ISSN 1936-5330

THE FEDERAL RESERVE BANK *of* KANSAS CITY
ECONOMIC RESEARCH DEPARTMENT

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January 2009
RWP 09-02

RESEARCH WORKING PAPERS

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Abstract

This note continues the discussion of the results reported by Ricardo Caballero and Eduardo Engel (1993), hereafter CE, and Ricardo Caballero, Eduardo Engel, and John Haltiwanger (1997), hereafter CEH, by responding to the results reported in Christian Bayer (2008). Russell Cooper and Jonathan Willis (2004), hereafter CW, find that the aggregate nonlinearities reported in CE and CEH may be the consequence of mismeasurement of the employment gap rather than nonlinearities in plant-level adjustment. Bayer reassesses this finding in the context of the CE model in the case where static employment gaps are observed and concludes that the CW result is not robust to alternative shock processes. We concur with Bayer's assessment that the nonlinearity finding is sensitive to the aggregate profitability shock process. We argue, however, that Bayer's finding does not imply that the mismeasurement problem goes away. Instead, the nonlinearity created by mismeasurement is directly related to the level of the aggregate shock. Once the empirical specification properly incorporates the aggregate shock, the nonlinearity test is robust to alternative shock processes and confirms the results in CW. More importantly, we demonstrate that the CW findings are robust to alternative shock proceses for the natural case of unobserved gaps as examined by CE and CEH.

JEL Classification: E24, J23, J64

Keywords: Aggregate Employment, Employment, Adjustment Costs

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I. Introduction

This note continues the discussion of the results reported by Ricardo Caballero and Eduardo Engel (1993), hereafter CE, and Ricardo Caballero, Eduardo Engel, and John Haltiwanger (1997), hereafter CEH, by responding to the results reported in Christian Bayer (2008). Russell Cooper and Jonathan Willis (2004), hereafter CW, find that the aggregate nonlinearities reported in CE may be the consequence of mismeasurement of the employment gap rather than nonlinearities in plant-level adjustment. Bayer reassesses this finding in the context of the CE model in the case where static employment gaps are observed and concludes that the CW result is not robust to alternative shock processes.

We concur with Bayer's assessment that the nonlinearity finding is sensitive to the aggregate profitability shock process. We argue, however, that Bayer's finding does not imply that the mismeasurement problem goes away. Instead, the nonlinearity created by mismeasurement is directly related to the level of the aggregate shock. Once the empirical specification properly incorporates the aggregate shock, the nonlinearity test is robust to alternative shock processes and confirms the results in CW. More importantly, we demonstrate that the CW findings are robust for the natural case of unobserved gaps as examined by CE and CEH. In the case of CE, we demonstrate that estimation of the quadratic hazard function through the CE procedure, which does not assume the cross-sectional distribution of gaps is observed, is robust to the variations in the stochastic process for the aggregate shocks. In the case of CEH, we demonstrate that finding of omitted variable bias in the CEH procedure in the presence of quadratic adjustment cost shocks is also robust to alternative shock processes.

This response consists of four components. In Section II, we present findings for the case of observable static employment gaps under alternative specifications of the aggregate shock processes. In Section III, we present additional evidence to better understand nonlinearities of the baseline quadratic labor adjustment cost model. In Sections IV and V, we present results using the estimation procedures of CE and CEH, respectively.

II. Robustness to Shock Process

Following the description in CW, there are three main components of the CE model. The first is the specification of the gap, z, between desired (log) employment, e^*, and actual (log) employment, e, for establishment i in period t,

$$z_{i,t} = e^*_{i,t} - e_{i,t-1} \tag{1}$$

where z represents the gap between plant employment at the beginning of period t and the level of employment it would choose if it could "costlessly" alter employment after observing shocks in period t.

The second equation is the specification of the adjustment rate or hazard function for establishments as a function of the employment gap,

$$\Lambda(z) = \lambda_0 + \lambda_2 z^2. \tag{2}$$

This specification nests a constant and a quadratic specification for $\Lambda(\cdot)$.

