (3). Even if an estimate for  $\theta$  is available, the employment gap cannot be accurately constructed without observing the hours target. The errors caused by having the correct  $\theta$  and using the mean level of hours to approximate the hours target are illustrated precisely by the observed static target results above for the aggregate hazards.

## 6.2 Omitted Variable Bias

An important problem in the estimation of  $\theta$  when the gap is not observed is omitted variable bias. To understand this point, recall the regression equation used by CEH, (6):

$$\Delta \tilde{e}_{i,t} = \delta - \theta \Delta \tilde{h}_{i,t} + \varepsilon_{i,t}.$$

From Table 2, the sign of the estimated value of  $\theta$  from (6) is opposite that obtained when the observed gap is used in the regression, as in (3). Since the CEH methodology relies on  $\theta$  to construct a measure of the gap, this difference is important to understand.

The error term in (6) contains the change in the employment target level. If changes in hours are uncorrelated with changes in employment targets, the sign on  $\theta$  will be determined

---

<sup>36</sup>See the appendix in Cooper and Willis [2002a] for the analytic derivation of  $\theta$ .

by the unconditional correlation between changes in hours and changes in employment. In the simulated data from our model, this latter correlation is 0.65. The driving force behind this positive correlation is the partial adjustment to changes in employment targets. When plants experience productivity shocks, they respond to changes in employment targets by changing both hours and employment in the same direction. This positive correlation between hours and employment implies the negative sign on  $\theta$  in (6), as reported in Table 2.

But there is no rationale for the assumption that changes in hours are uncorrelated with changes in employment targets. Because CEH acknowledge that hours and employment target changes could be correlated, they use only observations in which there are large changes in both hours and employment to estimate  $\theta$ . They argue that in these periods, the changes in employment targets are swamped by the effects of large changes in hours and employment. But in a model of convex adjustment costs, the only periods in which there are large changes in hours and employment are periods in which there are large changes in employment target levels.

This is evident in the simulated data: the correlation between changes in hours and changes in employment target levels is 0.98 in the full sample and 0.996 in the CEH-criterion subsample. Therefore, the CEH methodology produces a biased estimate of  $\theta$ . To obtain an unbiased estimate of  $\theta$  in a model of quadratic costs of adjustment, it is essential to control for changes in employment target levels.

The implications of the sign reversal are displayed in Figure 2, which shows a sample of employment changes, deviations in hours, and various measures of the employment gap from a simulation of the model. The upper panel displays the two measures of the actual gap, and the lower panel displays two measures of the gap constructed from CEH estimates of  $\theta$ . The differences between the gap measures are readily apparent once the scales of the two panels are taken into account. The series for employment changes and hours deviation are identical in the two panels. In the upper panel, the gap measures have a higher degree of variability than employment changes, indicative of the expected plant behavior of partial adjustment when faced with convex costs of adjustment. In the lower panel, employment changes greatly exceed the CEH gap measures; hence the large parameter estimates in Table 1b. Since hours and employment are positively correlated, the negative sign on  $\theta$  causes the constructed employment gap to be a dampened version of the change in employment. The actual measures of the gap and the CEH gap measures are positively correlated (approximately 0.41 for the big change subsample), but the conclusions to be drawn from analysis of these series are very different.

## 7 Conclusions

The point of this paper is to assess the findings of CE and CEH that aggregate employment dynamics depend upon the cross-sectional distribution of employment gaps. We argue that due to measurement problems, a researcher might indeed find that the cross-sectional distribution matters for aggregate time series even if adjustment costs are quadratic. Thus, the conclusion of CE and CEH that nonlinear adjustment at the plant-level is present in aggregate time series is not based on convincing evidence. So, despite the overwhelming evidence that plant-level adjustment is nonlinear, the question of whether this matters for aggregate employment dynamics remains an open issue.

Can we do better? Within the gap methodology, it is apparent that the CEH methodology is inferior to that employed by CE.<sup>37</sup> However, even the CE approach falls short, due primarily to state-contingent differences between the frictionless and static employment targets. We have seen that adding a state-dependent hours target to the model yields the appropriate frictionless target, though implementing this procedure with actual data is less clear.

There are competing approaches to estimating a parameterized version of an adjustment cost function nesting both convex and nonconvex costs that do not rely on gap measures. Examples of this, which now exist in the literature on investment, durables and price-setting, involve using indirect inference techniques to match the moments produced by simulations of a structural model with those observed.<sup>38</sup> Clearly, labor is next.

---

<sup>37</sup>We understand that data limitations led CEH to their formulation.