Given the definition of the employment gap and response function for establishments, the aggregate employment growth rate, ΔE, is calculated by integrating over all establishments,

$$\Delta E_t = \int_{-\infty}^{\infty} z\Lambda(z) f_t(z) dz, \tag{3}$$

where $f_t(z)$ is the period t probability density function of employment gaps across plants.

Assuming employment gaps are observed, the parameters of the hazard function can be estimated based on the relationship between aggregate employment growth and the cross-sectional distribution of the employment gap. By substituting equation (2) into equation

(3), we get the following estimation equation,

$$\Delta E_t = \lambda_0 m_{1,t} + \lambda_2 m_{3,t} + \varepsilon_t, \tag{4}$$

where $m_{j,t}$ is the jth uncentered moment of the cross-sectional distribution of the gap in period t.

To evaluate the procedures and inferences from the estimation of (4), CW study a data set created by the dynamic optimization problem of a plant facing quadratic adjustment costs.[1] The profitability at the plant-level is driven by aggregate and plant-specific shocks. Both of these processes are stationary.

The procedure for the estimation of the parameters of $\Lambda(\cdot)$ depends on what is observed to the researcher. Table 1 of CW presented two cases: (i) when z is observed and (ii) when z is constructed based upon a static target.[2] The points raised in Bayer's critique focus on the second case. We respond to those in this section.

The goal of this exercise in CW was to evaluate the use of the static target and thus the static gap in the CE procedure. As shown in Figure 1 of CW, the frictionless and static targets do not generally coincide. If (4) is estimated using the frictionless target, CW report that λ_2 is not significantly different from zero: with a quadratic hazard, the adjustment rate is constant.[3]

CE, however, rest their findings on the static rather than the frictionless target. Thus one goal of CW was to characterize the effects on the estimates of λ_2 from using the **static** rather than the frictionless gap measure.

[1] The underlying economic environment is explained in detail in CW.

[2] As the name suggests, the static target is constructed by assuming the optimizing plant never faces adjustment costs.

[3] This flat hazard is a theoretical result when shocks follow a unit root process. It holds quantitatively in our exercise with stationary shocks.

Table 1 below shows results for six different specifications of **aggregate** shocks assuming the cross-sectional distribution of gaps is observed and the gap is constructed from the static target. The issue brought up by Bayer pertains specifically to the aggregate shocks: variations in the distribution of the idiosyncratic shocks do not produce significant variations in the results reported in Table 1. The alternative shock specifications in Table 1 represent different approaches at specifying a continuous log-normal AR(1) aggregate process in a discrete state space with an autocorrelation of 0.72 and standard deviation of 0.028.

The first row of Table 1 provides the estimates based on the shock process used in CW. The state space for these shocks was created by equally spacing points within (+/-) two standard deviations of the mean. The results represent the average estimates across 1000 simulated panels, where each panel consists of 1000 plants and 1000 periods. The standard deviation of the parameter estimates across the 1000 panels is shown in parentheses. The second row shows that the results are robust to the use of a finer state space of 51 points equally spaced within (+/-) two standard deviations of the mean.

The results in the third row were constructed with 11 equally spaced aggregate shocks within (+/-) three standard deviations of the mean. As suggested in Bayer's critique, the nonlinearity found in the first specification appears to have vanished. The fourth row delivers a similar result when the fineness of the grid is increased to 51 points.

This result confirms earlier statements that these estimates are particularly sensitive to the specification of shocks.[4] The note by Bayer helps to make clear the statistical nature of this sensitivity.

First, it is important to emphasize that the results are robust to variations in the stochastic process for the idiosyncratic shocks, but do depend on the process for the aggregate shocks. So the variations in the shock process reported in this table pertain to aggregate not idiosyncratic shocks. Second, another interpretation of the differences in results is the

[4] This point is consistent with results reported in an earlier working paper, cited in footnote #26 of CW.