<sup>38</sup>We have tried without success to use that approach to match the aggregate regression results reported in CEH. An alternative is to use VARs, following Sargent [1978], and also to structure the estimation around plant-level reduced-form regressions as in Cooper and Haltiwanger [2000]. This is in process.

## 8 References

- Bils, M., "The Cyclical Behavior of Marginal Cost and Price," *American Economic Review*, 77 (1987), 838–55.
- Caballero, R. and E. Engel, "Microeconomic Adjustment Hazards and Aggregate Dynamics," *Quarterly Journal of Economics*, 108 (1993), 359–83.
- Caballero, R., E. Engel and J. Haltiwanger, "Aggregate Employment Dynamics: Building From Microeconomic Evidence," *American Economic Review*, 87 (1997), 115–37.
- Cooper, R. and J. Haltiwanger, "On the Nature of Capital Adjustment Costs," NBER Working Paper # 7925, September 2000.
- Cooper, R. and J. Willis, "The Economics of Labor Adjustment: Mind the Gap," Federal Reserve Bank of Minneapolis, Staff Report 310, 2002a.
- Cooper, R. and J. Willis, "The Cost of Labor Adjustment: Inferences from the Gap," Federal Reserve Bank of Kansas City, Research Working Paper 02-11, 2002b.
- Davis, S. and J. Haltiwanger, "Gross Job Creation, Gross Job Destruction and Employment Reallocation," *Quarterly Journal of Economics*, 107 (1992), 819–63.
- Davis, S., J. Haltiwanger and S. Schuh, *Job Creation and Destruction*, Cambridge Mass.: MIT Press, 1996.
- Hamermesh, D., "Labor Demand and the Structure of Adjustment Costs," *American Economic Review*, 79 (1989), 674–89.
- Hansen, G., "Technical Progress and Aggregate Fluctuations," *Journal of Economic Dynamics and Control*, 21 (1997), 1005–23.
- Nickell, S., "Fixed Costs, Employment and Labour Demand over the Cycle," *Economica*, 45 (1978), 329–45.
- Olley, S. and A. Pakes, "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64 (1996), 1263–97.
- Sargent, T., "Estimation of Dynamic Labor Demand Schedules under Rational Expectations," *Journal of Political Economy*, 86 (1978), 1009–44.
- Shapiro, M., "The Dynamic Demand for Capital and Labor," *Quarterly Journal of Economics*, 101 (1986), 513–42.
- Tauchen, G., "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20 (1986), 177–81.

Table 1a: Aggregate Implications,

Observed Gap	$\lambda_0$	$\lambda_1^+$	$\lambda_1^-$	$\lambda_2$	$R^2$
Frictionless	0.35				1.00
	(0.0001)				
	0.35	-0.009	0.010		1.00
	(0.0003)	(0.003)	(0.003)		
	0.35			0.003	1.00
	(0.0002)			(0.010)	
Static	0.22				0.90
	(0.002)				
	0.17	0.27	0.27		0.90
	(0.01)	(0.08)	(0.08)		
	0.19			0.69	0.90
	(0.0069)			(0.19)	
0.19	0.003	0.003	0.69	0.90	
(0.007)	(0.023)	(0.023)	(0.19)		

Notes: Results from estimation of (15). Standard errors in parentheses.

Table 1b: Aggregate Implications of CEH Procedure,

CEH Gap	$\lambda_0$	$\lambda_1^+$	$\lambda_1^-$	$\lambda_2$	$R^2$
Full sample	1.71				0.70
	(0.04)				
	1.19	26.79	25.95		0.70
	(0.17)	(9.06)	(9.03)		
	1.46			547.38	0.70
	(0.09)			(181.96)	
Big change	1.00				0.17
	(0.07)				
	0.40	31.46	42.19		0.17
	(0.48)	(30.84)	(30.14)		
	0.63			1226.55	0.17
	(0.26)			(816.93)	

Notes: Results from estimation of (15). Standard errors in parentheses.

Table 2: Estimate of Aggregate Hazard using CE Procedure

	$\tilde{\lambda}_0$	$\tilde{\lambda}_2$	$z_0$	$\sigma_I$	Stat
CE proc.: BLS data	0.019	0.53	-0.82	0.059	0.007
CE proc.: Simulated data	0.014	0.37	-0.83	0.090	0.0046

Notes: Results from estimation of (11) using simulated data.

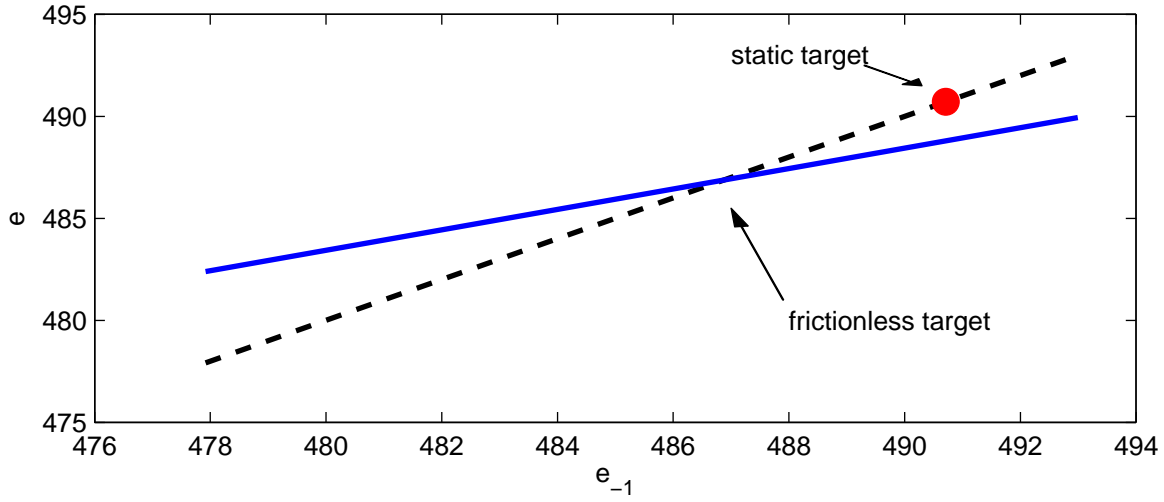
Table 3: Estimate of  $\theta$

	$\theta$	$R^2$
Observed Gap		
Frictionless target	2.35 (0.001)	0.83
Static target	4.72 (0.0001)	0.999
CEH Gap		
Full sample	-0.90 (0.001)	0.42
Big change	-1.26 (0.001)	0.86

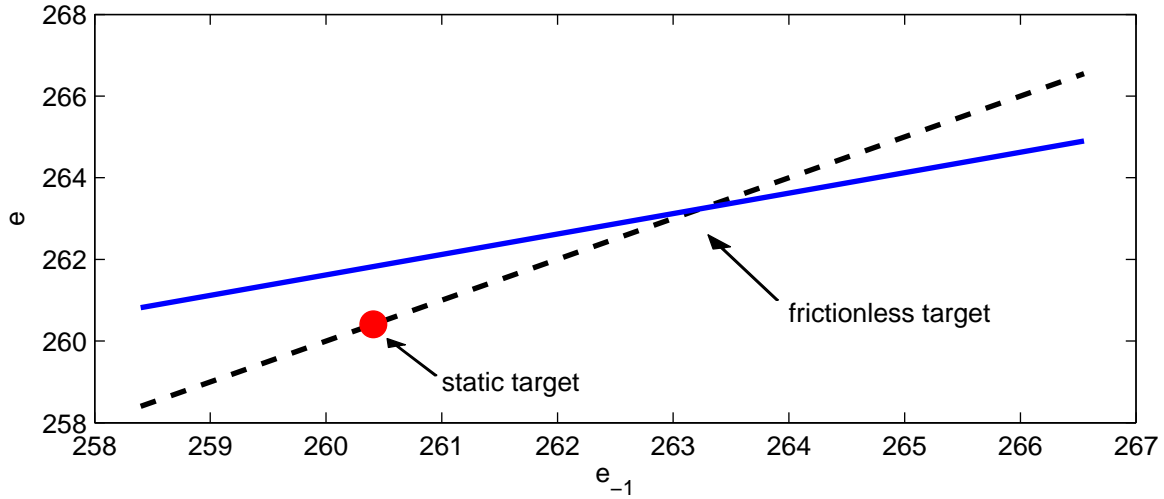
Notes: Results from estimation of (15). Standard errors in parentheses.