Table 1. Hazard Estimates using Observed Static Target

Static Gap	λ_0	λ_2	R^2
CW (2 std, 11 points)	0.204	0.413	0.898
	(0.008)	(0.198)	
CW (2 std, 51 points)	0.204	0.439	0.898
	(0.008)	(0.203)	
Bayer (3 std, 11 points)	0.223	0.009	0.899
	(0.007)	(0.153)	
Bayer (3 std, 51 points)	0.223	0.014	0.899
	(0.007)	(0.162)	
Importance Sampling (11 points)	0.194	0.622	0.897
	(0.008)	(0.213)	
Importance Sampling (51 points)	0.216	0.145	0.899
	(0.007)	(0.175)	

Notes: This table displays coefficients from a regression of aggregate employment growth on the first and third uncentered moments of the static employment gap distribution. Each row differs in the underlying representation of an AR(1) aggregate shock process with autocorrelation of 0.72 and standard deviation of 0.028. The results represent averages across estimates from 1000 simulated datasets, where each dataset consists of 1000 establishments and 1000 periods. The standard deviation of estimates across the datasets are presented in parentheses.

sensitivity of the estimates to the approximation of the process (rather than the process itself). From this perspective, the differences in findings come from a choice in how best to approximate a stochastic process.

To study this further, the last two rows of the table use a technique called importance sampling.[5] Instead of spacing points equally apart in the state space, importance sampling puts the points so that each partition has equal probability. In row 5 of Table 1, λ_2 is significant for this alternative representation with 11 points. As more points are added using importance sampling, not only does the fineness of the grid increase, but the coverage area of the state space also increases by extending the endpoints. With 51 points in the state space using the importance sampling representation, the results in row 6 show that λ_2 remains positive, but is no longer significant. The coverage area of the state space for this specification is (+/-) 2.3 standard deviations.

[5]See Jérôme Adda and Russell Cooper (2003) for a discussion of this technique and cites to related work.

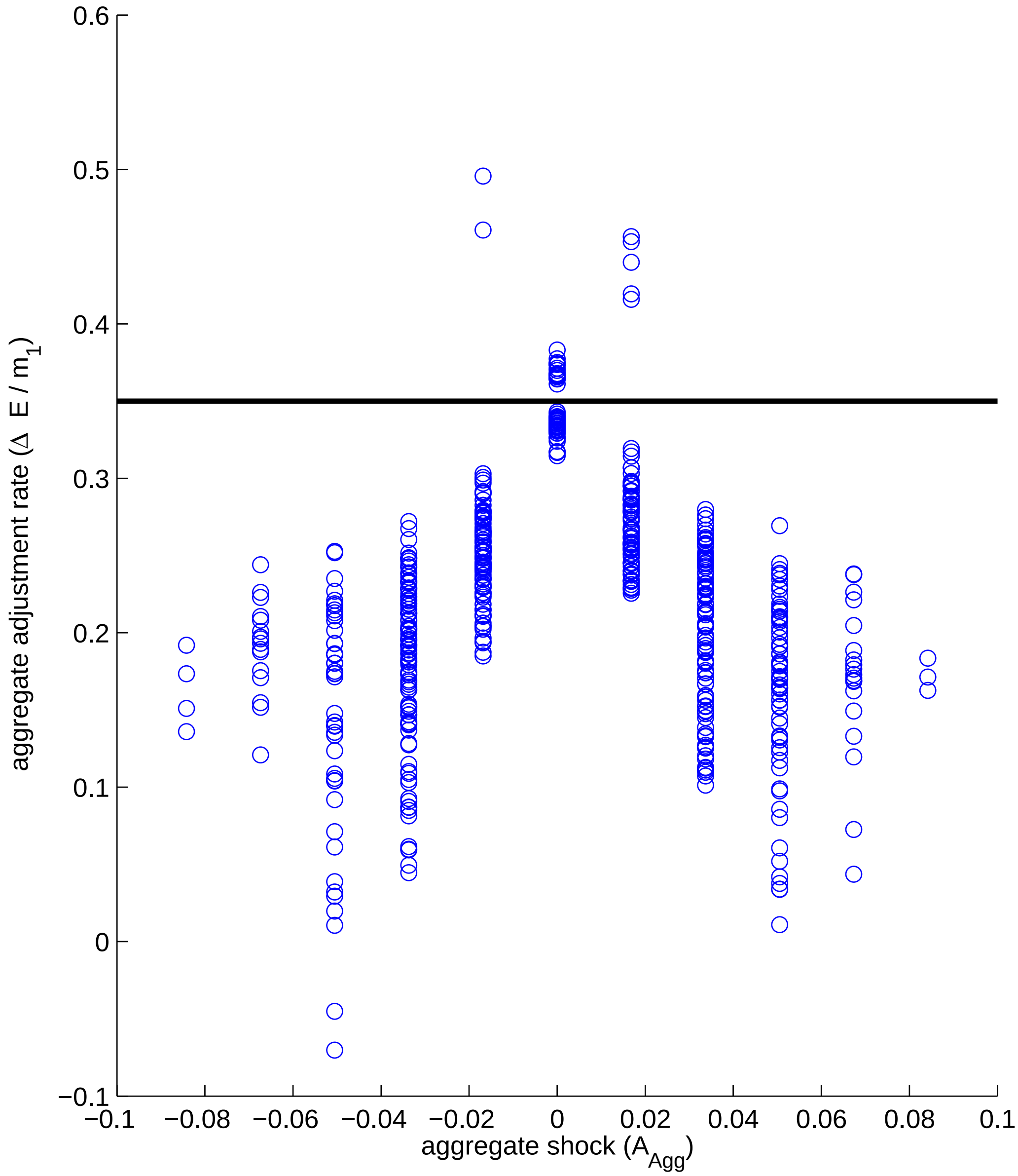
III. Where Did the Non-linearity Go?

The results summarized in Table 1 show the sensitivity of the estimate of λ_2 to the aggregate shock process. Given the importance of large movements in the aggregate economy for the CE results, it is surprising that the nonlinearity seems to disappear in the presence of extreme aggregate shocks.

To understand this further, we study a simulation in which the aggregate shock takes values in a space within 3 standard deviations of the mean, as in the third row of Table 1. In that case the non-linearity in (4) seemed to disappear. However, Figure 1 reveals an important relationship between the adjustment response of employment, measured as the aggregate change in employment divided by the mean of the cross-sectional gap distribution, and the level of the aggregate shock. As a benchmark, the horizontal line illustrates the adjustment response for this model when the gap is measured using the frictionless target, $\lambda_0^f = 0.35$. When the static target is used to construct the gap, we see that the average adjustment rate, displayed as a scatterplot, is much lower for extreme values of the aggregate shock. The intuition for this relationship is based on the mismeasurement of the static gap. In response to large (absolute) aggregate shocks, firms facing quadratic adjustment costs and stationary shock processes make smaller proportional employment adjustments, when measured using a static target, because the dynamic marginal benefit of employment adjustment is smaller when the shock is mean reverting than when it is viewed as permanent, as in a static solution.

Figure 1 also reveals a second relationship related to the observed aggregate adjustment rate for a given level of the aggregate shock. For each of the 11 possible realizations of the aggregate shock, there is a dispersion of aggregate adjustment rates. The implication is that the aggregate employment growth rate is not perfectly explained by the mean of the employment gap distribution, conditional on the level of the aggregate shock. Consequently, higher order moments of the gap distribution may be informative on the degree of mismeasurement

Figure 1. Nonlinear Response of Employment Adjustment to Aggregate Shock



Notes: This figure displays the relationship between aggregate employment adjustment rates measured using the static gap and the aggregate shock. The relationship is illustrated using a scatterplot of the aggregate observations from a simulation of the CW model where the aggregate shock is represented by an 11-point grid equi-spaced between (+/-) 3 standard deviations of the mean. For illustration purposes, observations with a mean value of the gap distribution close to zero were omitted. The solid line represents the adjustment rate measured using the frictionless employment gap.

for the aggregate adjustment rate.