Figure 1: Employment and Hours Policy Functions

Employment policy function conditional on high productivity shock



Employment policy function conditional on low productivity shock



Hours worked corresponding to employment target levels

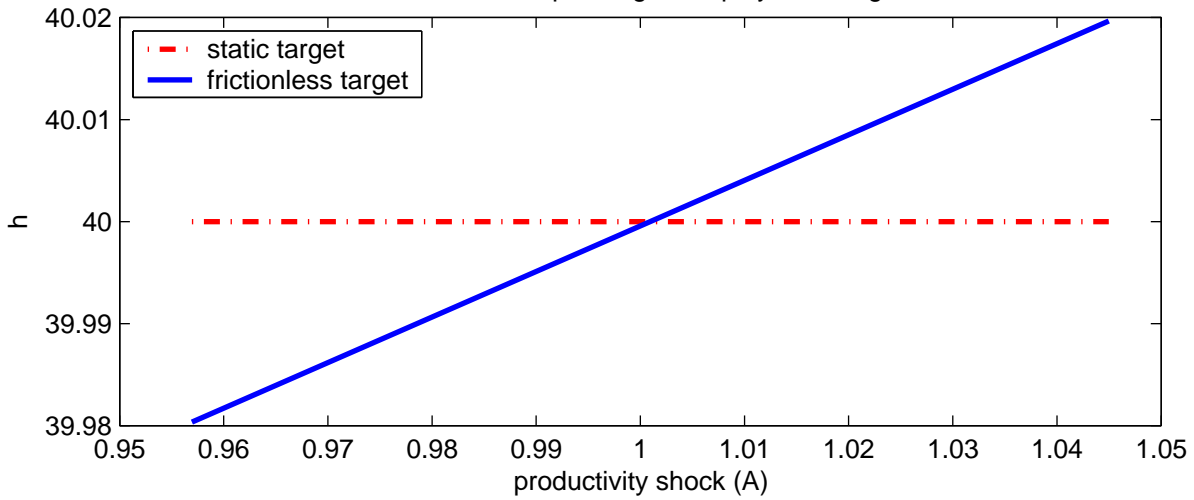
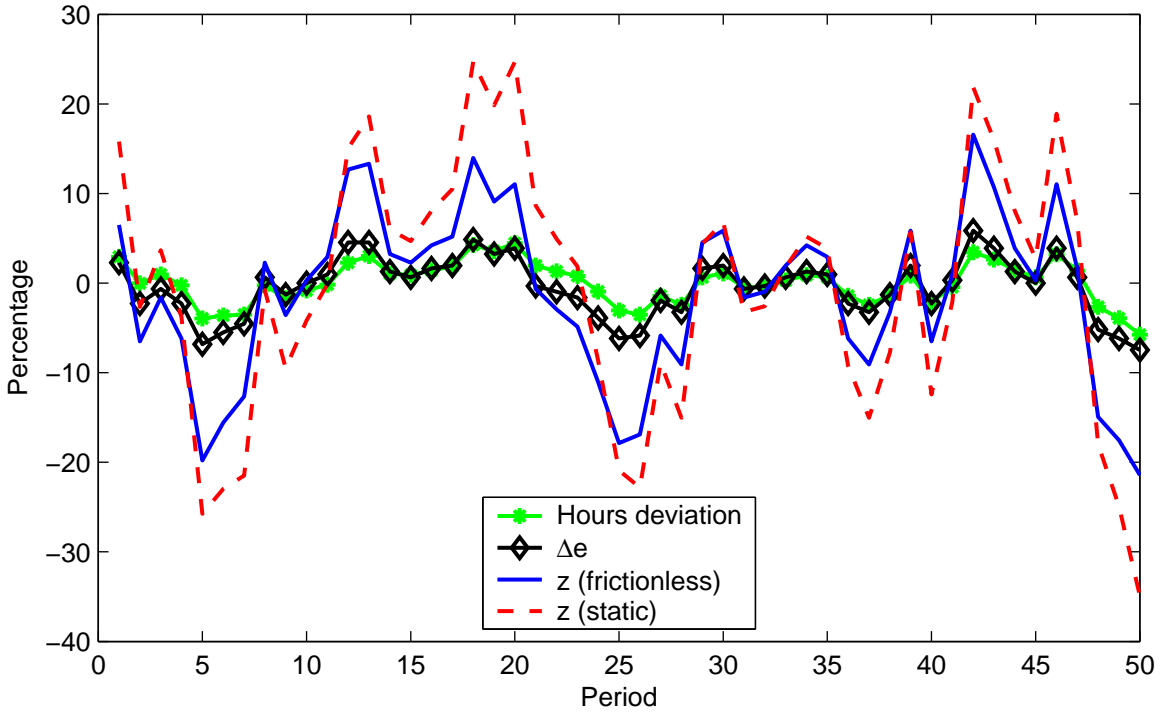




Figure 2: Simulation results

Simulated employment gaps: measured directly



Simulated employment gaps: measured using CEH methodology