We can quantify the relationship observed in Figure 1 by augmenting equation (4) to control for the level of the aggregate shock. To capture the decreasing average adjustment with respect to the absolute level of the shock, we add an interactive term of the mean level of the gap, m_1, and the absolute log-level of the aggregate shock, A_{Agg} and estimate

$$\Delta E_t = \quad \lambda_0 m_{1,t} \quad + \lambda_2 m_{3,t} + \gamma m_{1,t} \left| A_{Agg,t} \right| + \tilde{\varepsilon}_t.^6 \tag{5}$$

The results of the estimation for different specifications of the shocks processes are shown in Table 2.

There are a couple of important differences between these results and those reported in Table 1. First, the estimate of λ_2 is both large and significant for all shock specifications. Second, the interaction between the mean level of the gap and the aggregate shock is significant: the estimate of γ is negative, corresponding to the pattern in Figure 1. Third, the R^2 for these regressions is higher than the corresponding regressions in Table 1 that omit the interactive term.

Based on this new evidence, the inferences about the presence of non-linearities drawn from Table 1 are not quite right. It appears that adding more dispersion to the aggregate shocks did not really eliminate the non-linearity. Instead, it was hidden by the choice of a particular hazard specification and revealed through (5).

Table 2. Exploring Nonlinearities in Hazard Estimates using Observed Static Target

Static Gap	λ_0	λ_2	γ	R^2
CW (2 std, 11 points)	0.222	4.297	-4.262	0.953
	(0.008)	(0.386)	(0.143)	
CW (2 std, 51 points)	0.221	4.427	-4.348	0.953
	(0.008)	(0.389)	(0.146)	
Bayer (3 std, 11 points)	0.225	3.512	-3.575	0.949
	(0.009)	(0.393)	(0.176)	
Bayer (3 std, 51 points)	0.223	3.692	-3.696	0.948
	(0.009)	(0.400)	(0.183)	
Importance Sampling (11 points)	0.217	4.940	-4.740	0.953
	(0.008)	(0.420)	(0.153)	
Importance Sampling (51 points)	0.232	4.031	-4.002	0.950
	(0.008)	(0.400)	(0.163)	

Notes: This table displays coefficients from a regression of aggregate employment growth on the first and third uncentered moments of the static employment gap distribution and an interaction of the first moment with the absolute value of the aggregate shock. Each row differs in the underlying representation of an AR(1) aggregate shock process with autocorrelation of 0.72 and standard deviation of 0.028. The results represent averages across estimates from 1000 simulated datasets, where each dataset consists of 1000 establishments and 1000 periods. The standard deviation of estimates across the datasets are presented in parentheses.

IV. CE Procedure

The discussion in Section II suggests that estimates of the quadratic hazard are sensitive to the presence of large aggregate shocks. Yet, the contrary perspective is stressed in the results reported in CE. Figure IV in CE indicates that the explanatory power of the quadratic hazard occurs when there are "sharp contractions and brisk expansions". How then do we reconcile these findings?

We study this issue by providing estimates of the model based upon the CE procedure. This procedure is the one that CE used to estimate a hazard function from manufacturing data from the Bureau of Labor Statistics (BLS). In contrast to the results summarized in Table 1, which use an observed static target and the actual cross-sectional distribution of the gaps, in actual data neither the targets nor the cross-sectional difference in the gaps are observable.

To confront actual data, CE created an estimation procedure from observations of aggregated employment growth and hours. As discussed in Section I.C of CW, CE work with three equations. The first represents aggregate employment growth:

$$\Delta E_{t+1} = \int_{-\infty}^{\infty} (\Delta E^*_{t+1} - z)\Lambda(z - \Delta E^*_{t+1}) f_t(z) dz. \tag{6}$$

This expression requires two inputs to characterize employment adjustment as a function of the plant-level employment gap: the growth rate of the aggregate employment target, ΔE^*_{t+1}, and a hazard function, $\Lambda(\cdot)$. The target employment growth is constructed as a function of aggregate employment growth and aggregate hours growth using an assumed value of θ

$$\Delta E^*_t = \Delta E_t + \theta \Delta H_t. \tag{7}$$

Growth in the aggregate employment target is used in (6) to generate a predicted level of employment growth, given $f_t(z)$. The cross-sectional distribution, which is not observed, is created recursively from an initial cross-sectional distribution after applying the hazard and allowing for additional idiosyncratic shocks. For this exercise, as in CE, the hazard function is

$$\Lambda(z) = \lambda_0 + \lambda_2 (z - z_0)^2, \tag{8}$$

where z_0 is another parameter to be estimated.

In estimations using simulated data from the CW model, it is important to note that z_0 is not well identified. In (8), z_0 represents the level of the gap corresponding to the minimum of the hazard function. From the estimation using BLS data, the estimated value of $z_0 = -0.82$. For estimations using simulated data, a value for the z_0 parameter at -0.82 has been imposed because there appear to be multiple local minima when this parameter is estimated.[7] Often

[7] In CW we used a local minimizer which found the reported estimates in the neighborhood of the CE estimates.

Table 3. Hazard Estimates Using CE Procedure

	z_0	λ_0	λ_2	σ_I	SSR
BLS data	-0.82	0.019	0.53	0.059	0.007
CW (2 std, 11 points)	-0.82	0.012	0.477	0.069	0.012
		(0.002)	(0.016)	(0.006)	
CW (2 std, 51 points)	-0.82	0.018	0.471	0.066	0.012
		(0.003)	(0.016)	(0.006)	
Bayer (3 std, 11 points)	-0.82	0.012	0.481	0.072	0.015
		(0.002)	(0.016)	(0.006)	
Bayer (3 std, 51 points)	-0.82	0.012	0.483	0.072	0.014
		(0.002)	(0.016)	(0.006)	
Importance Sampling (11 points)	-0.82	0.013	0.468	0.066	0.010
		(0.002)	(0.015)	(0.006)	
Importance Sampling (51 points)	-0.82	0.012	0.480	0.070	0.013
		(0.002)	(0.016)	(0.006)	

Notes: This table displays estimates of the hazard function and volatility of the idiosyncratic shock for the model specified by CE. The final column displays the sum of squared residuals (SSR) from the CE estimation procedure. The first row represents estimates produced by CE using BLS data. The other rows represent estimates based on simulated data, where each row differs in the underlying representation of an AR(1) aggregate shock process with autocorrelation of 0.72 and standard deviation of 0.028. The results using simulated data represent averages across estimates from 1000 simulated datasets, where each dataset consists of 1000 establishments and 1000 periods. The standard deviation of estimates across the datasets are presented in parentheses.

these other minima have negative values of λ_2 and extreme values for z_0. Further, these other minima have values for the mean and volatility of the cross-sectional gap distribution quite different from the value inferred from the BLS estimates. Finally, these minima have values of the objective function which are quite close, suggesting an identification problem.

Setting z_0 equal to -0.82 thus serves two purposes. First, it ensures that the mean and volatility of the cross-sectional gap distribution are close to that in the data. Second, it resolves the identification problem.[8] Appendix A provides local estimation results for z_0, finding very little change in estimates under alternative shock processes.

[8] Interestingly, the lack of asymmetry in our theoretical model would seem to imply that z_0 should be equal to 0. Yet when we conducted estimations setting $z_0 = 0$, the resulting sum of squared residuals was much higher than when $z_0 = -0.82$. This outcome is also true for estimates based on BLS data. Russell Cooper and Jonathan Willis (2009) studies in more detail the inferences about adjustment costs from estimating (8).

Table 3 shows our results from this exercise. The first row of Table 3 displays estimates produced by CE using BLS data. Rows 2 and 3 report estimates based on simulated data from the CW model where the aggregate shock process contains equally spaced points within (+/-) two standard deviations of the mean. Using both BLS data and simulated data, the parameter estimates indicate a significant non-linearity in the hazard.

In light of the concerns with dispersion of aggregate shocks seen in Table 1, it is important to see the robustness of the results for alternative specifications of the aggregate shocks. The remaining rows represent the identical specifications of the aggregate shock process as in Table 1. As before, these results represent averages over estimates from 1000 simulations each containing 1000 establishments and 1000 periods.

The results here are striking: changes in the dispersion of aggregate shocks has no effects on the estimates of λ_2 from the CE procedure. Thus, in contrast to the results reported in Table 1, changes in the dispersion of aggregate shocks have no impact on the inference from the CE procedure.

V. CEH procedure

The estimation in CEH is based upon a second hypothesized relationship between a closely related measure of the employment gap, $z^1_{i,t}$, and plant-specific deviations in hours, h:

$$z^1_{i,t} = \theta(h_{i,t} - \bar{h}). \tag{9}$$

Here $z^1_{i,t}$ is the gap in period t **after** adjustments in the level of e have been made: $z^1_{i,t} = z_{i,t} - \Delta e_{i,t}$.

Intuitively, θ should be positive in a model with quadratic adjustment costs. As profitability rises, hours and the desired number of workers will both increase. The gap decreases as workers (e) are added and hours fall closer to the average level, $\bar{h}$. Thus, the supposed relationship between this measure of the gap and hours deviations seems reasonable, both in terms of the response of these variables to a shock and in terms of transition dynamics. Note, though, that the correlation between hours and workers is somewhat complicated: the productivity shock leads to positive initial comovement between e and h but, in the adjustment process, the comovement is negative.

The issue is estimating θ. Using (1) in (9) and taking differences yields:

$$\Delta e_{i,t} = -\theta \Delta h_{i,t} + \Delta e^{*}_{i,t} \tag{10}$$

Adding a constant (δ) and noting that $\Delta e^{*}_{i,t}$ is not observable, CEH estimate θ from:

$$\Delta e_{i,t} = \delta - \theta \Delta h_{i,t} + \varepsilon_{i,t}. \tag{11}$$

As CEH note, estimation of this equation may suffer from omitted variable bias since the error term (principally $\Delta e^{*}_{i,t}$) is likely to be correlated with changes in hours. That is, a positive shock to profitability may induce the plant to increase hours (at least in the short run) and will generally cause the desired level of employment to increase as well. CEH argue that this problem is (partially) remedied by looking at periods of large adjustment since then the changes in hours and employment will overwhelm the error. CW find that in the presence of quadratic adjustment costs, omitted variable bias leads to an estimate of θ with the wrong sign even when only periods of large adjustment are used in the estimation.

Robustness results of the CW finding of omitted variable bias are presented in Table 4. Similar to the findings with respect to the CE estimation methodology, changes in the dispersion of aggregate shocks have no effects on the omitted variable bias present the estimates of θ using the CEH methodology.

Table 4. Examining robustness of estimates of θ

	Observed gap Static	CEH gap Full sample	CEH gap Big change
CW (2 std, 11 points)	4.72	-0.90	-1.26
	(0.000)	(0.004)	(0.014)
CW (2 std, 51 points)	4.72	-0.90	-1.26
	(0.000)	(0.004)	(0.012)
Bayer (3 std, 11 points)	4.72	-0.92	-1.29
	(0.000)	(0.005)	(0.012)
Bayer (3 std, 51 points)	4.72	-0.92	-1.33
	(0.000)	(0.004)	(0.012)
Importance Sampling (11 points)	4.72	-0.87	-1.21
	(0.000)	(0.004)	(0.011)
Importance Sampling (51 points)	4.72	-0.89	-1.26
	(0.000)	(0.004)	(0.012)

Notes: This table is a robustness check of Table 3 in CW (AER 2004). The results represent averages across estimates from 1000 simulated datasets, where each dataset consists of 1000 establishments and 1000 periods. The standard deviation of estimates across the datasets are presented in parentheses.

VI. Conclusion

Our response to Bayer consists of four points. First, the results reported in CW for the estimation of the static gap model are sensitive to the dispersion of aggregate shocks. Second, even when there is no apparent nonlinearity in a quadratic hazard specification, we find that the nonlinearity is present once we condition parameters of the quadratic hazard on realizations of the aggregate shocks. Third, in a more realistic exercise in which the researcher does not observe the gap nor the cross-sectional distribution, following the procedure of CE, we find evidence of nonlinearity in the estimated aggregate hazard. This is true even when the dispersion of shocks does not produce a nonlinearity using a simple quadratic hazard and the observed static gap. Fourth, we find that the CEH procedure continues to suffer from omitted variable bias under alternative specifications of the shock process.

Our assessment of the gap methodology remains as before. While some results appear to be very sensitive to the specification of the shock distribution, the methodologies used by

CE and CEH for the natural case of unobserved gaps continue to suffer from measurement error and omitted variable bias, respectively, in the presence of quadratic adjustment costs. The argument for using other approaches to study dynamic discrete choice models remains as inference from the gap approach is tenuous.

Appendix A

In the results in Section IV, we set $z_0 = -0.82$ because it is not well identified globally. Table 5 provides an examination of sensitivity of the local estimation properties of z_0 under alternative aggregate shock specifications. To get the estimates, we use Matlab's Neld-Mead simplex hillclimber (fminsearch). For a starting value, we use the parameter estimates from the previous exercise where z_0 was set at -0.82.[9] The findings are that there is very little change in the local estimates of z_0 across the different aggregate shock specifications.

Table 5. Examining the local estimation properties of z_0

	λ_0	λ_2	z_0	σ_I	SSR
BLS data	0.019	0.53	-0.82	0.059	0.007
CW (2 std, 11 points)	0.012	0.480	-0.817	0.069	0.012
	(0.002)	(0.019)	(0.010)	(0.006)	
CW (2 std, 51 points)	0.018	0.456	-0.834	0.067	0.011
	(0.003)	(0.028)	(0.018)	(0.006)	
Bayer (3 std, 11 points)	0.012	0.481	-0.820	0.073	0.015
	(0.002)	(0.018)	(0.010)	(0.006)	
Bayer (3 std, 51 points)	0.012	0.484	-0.819	0.072	0.014
	(0.002)	(0.018)	(0.010)	(0.007)	
Importance Sampling (11 points)	0.013	0.475	-0.813	0.066	0.010
	(0.002)	(0.018)	(0.010)	(0.006)	
Importance Sampling (51 points)	0.012	0.482	-0.818	0.071	0.013
	(0.002)	(0.017)	(0.009)	(0.006)	

Notes: This table displays estimates of the hazard function and volatility of the idiosyncratic shock for the model specified by CE. The final column displays the sum of squared residuals (SSR) from the CE estimation procedure. The first row represents estimates produced by CE using BLS data. The other rows represent estimates based on simulated data, where each row differs in the underlying representation of an AR(1) aggregate shock process with autocorrelation of 0.72 and standard deviation of 0.028. The results using simulated data represent averages across estimates from 1000 simulated datasets, where each dataset consists of 1000 establishments and 1000 periods. The standard deviation of estimates across the datasets are presented in parentheses.

[9] We also estimate using two additional starting values that are the original starting values changed by +/- 2%.

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